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THE INFLUENCE OF THE DEFLECTION OF THE VERTICAL ON GEODETIC SURVEYS IN BRAZIL

A influência do desvio da vertical nas medições geodésicas no Brasil

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Abstract:

The densification of geodetic surveys using classical positioning techniques such as total stations may be necessary due to the quality of Global Navigation Satellite System (GNSS) positioning in urban canyons. However, the correction of distances and angles due to the deflection of the vertical (DV) is usually neglected in commercial softwares and internal software of total stations. Given that context, this research seeks to estimate the influence of DV on the horizontal geodetic positioning with total station in the Brazilian territory. Secondarily, it seeks to demonstrate the practical application of DV in the densification of geodetic networks. It is important to note that land surveys in Brazil must be connected to a geodetic network; therefore, the neglect of DV may degrade the positional quality of geodetic surveys. Results obtained indicate differences in horizontal geodetic positions of up to 45 ppm. Considering the desired positional quality of the geodetic network, such values demonstrate the importance of a proper correction for the DV.

Keywords: geodetic networks; deflection of the vertical; distance measurement; error propagation; horizontal positioning.

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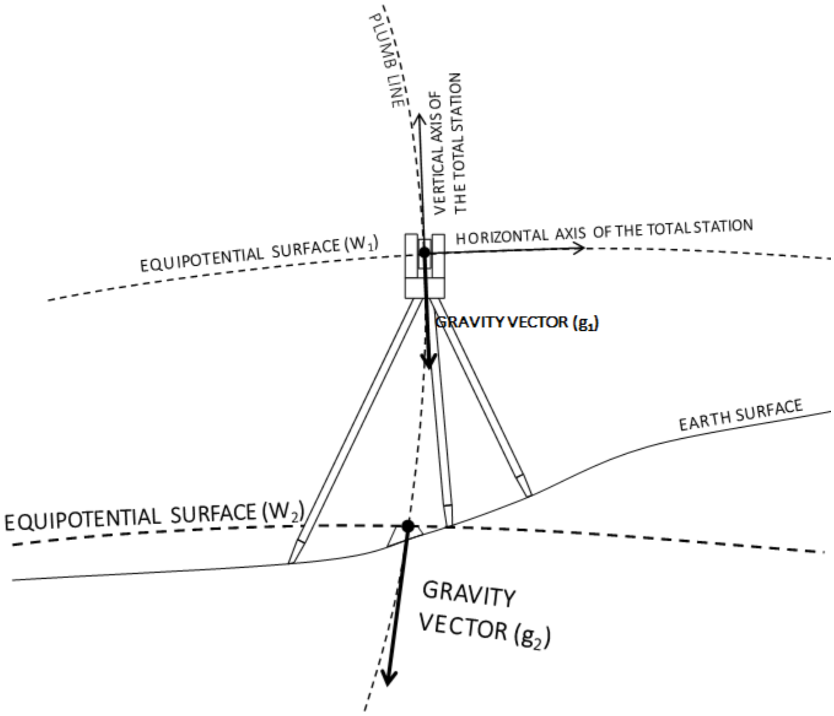
1. Introduction

The establishment of geodetic networks using GNSS (Global Navigation Satellite System) technology is widely used worldwide. However, there are restrictions due to signal blockage and multipath, for example, when used in urban environments (Pissardini *et al*. 2017), which may affect the final positioning result. The choice of the geodetic markers location takes into account the environments with full technical conditions for the GNSS positioning, this is, areas free from obstacles to signal tracking, in order to ensure homogeneity and positional quality. However, the positioning in urban environments requires the densification of geodetic network markers close to the survey locations, allowing economy, reduction in the propagation of errors, and optimizing field-operating procedures (Klein *et al*. 2017). There are cases where the densification of the geodetic network by positioning techniques with total station is necessary, due to the quality of the GNSS positioning in urban canyons (Deambrogio and Julien 2013; Klein *et al*. 2017; Osada *et al*. 2017). It is emphasized that even in rural environments there may be inadequate conditions for GNSS positioning (INCRA 2013).

During data acquisition with total station, a *sine qua non* condition for taking a measurement is that the vertical axis must have the direction of the gravity vector at the point of intersection of the vertical and horizontal axes of the total station; thus referring all subsequent measurements to this reference. Terrestrial geodetic observations are related to the local vertical, and thus delivers results orientated in local gravity vector orientation $\boldsymbol{g}$ (Seeber 2003 p. 21; Torge and Müller 2012 p. 203).

It is important to highlight that at each point on Earth it will pass an equipotential surface (Wi), which is a reflex of the effects of the gravity field due to the irregularities in the density of the materials that constitute the planet. At any point of this surface, the plumb line (or line of force) that passes through it will be perpendicular to the surface, and the derivative of the geopotential (W) in a given direction will be equal to the gravity component in this direction (Figure 1). In this way, the gravity vector in any given point is tangent to the plumb line at the point, thus, “direction of the gravity vector”, “vertical” and “direction of the plumb line” are synonyms (Hofmann-Wellenhof and Moritz 2005 p. 45).

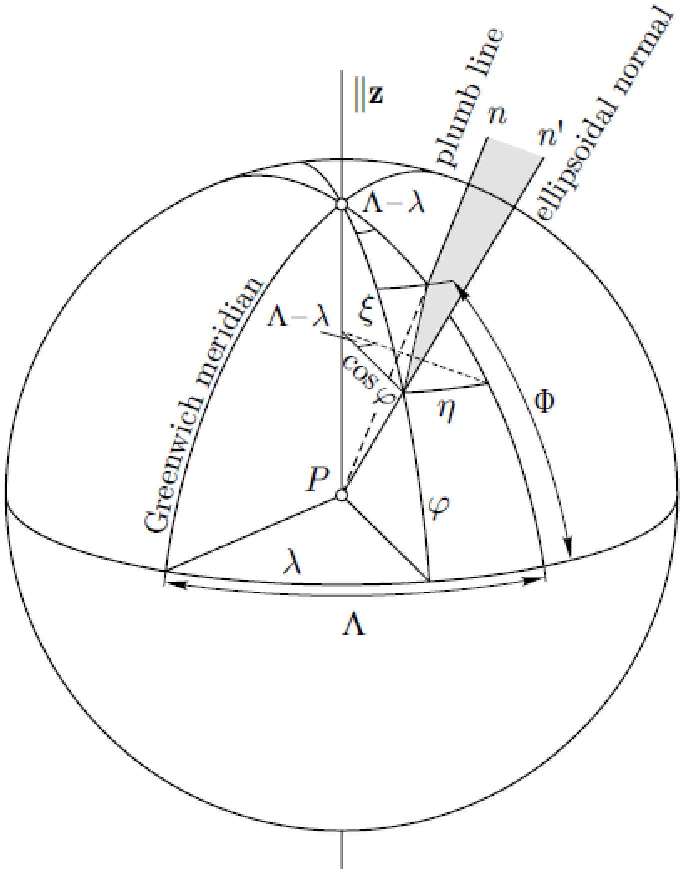


Source: the authors

Figure 1: Total station axes and gravity vector.

Seeber (2003 p. 10) and Monico (2008 chapter 3) showed that the GNSS positioning is associated with the ellipsoid as a reference surface, and, consequently, with the normal direction (ellipsoidal normal) of the station point. In this research, we will adopt the term normal to identify the ellipsoidal normal.

The rotation between the plumb line and the normal at the same point on the terrestrial surface is called Deflection of the Vertical, θ (Featherstone, 1999; Seeber, 2003, p. 26; Zanetti *et al*., 2008), spatially illustrated in Figure 2. The θ is represented by two components: prime vertical (η) in the west-east direction and meridian component (ξ) in the south-north direction.



Source: Hofmann-Wellenhof and Moritz, 2005, p. 92.

Figure 2: Deflection of the vertical and its components.

In Figure 2:

Λ, Φ: Astronomical longitude and latitude of the observation point, respectively

φ, λ: Geodetic (ellipsoidal) latitude and longitude of the observation point, respectively

P: observation point

η: prime vertical component at the observation point

ξ: meridian component at the observation point

Equations from 1 to 4 relate components of θ and position coordinates.

$$\xi = \Phi - \varphi \tag{1}$$

$$\eta = (\Lambda - \lambda) \cdot \cos \varphi \tag{2}$$

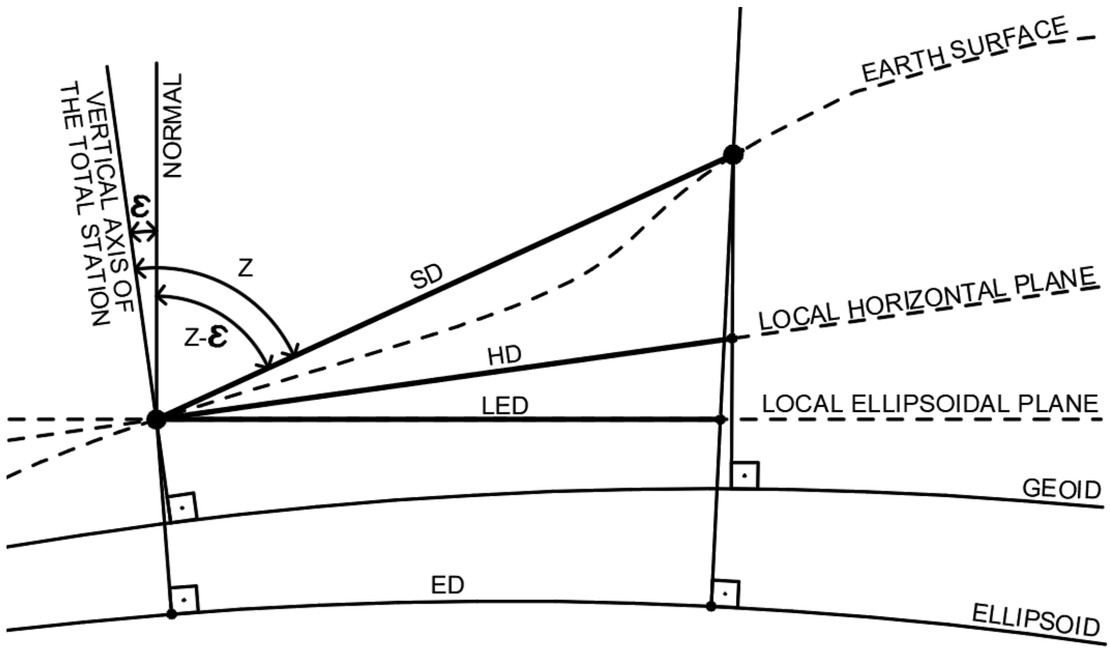
$$\theta = \sqrt{\xi^2 + \eta^2} \tag{3}$$

$$\varepsilon = \xi \cdot cos\ Az + \eta \cdot sin\ Az \tag{4}$$

Where ε is the deflection of the vertical component in a given azimuth, Az. Featherstone and Rüeger (2000) assert:

> "In terrestrial surveying, the deviation of the vertical has six primary uses:
> 1. transformation between astronomical coordinates and geodetic coordinates;
> 2. conversion between astronomic or gyro azimuths and geodetic azimuths;
> 3. reduction of measured horizontal directions (and angles) to the ellipsoid;
> 4. reduction of measured zenith angles to the ellipsoid;
> 5. reduction of slope electronic distance measurements (EDM) to the ellipsoid using zenith angles; and
> 6. determination of height differences from zenith angles and slope distances."

When integrating total station observations with satellite positioning, the measurements (horizontal angles, zenith angles and slope distances) need to consider the deflection of the vertical to rotate local horizontal plane to local ellipsoidal plane (Figure 3). Torge (2012 p.249) states that the relevance of ellipsoidal calculations has decreased, but it is also possible to reduce the slope distances to the ellipsoid. In this reduction it is essential to consider the deflection of the vertical. However, the deflection of the vertical is usually neglected in commercial softwares and internal software of total stations (Zanetti *et al.* 2008; Featherstone, 1999). Kao (1992) states that large deviations from the vertical, neglected in the reduction of observations, affect the positional quality of high precision networks.



Source: the authors.

Figure 3: rotation beetween local horizontal plane to local ellipsoid plane with deflection of the vertical.

Therefore, the transformation of observations between local systems to the global geocentric system without considering the rotation from deflection of the vertical, will result in a systematic error, and, consequently, will propagate in the geodetic positioning, generating errors of difficult identification and consistent adjustments (Ghilani 2010 p. 487 and 495).

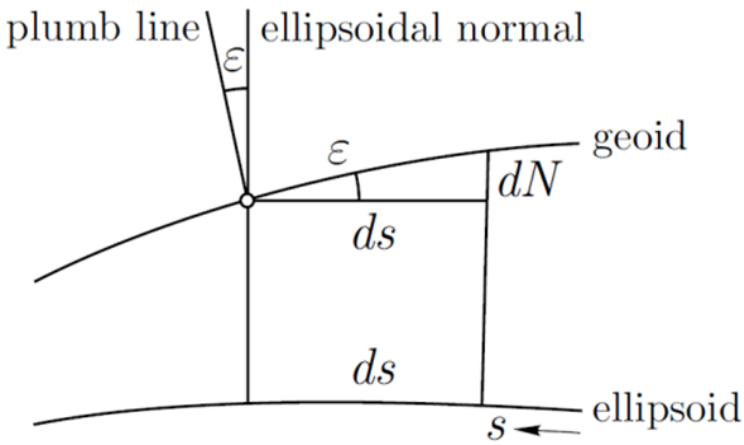
Within this context, in this research the influence of the deflection of the vertical in the horizontal geodetic positioning on Brazilian territory is analyzed in a pioneering way, especially in the geodetic network densification.

2. Determining the Deflection of the Vertical

As stated by Sabri, Sudarsono and Indriana (2019), the θ can be determined by geometric and physical observations. For geometric observations, the θ is obtained by comparing the astronomical and geodetic coordinates (astrogeodetic method). For physical observations, the deflection of the vertical is determined by gravimetric observations associated with GNSS observations.

According to Zanetti *et al*. (2008) there are other methods to obtain the θ: (i) determination by means of digital zenith cameras, where star observation and GNSS positioning are made; and (ii) determination through the Procrustes transformation, where some points are measured by geodetic coordinates with GNSS and additionally by total station in a local system in order to solve the rotation matrix.

Although less common (Sabri, Sudarsono and Indriana 2019), there is the relation between the θ and the geoidal undulation, where the ε is obtained as function of the geoidal undulation change (Figure 4).



Source: Hofmann-Wellenhof and Moritz (2005 p. 116).

Figure 4: Relationship between the ε, the geoidal undulation difference (dN) and ellipsoidal distance (ds) .

Equation 5 demonstrates the mathematical relationship between the variation of the geoidal undulation (dN) and the ellipsoidal distance (ds), where ε is in meters per meter (thus equivalent to radians) and its sign indicates the direction to be considered at the zenith angle.

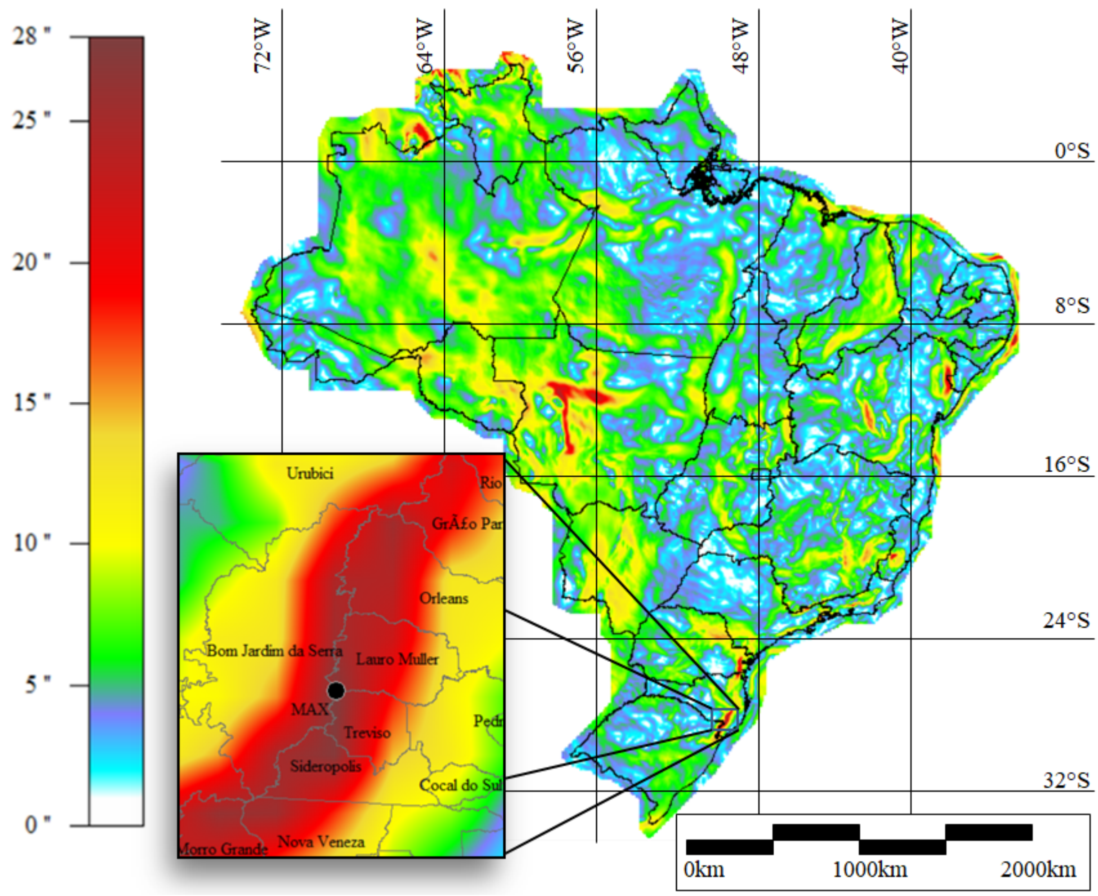
$$\varepsilon = -\frac{dN}{ds} \tag{5}$$

Vitti (2017) shows an application using equation (5) through leveling and geodetic GNSS positioning for the calculation of component ε.

In this relationship, the accuracy of ε directly depends upon the quality of the geoidal model. However, this relation does not require astronomical or direct gravimetric observations, nor the availability of zenith cameras. In this way, it is possible to estimate the influence of the θ in the design stage of the surveys. In this research, we will seek in the Brazilian territory the average and the maximum value of the θ, evaluating the common and critical impacts of its neglect in the horizontal geodetic positioning.

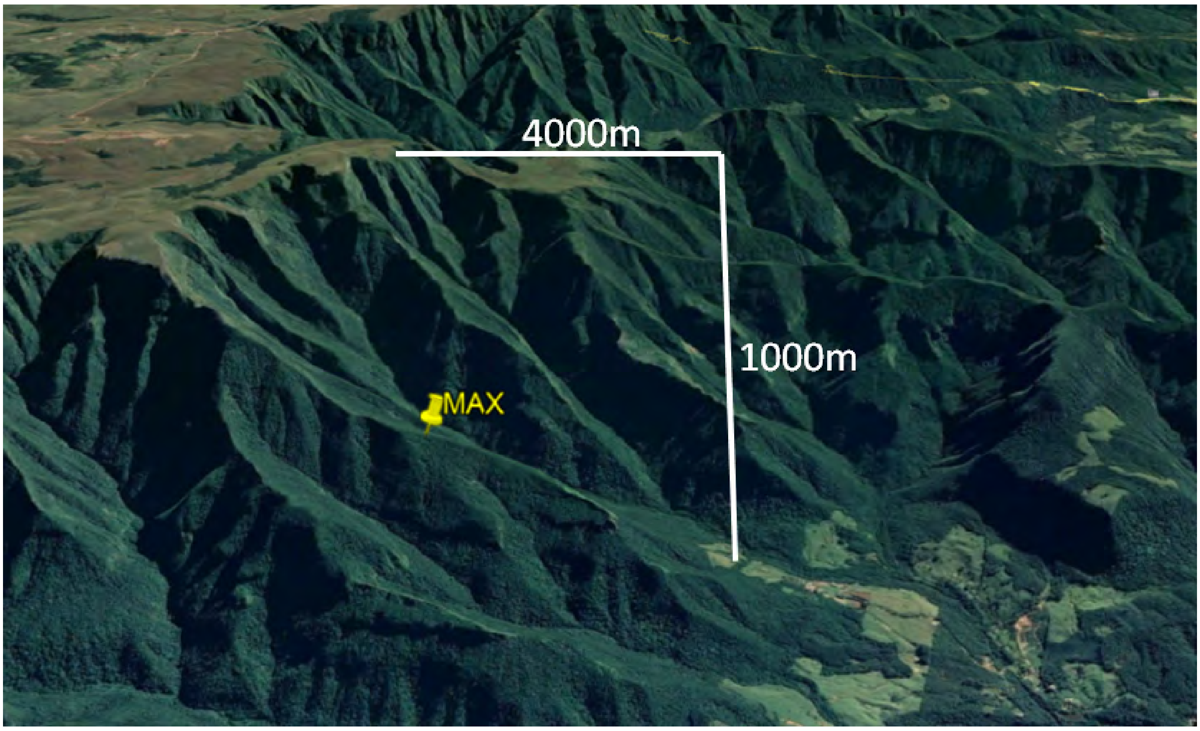
The global gravitational model (GGM) adopted was the EGM2008 (Earth Gravitational Model 2008) represented by spherical harmonics up to the maximum degree 2190 corresponding a spatial resolution of about 9 km (ICGEM 2019), widely used as reference for studies and comparisons of the θ (Barzaghi, Carrion, Pepe and Prezioso 2016; Hirt 2010a; Osada 2016 and 2017; Sabri, Sudarsono and Indriana 2019). The International Centre for Global Earth Models (ICGEM 2020) provides a web service to directly obtain the deflection of the vertical (θ) and its components (η, ξ). The ICGEM is one of the five services coordinated by the International Gravity Field Service (IGFS) of the International Association of Geodesy (IAG) which provides the community with calculation services on geopotential models (Ince *et al.* 2019). As data input, it is possible to inform the geodetic (latitude and longitude) coordinates and height, besides allowing to choose the degree of spherical harmonic. For this research, the maximum degree (2190) was used. Using Shuttle Radar Topography Mission data (Farr *et al.* 2007), a digital elevation model was generated with spatial resolution equivalent to EGM2008 of 5' x 5' (≈9 x 9km) to extract values from the EGM2008 in a tide-free system. For more information on how EGM calculates deflection, see Barzaghi, Carrion, Pepe and Prezioso (2016).

Figure 5 illustrates the magnitude of deflection of the vertical in the Brazilian territory. It can be observed values ranging from 0" until up to 28" in specific regions. The average θ was 5.7" and the maximum was 28.4" in the south of Brazil, along the *Serra do Mar* region. The maximum θ location is a canyon region with a high slope (Figure 6). But other regions with high θ values are not associated with large height variations. Usually, total stations have an angular accuracy better than 5". Therefore, the neglect of the θ may become a source of error greater than the angular standard deviation of the instrument.



Source: Made by the authors, data extracted from ICGEM (2020).

Figure 5:Magnitude of deflection of the vertical in Brazil.



Source: Adapted from Google Earth (2020).

Figure 6: maximum θ location is a canyon region.

Since we have the g_1 gravity vector in function of the W_1 equipotential surface passing through the center of the equipment, and the g_2 gravity vector in function of the W_2 equipotential surface passing at the point above the terrestrial surface (Figure 1), we will have different values of the θ considering the center of the instrument or the geodetic mark, which are at different heights.

In order to analyze whether this effect is significant for this case study, height variations were simulated from a reference point, in the region with the greatest θ and which presents strong gradients in the GGM (Figure 5). It can be observed in Table 1 that in a height difference of 50 m, the θ varied slightly more than 0.1". In practice, the height of the instrument will be close to 1.5 m, which produces changes only in the milliarcsecond, showing that this effect is negligible for the purposes considered here. It is important to highlight that the geopotential model EGM2008 does not use a digital terrain model with high spatial resolution, so the values can be slightly higher and lower, but this does not compromise the conclusion of the research because we seek to analyze only the order of magnitude for the Brazilian territory.

Table 1: Variation of the θ as function of height for maximum θ point.

Ortometric Height	Height Variation	θ	θ Variation
990 m		28.4"	
991.5 m	+1.5 m	28.4"	0.0"
1000 m	+10 m	28.4"	0.0"
1040 m	+50 m	28.3"	-0.1"

Source: Made by the authors, data extracted from ICGEM (2020)

Another important aspect that must be analyzed is the variation of the among the points occupied by the total station and the reflecting target. Different locations will have different gravity vector directions, as well as different directions for the normal. In order to analyze the magnitude of these variations, we also simulated in the region of maximum θ the variation of the ε in distances of 0 to 5 km in the geodetic azimuth of 114°. This azimuth is the direction generated by the components η =+25.8" and ξ = -11.8", obtained in the highlighted point of Figure 5.

Table 2 shows that even at distances of 2 km (uncommon in measurements with total station), the variation of the was less than 0.2". In a distance of 5 km, the variation was close to 3", which is a considerable amount; however, it occurs in an impractical distance for the geodetic network densification, due to the urban obstacles and the usual range of total stations currently available on the market. The current GGM have low resolution (e.g. EGM2008) and the deflection of the vertical is influenced by the variation in masses present in a few hundred meters (Featherstone and Rüeger 2002). Therefore, the values of θ in regions with large variations in relief may be slightly higher or lower than those identified. However, such variation can be neglected for the purposes of this research.

Table 2: Variation of the as function of the distance between total station and reflector in maximum θ region.

DISTANCE	ε	VARIATION
0 m	28.4"	
100 m	28.4"	0.0"
200 m	28.5"	0.1"
500 m	28.5"	0.1"
1 km	28.5"	0.1"
2 km	28.2"	-0.2"
5 km	25.5"	-2.9"

Source: Made by the authors, data extracted from ICGEM (2020).

3. The influence of the deflection of the vertical on the horizontal geodetic positioning

As seen before, the θ affects the linear and angular reductions to the ellipsoid. If the geodetic azimuth is in a certain direction in such a way that the θ is in the plane defined by the vertical axis of the total station and the line of sight (Figure 1), its effect is maximum on the linear reduction, and therefore, $\varepsilon = \theta$. Otherwise, if the θ is in the plane defined by the vertical and horizontal axis of the total station, its effect is maximum on the angular reduction. It should be noted that the influence of θ on the horizontal positioning, will always have the same magnitude, despite the direction of the given azimuth, being able to be distributed in the linear and angular reductions. In this research, it will be considered the case in which the effect is maximum on the linear reduction, which consequently will produce the horizontal positioning error. In this way the analysis is simplified and faithfully represents the influence.

In order to evaluate the influence of the θ on the horizontal geodetic positioning, it will not be considered the reduction factor in function of the survey height, assuming that the observations are measured on the ellipsoid. In addition, the effect of the ellipsoid curvature will not be considered, since it is negligible for distances of up to a few kilometers. It is important to highlight that these sources of systematic errors, when significant, must be treated independently of the θ, being outside the scope of this research. Details on the subject can be found in França (2015). Other important sources of errors in long-range distances, such as atmospheric refraction (Rüeger

1996; Torge and Müller 2012 p. 26 and 5.5.2), must be addressed individually and are not the subject of this research. The reduction of the *SD* to the Local Ellipsoidal Distance (*LED)*, according to the Figure 3 is given by:

$$LED = SD \cdot \sin(Z + \varepsilon) \tag{6}$$

Therefore, due to the error propagation (Ghilani 2010, chapter 6), the ε will cause a variation in the value of *LED* given by:

$$\Delta LED_{\varepsilon} = SD \cdot |\cos Z| \cdot \varepsilon \tag{7}$$

Considering that the unit of measurement of ε in Equation (7) is in radians.

4. Results

As seen in Equation 7, the influence of the deflection of the vertical on the local ellipsoidal distance (ΔLED_{ε}) depends on the zenith angle, the ε and the measured slope distance. In order to quantify this influence, we will adopt some scenarios of possible occurrence in practice, considering $\varepsilon = \theta$ so that the entire effect affects the *LED*, facilitating the analysis without limiting the conclusion.

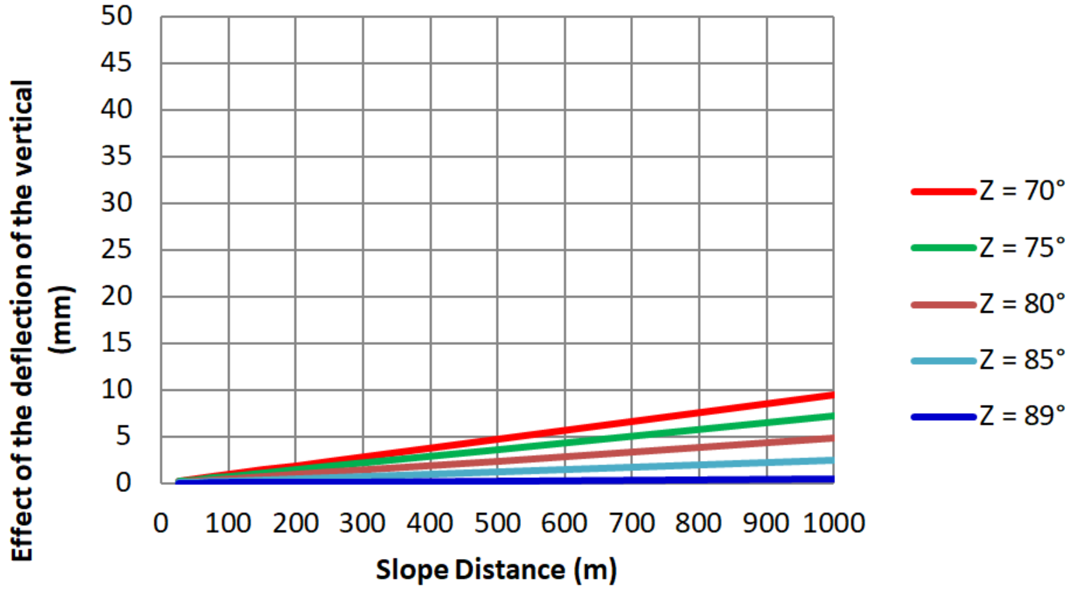
For the zenith angles, the closer to 90° (horizontal line of sight), the lower will be the influence of the θ. Klein *et al.* (2017) show a real case of geodetic network implementation in an urban area, where geodetic markers were placed in strategic points of high visibility, such as on top of hills and buildings (Figure 7). From possible line of sights of the geodetic network markers on the ground, Figure 7 shows zenith angles measured at 68° and 69° (A014 for P5 and A002 for P2, respectively). In this way, it was considered zenith angles from 89° to 70°, even though it is physically possible to measure zenith angles of the order of approximately 40°, but in practice it is very unlikely.

For slope distances, it was considered a variation of 100 m up to 1000 m, since it is unlikely new geodetic markers outside this range in the densification of geodetic networks with total station.

Source: Adapted of Google Earth (2020).

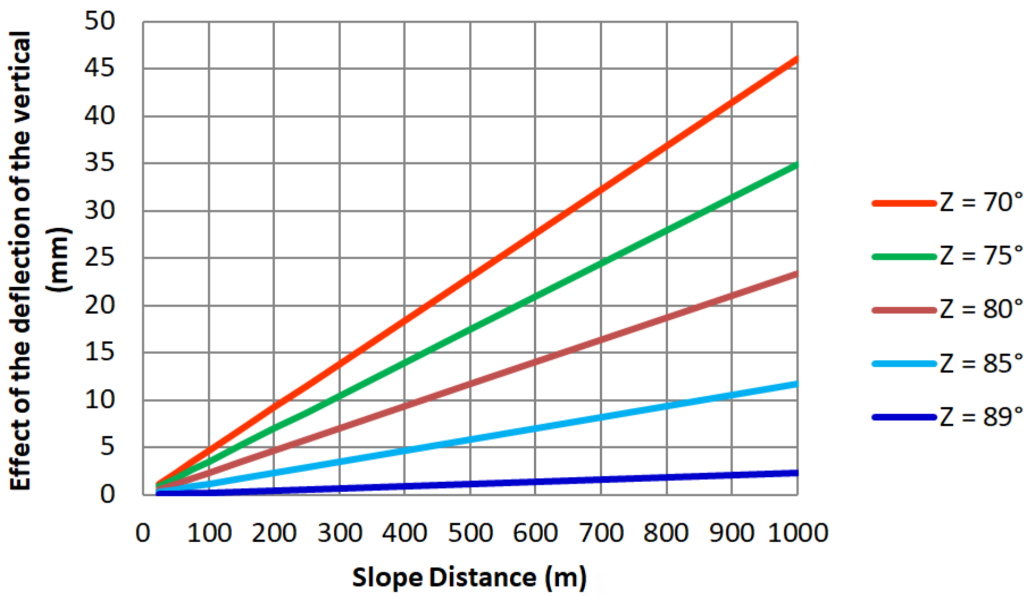
Figure 7: Real examples of surveys with zenith angles away from the horizon for geodetic networks in urban areas.

For the values of the θ in Brazil, it was adopted the average value (5.7”) and the maximum value (27.8”) obtained from EGM2008. Results of these scenarios can be seen in Figures 8 and 9, respectively. It can be observed that in critical situations, such as observations with 1000 m and zenith angle of 70° with the average value of θ, the influence on the LED is close to 10 mm; while with the maximum θ, the influence on the LED exceeds 45 mm.



Source: the authors.

Figure 8: Effect of the average θ on the horizontal positioning in Brazil.

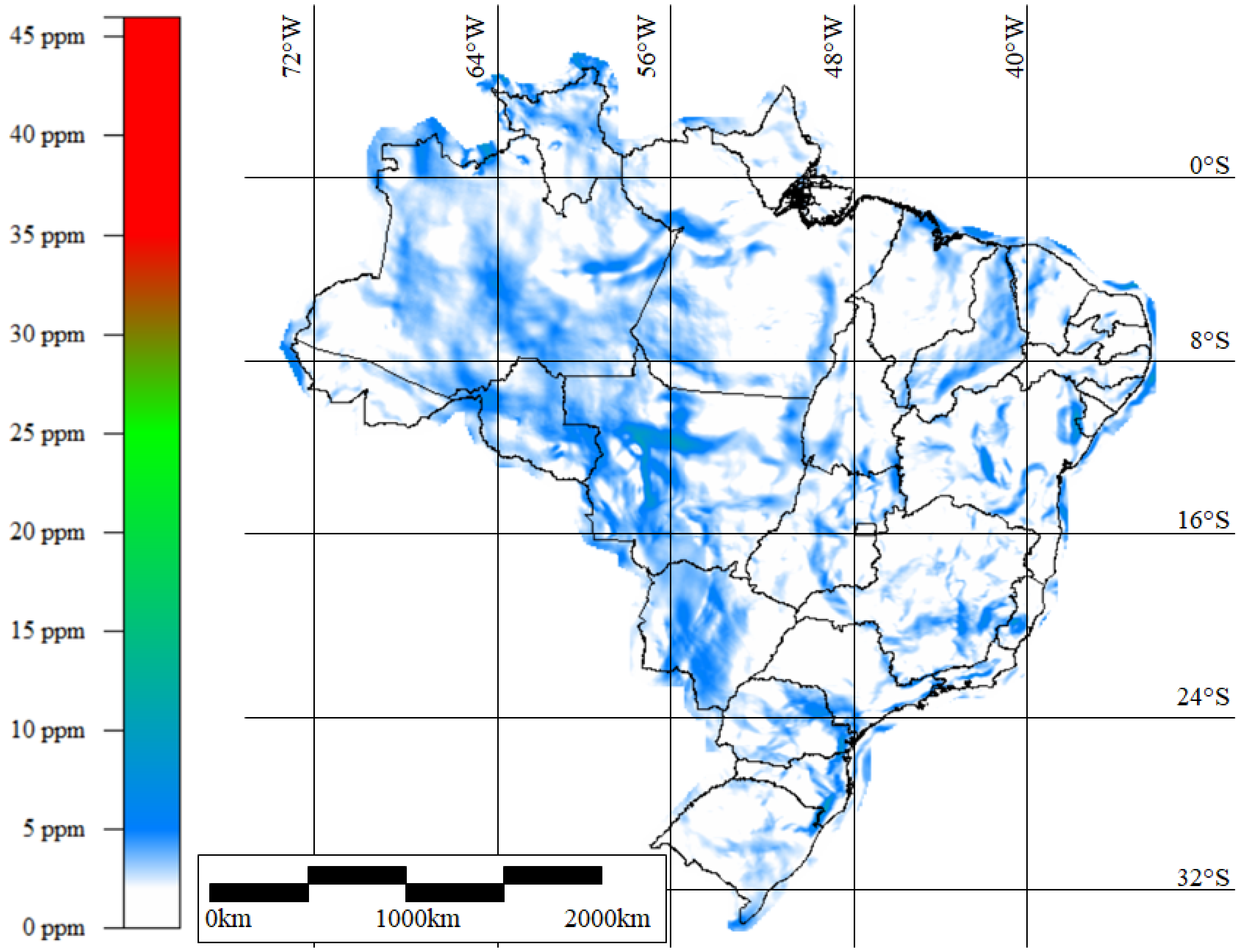


Source: the authors

Figure 9: Effect of the maximum θ on the horizontal positioning in Brazil.

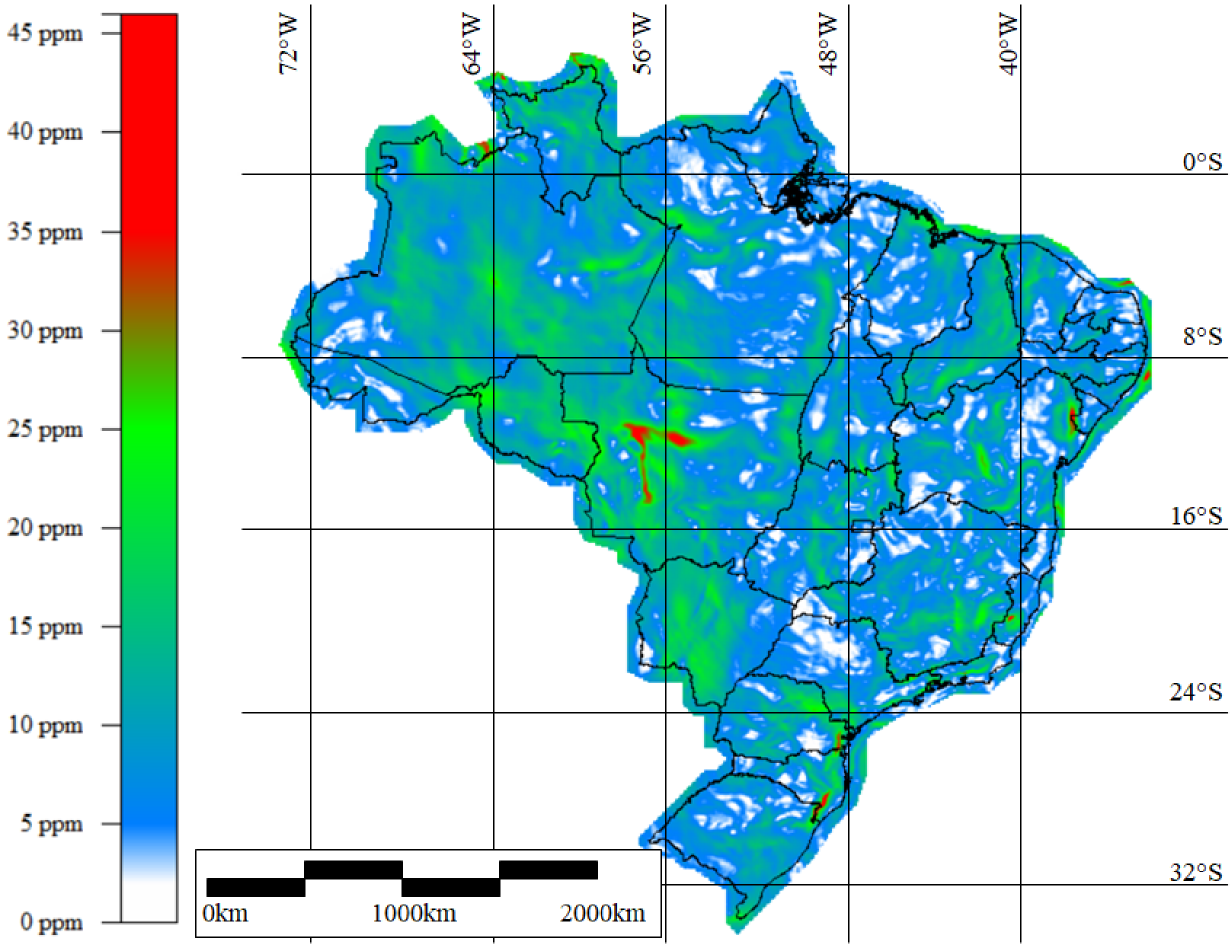
In order to illustrate the behavior of the influence of the θ on distance, and consequently on geodetic horizontal positioning for the entire Brazilian territory, it were calculated the values of in parts per million (ppm) for zenith angles of 85° and 70° (Figures 10 and 11). It should be pointed out that both the map of the θ values (Figure 5), and the results obtained for the influence of the θ on the horizontal geodetic positioning (Figures 10 and 11), are presented for the first time with focus on the Brazilian territory.

In summary, it can be stated that the θ may have an influence of centimetric size on LED obtained with total station, even though, usually, this influence will be in the order of a few millimeters. However, the conduction of geodetic surveys for several kilometers, for example, through traverse survey, may result in cumulative systematic errors of a centimetric order due to the neglect of the θ; thus affecting the quality of land surveys in Brazil.



Source: the authors

Figure 10: The effect of the θ on LED (in ppm) for a zenith angle of 85° in Brazil.



Source: the authors.

Figure 11: The effect of the θ on LED (in ppm) for a zenith angle of 70° in Brazil.

5. Conclusion

Integrating technologies such as GNSS and total station is necessary, due to the restrictions of each one. This requires knowledge of the errors involved, avoiding the propagation of systematic errors that can be eliminated, or, at least, minimized. This is the case with the deflection of the vertical investigated in this research.

With the elaboration of a map of the deflection of the vertical values in the Brazilian territory through the EGM2008 model, it was possible to identify that, in Brazil, the average value of the θ is 5.7" and the maximum value is 28.4". It was also simulated variations of the θ as function of height, as well as variations in function of the distance between the total station and the reflecting target. In both simulations, spatial variations of the deflection of the vertical appeared to be insignificant for most applications of the geodetic surveys. For applications that require higher precision, further studies are needed, as well as a more accurate computation of the deflection of the vertical.

Analyzing equation (7), it is observed that for lines of sight with zenith angle near 90°, the effect of the deflection of the vertical on the local ellipsoidal distance will be smaller. Therefore, in urban areas with lines of sight more inclined in relation to the horizon, with zenith angle much smaller or much greater than 90°, the influence of DV requires special attention. The results obtained indicate that the influence of the θ on the horizontal geodetic positioning in Brazil may result in systematic errors of up to 45 ppm for zenith angles of 70°. Therefore, this error size is about twenty times larger than the standard accuracy of total stations currently available (± 2mm + 2 ppm).

In this way, error propagation generated by the neglect of the deflection of the vertical may put at risk the quality of the land surveys in urban areas, such as in the case of the 80 mm positional accuracy required by Decree 9310/2018 (Brasil 2018), which established the general standards and applicable procedures for Urban Land Regularization.

The situation is aggravated by the fact that most of the municipalities in Brazil do not have geodetic networks, where for each survey of an urban property it is necessary the transportation of coordinates by traverse survey (if GNSS positioning is not recommended), which can generate inconsistencies between the property boundaries and legal disputes.

Therefore, since the reduction of distances and angles due to the deflection of the vertical is usually neglected by commercial softwares and internal software of total stations, it is recommended to estimate its components in the mathematical model, for example, by the least squares method. Furthermore, in order to minimize the influence of the deflection of the vertical on the horizontal geodetic positioning with total station, it is suggested new approaches for establishment of geodetic markers, avoiding the measurement of long distances and/or with zenith angles away from 90°.

In the release of the next geoidal or quasi-geoidal model in Brazil, it is recommended to provide the values of the deflection of the vertical components (η and ξ), along with the geoidal undulations and/or height anomalies, given its practical importance as demonstrated by the results of this research. Prešeren *et al.* (2018) states that the adoption of local geoid models, bring better results in deflection of the vertical.

Finally, it is recommended new studies of the variation of the deflection of the vertical in regions with great variation of altitude using the Residual Terrain Model technique and also with GGM of higher spatial resolution, as example researched by Hirt (2010b).

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AUTHOR'S CONTRIBUTION

Rovane Marcos de França: planning, methodology, research, data processing, writing, editing.

Ivandro Klein: planning, conceptualization, methodology, review, supervision.

Luis Veiga: conceptualization, review, supervision.

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Erratum

On the first page, where it reads:
A influência da deflexão da vertical nas pesquisas geodéticas no Brasil
Should read:
A influência do desvio da vertical nas medições geodésicas no Brasil

On page 4, where it reads:
Therefore, the transformation of observations between local systems to the global geocentric system without considering the rotation from deflection of the vertical, will result in a systematic error, and, consequently, will propagate in the geodetic positioning, generating errors of difficult identification and consistent adjustments (Ghilani 2010 p. 487 and 495).

Should read:
Therefore, the transformation of observations between local systems to the global geocentric system without considering the rotation from deflection of the vertical, will result in a systematic error, and, consequently, will propagate in the geodetic positioning, generating errors of difficult identification and inconsistent adjustments (Ghilani 2010 p. 487 and 495).

On page 5, where it reads:
As stated by Sabri, Sudarsono and Indriana (2019), the θ can be determined by geometric and physical observations. For geometric observations, the θ is obtained by comparing the astronomical and geodetic coordinates (astrogeodetic method). For physical observations, the deflection of the vertical is determined by gravimetric observations associated with GNSS observations.

Should read:
As stated by Sabri, Sudarsono and Indriana (2019), the θ can be determined by geometric and physical observations. For geometric observations, the θ is obtained by comparing the astronomical and geodetic coordinates (astrogeodetic method). For physical observations, the θ is determined by gravimetric observations associated with GNSS observations.

On page 7, where it reads:

Since we have the g_1 gravity vector in function of the W1 equipotential surface passing through the center of the equipment, and the g_2 gravity vector in function of the W2 equipotential surface passing at the point above the terrestrial surface (Figure 1), we will have different values of the θ considering the center of the instrument or the geodetic mark, which are at different heights.

Should read:

Since we have the g_1 gravity vector in function of the W_1 equipotential surface passing through the center of the instrument, and the g_2 gravity vector in function of the W_2 equipotential surface passing at the geodetic mark above the terrestrial surface (Figure 1), we will have different values of the θ considering the center of the instrument or the geodetic mark, which are at different heights.

On page 8, where it reads:

Another important aspect that must be analyzed is the variation of the among the points occupied by the...

Should read:

Another important aspect that must be analyzed is the variation of the ε among the points occupied by the...

On page 8, where it reads:

Table 2 shows that even at distances of 2 km (uncommon in measurements with total station), the variation of the was less than....

Should read:

Table 2 shows that even at distances of 2 km (uncommon in measurements with total station), the variation of the ε was less than...

On page 8, where it reads:

Table 2: Variation of the as function of the distance between total station and reflector in maximum θ region.

Should read:

Table 2: Variation of the ε as function of the distance between total station and reflector in maximum θ region.

On page 9, where it reads:

For the zenith angles, the closer to 90° (horizontal line of sight), the lower will be the influence of the θ. Klein *et al.* (2017) show a real case of geodetic network implementation in an urban area...

Should read:

For the zenith angles, the closer to 90° (horizontal line of sight), the lower will be the influence of the θ. Klein *et al.* (2017) show a real case of geodetic network densification in an urban area...

On page 11, where it reads:

In order to illustrate the behavior of the influence of the θ on distance, and consequently on geodetic horizontal positioning for the entire Brazilian territory, it were calculated the values of in parts per million (ppm) for zenith angles of 85° and 70° (Figures 10 and 11)...

Should read:

In order to illustrate the behavior of the influence of the θ on distance, and consequently on geodetic horizontal positioning for the entire Brazilian territory, it were calculated the values of $\Delta LED\varepsilon$ in parts per million (ppm) for zenith angles of 85° and 70° (Figures 10 and 11)...]

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