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FORCED CONVECTION TO LAMINAR FLOW OF LIQUID EGG YOLK IN CIRCULAR AND ANNULAR DUCTS

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Abstract - The steady-state heat transfer in laminar flow of liquid egg yolk - an important pseudoplastic fluid food - in circular and concentric annular ducts was experimentally investigated. The average convection heat transfer coefficients, determined by measuring temperatures before and after heating sections with constant temperatures at the tube wall, were used to obtain simple new empirical expressions to estimate the Nusselt numbers for fully established flows at the thermal entrance of the considered geometries. The comparisons with existing correlations for Newtonian and non-Newtonian fluids resulted in excellent agreement. The main contribution of this work is to supply practical and easily applicable correlations, which are, especially for the case of annulus, rather scarce and extensively required in the design of heat transfer operations dealing with similar shear-thinning products. In addition, the experimental results may support existing theoretical analyses.
Keywords: Egg yolk; Heat transfer; Laminar flow; Nusselt number.

INTRODUCTION

The major technical applications of laminar flow forced convection heat transfer in closed conduits are in the analysis and design of heat exchangers. The convective heat transfer coefficients for non-Newtonian flows in circular and annular ducts are of particular interest in processes involving double-pipe and triple-pipe heat exchangers, whose design is not a trivial operation. This problem mainly concerns food or chemical industries, when products to be treated may exhibit complex rheological behavior, such as fruit purees, emulsions and polymeric melts, which present high apparent viscosities.

Tubular heat exchangers are one of the commonest types of heat exchangers in the food industry and the typical areas of application are pasteurization and sterilization of fruit concentrates and vegetable purees (Sannervik et al., 1996). In this way, non-dimensional correlations for heat transfer in fully developed laminar flows of pseudoplastic fluids in circular and concentric annular geometries during forced convection at the thermal entrance are essential in the modeling, optimization and design of these processes.

Many investigations have suggested a number of analytical, semi-analytical and empirical correlations for circular ducts; however, especially for the case of annular regions, whose studies are primarily devoted to numerical analyses, experimental data are of particular importance in order to confirm theoretical evaluations.

As observed by Cho and Hartnett (1982), since the early theoretical work of Graetz, in 1883 and

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1885, Nusselt, in 1910, and Lévêque, in 1928, the analysis of the laminar heat transfer performance in the thermal entrance region of Newtonian fluids flowing in circular tubes has been carried out in different ways by many investigators, for various boundary conditions. Some of these results are summarized in Shah and London (1978), Shah and Bhatti (1987) and Kays and Crawford (1993). Additionally, also available in the literature is a great deal of information on the effects of viscous dissipation (Krishnan and Sastri, 1978; Lawal and Mujumdar, 1985) and fluid axial conduction (Vick and Ozisik, 1981; Bilir, 1992; Olek, 1998). In the last fifty years, many extensions of the widely known Graetz problem (or Graetz-Nusselt problem) and the Lévêque solution have been proposed; in addition, the analyses of the accuracy of analytical and numerical results have received attention. We can cite some important contributions to this field, such as Lyche and Bird (1956) who extended the Graetz-Nusselt problem to Power-Law flow and Pigford (1955), who first modified the Lévêque approach to Power-Law fluids. Subsequently, Metzner et al. (1957) used a consistency empirical radial correction in the extended Lévêque equation, as that one used by Sieder and Tate (1936) to account for radial variations in fluid consistency, while Christiansen and Craig (1962) have shown that the effects of radial viscosity variation may be treated by an analytical method, described by an exponential relationship. Popovska and Wilkinson (1977), Gori (1978), Cotta and Ozisik (1986), Mansour (1989), Prusa and Manglik (1994) and Khellaf and Lauriat (1997) also studied the heat transfer by forced convection in the thermal entrance of circular pipes for hydrodynamically developed flows of Power-Law fluids in the case of negligible viscous dissipation and axial heat conduction in the fluid, under conditions of constant surface temperatures, while Bird et al. (1977) and Cotta and Ozisik (1986) considered prescribed wall heat fluxes. Natural convection effects introduced by fluid density variations were studied by Mahalingam et al. (1975) and Basset and Welty (1975), among others, for uniformly constant heat flux at the wall, while for constant wall temperature average Nusselt correlations were proposed, for example, by Metzner and Gluck (1960), Oliver and Jenson (1964), Gori (1978) and Rodriguezluna et al. (1987). The effect of heat generation by viscous dissipation was included by Shih and Tsou (1978), Richardson (1979), Lawal and Mujumdar (1989), Ramachandran (1993), Manglik and Prusa (1995) and Barletta (1997), and convection in the region where laminar flow is fully developed both thermally and hydrodynamically was studied by Skelland (1967) and Barletta (1997), whereas the simultaneous development of the laminar flow and the convective heat transfer in short circular ducts have been experimentally analyzed by Fontes and Gasparetto (2001).

On the other hand, works on the problem of convective heat transfer in the thermal entrance region of annular ducts during flow of Power-Law fluids are rather scarce, while for Newtonian fluids an extensive literature exists both on forced convection (Lundberg et al., 1963; Moghadam and Aung, 1990; Barletta et al., 2004) and mixed convection (Nonino and Delgiudice, 1996; Barletta and Lazzari, 2005), including the effects of eccentricity (Cheng and Hwang, 1968; Trombetta, 1971; El-Shaarawi et al., 1998), viscous dissipation (Rokerya and Igbal, 1971) and inner or outer cylinder rotation (Childs and Long, 1996). For Power-Law flows, investigations are almost reduced to the case of hydrodynamic and thermal fully developed conditions, and the solutions of the problem are basically obtained through finite-difference techniques, except for the works of Tanaka and Mitsuishi (1975) and Nouar et al. (2000); the former not only analyzed numerically but also experimentally the problem of combined forced and free convection in the thermal entrance of a fully developed laminar flow in a horizontal annulus under the conditions of a constant-temperature inner wall and an insulated outer wall, whereas the latter considered the outer and inner cylinders uniformly heated with a constant heat flux density. The relatively recent studies of Manglik and Fang (2002) and Manglik and Fang (1995) considered the forced convection heat transfer in eccentric annular ducts with an insulated outer surface and uniform axial heat flux or constant wall temperature for the inner surface. Barletta (2000) investigated analytically the forced convection in a vertical annular duct with the boundary surfaces supposed to be isothermal with unequal temperatures. Batra and Sudarsan (1992) considered the case of the simultaneous development of the hydrodynamic and thermal boundary layers, in which one wall of the annulus was isothermal and the other adiabatic, while Capobianchi and Irvine (1992) examined fully developed heat transfer characteristics of a modified Power-Law fluid. Finally, we can cite Hong and Matthews (1969), who probably first solved analytically the heat transfer to Power-Law fluids in the thermal entrance region of annuli.

Recognizing the need of experimental results, extremely scarce in the literature and essential to validate theoretical analyses, the aim of this work was to obtain heat transfer coefficients, as well as average Nusselt numbers, in the laminar flow of egg yolk – a pseudoplastic fluid with important potential for the international market – through circular and annular ducts with constant surface temperatures, calculated from experimentally measured inlet and outlet bulk temperatures. The results were compared with existing correlations for Newtonian and non-Newtonian fluids and the influence of the annulus aspect ratio on the Nusselt number was considered.

MATERIALS AND METHODS

Samples: Liquid Egg Yolk

The liquid egg yolk for this study was obtained directly from the processing line of an egg breaking plant. The moisture content of the initial batch was determined by the gravimetric method using a vacuum oven (48 hrs, 333 K, 100 mmHg), resulting in 54.04(± 0.43)% of moisture (wet basis). The sample pH was 6.40 ± 0.04.

Thermophysical Properties of Liquid Egg Yolk

Specific heat, thermal conductivity, density and rheological parameters of liquid egg yolk at various temperatures were determined using the equations described by Gut et al. (2005). The calculated thermophysical properties are shown in Table 1.

All the fluid properties were estimated as the arithmetic average of inlet and outlet bulk temperatures, $T_{ba} = (T_{b1} + T_{b2})/2$, in which T_{b1} and T_{b2} (ºC) are, respectively, the inlet and outlet bulk temperatures of the fluid.

Experimental Apparatus for Determination of the Bulk Temperatures

The experimental apparatus used for measuring average heat transfer coefficients in horizontal ducts of circular and annular cross sections was similar to the equipment used by Telis-Romero et al. (2006) when studying hydrodynamics of egg yolk flowing in a circular pipe.

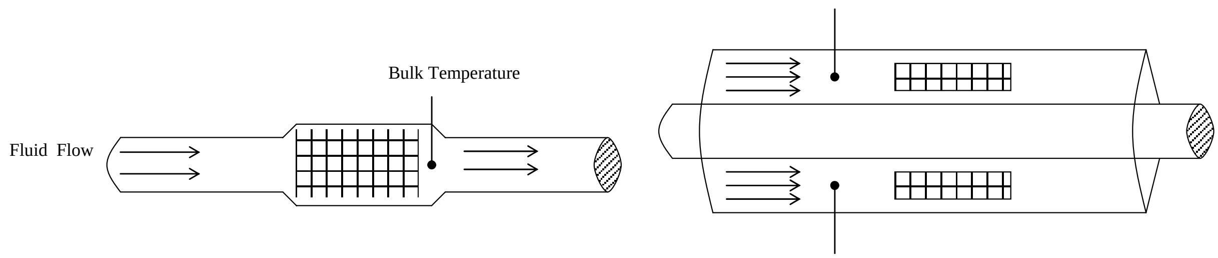
Two heat transfer units were used. One of the units was made with four horizontal steel circular tubes, with nominal diameters of 1/4 in, 1/2 in, 1 in and 1½ in; the total length of this test section was 1.2 m, providing a maximum length-to-diameter ratio (L/D) of 187.5. The other unit consisted of two sections of triple-pipe heat exchangers, with different radius ratio for the inner annulus and a total length of 1.2 m. The triple-pipe heat exchanger was formed by three coaxial cylinders, in which water from a thermostatic bath flowed in the tube and the outer annulus, while the liquid egg yolk flowed in the inner annulus. The external diameter of the tube (D_1) was fixed at 0.0137 m, while the internal diameter of the cylinder forming the inner annulus (D_2) was 0.0274 m, 0.0381 m, 0.0590 and 0.0727 m, thus providing the respective values of 0.0137 m, 0.0244 m, 0.0453 and 0.0590 m for the hydraulic diameter (D_h), and the respective values of 0.500, 0.360, 0.059 and 0.0727 m for the annulus aspect ratio (κ). The maximum length-to-diameter ratio (L/D_h) was 87.6.

The heat transfer sections were submerged in a large thermostatic bath (Model MA-184, Marconi, SP, Brazil), in which water flowed at a high mass flow rate and at a constant temperature ($\cong 66^{\circ}\mathrm{C}$). The experiments were carried out during samples heating by using water in the thermostatic bath. A distance of 1.0 m provided the developing length of the flow regime, guaranteeing the establishment of the velocity profile at the entrance of the heating zone. Temperature transducers (Model TT-302, Smar, SP, Brazil) were used to measure temperature at the beginning and at the end of the test section. A static mixer was placed at the end of the two pieces of the equipment to homogenize the final temperature of the liquid egg yolk, according to Figure 1. Copper-constantan thermocouples were soldered into grooves along the heating test sections to measure tube wall temperature. In all the experiments, the wall temperature difference between the first and the last copper-constantan thermocouple was approximately 0.6 ºC. When readings were taken, an effort was made to keep the average temperature of the fluid constant for several values of velocity so that its effect could be isolated from that of other factors, as suggested by Sieder and Tate (1936). The egg yolk was pumped by means of a peripheral pump (Model P-500, KSB, SP, Brazil). A flow meter (Model LD100, MLW, Germany) was used to initially adjust the desired flow rate in each experiment, but exact measurements were obtained by weighing fluid samples collected at determined time intervals. A data logger running a data acquisition and control program monitored temperatures and pressures. The average flow velocities were varied from 0.14 to 2.00 $m.s^{-1}$, adding up to one hundred thirty-six experimental values for the circular cross section and two hundred eighty-eight points for the annular geometry.

Table 1: Thermophysical properties of liquid egg yolk (0 < T < 61°C) from Gut et al. (2005).

Property	Correlation	R^2	Units
Density	$\rho = 1.1332 \cdot 10^3 - 0.057 \cdot T$	0.9960	ρ: kg/m^3 T: °C
Specific Heat	$Cp = 2.6290 \cdot 10^3 + 2.39 \cdot T$	0.9996	Cp: J/kg·K T: °C
Thermal Conductivity	$k = 0.390 + 4.0 \cdot 10^{-4} \cdot T$	0.9914	k: W/m·K T: °C
Consistency index (Power-law)*	$K = 8.182 \cdot 10^{-9} \cdot \exp\left(\frac{44{,}195}{R \cdot T}\right)$	0.9985	K: Pa·s^n T: K R: 8.31451 (J/mol·ºC)
Behavior index (Power-law)*	$n = 0.277 \cdot T^{0.198}$	0.8466	n: (dimensionless) T: K

* Valid for shear rate between 70.2 to 512.4 s^{-1}



a) Tubular duct b) Annular duct

Figure 1: Schematic apparatus used for measuring average heat transfer coefficients in horizontal ducts of tubular (a) and annular (b) cross sections.

The performance of the apparatus was checked using a CMC solution (1% w/w), which was pumped through the equipment at 68 different flow rates in the laminar range. At each flow rate, inlet and outlet temperatures in a straight pipe section of 1.80 m were measured with ten repetitions made at five-minute intervals, in a procedure similar to that used by Polizelli et al. (2003) and Telis-Romero et al. (2005) when investigating friction losses in fittings during non-Newtonian fluid flow. The thermophysical properties of CMC solution were obtained from the work of Carezzato et al. (2007).

Calculation of the Heat Transfer Coefficients and Development of the Average Nusselt Correlation

Considering a fluid flowing through a circular tube of diameter D, in which there is a heated wall section of length L, whose inside surface temperature is $T_0(z)$, a steady-state energy balance over a length L of the tube (Bird et al., 2002) is made by stating that the heat through the walls plus the energy entering at $z=0$ by convection equals the energy leaving the tube at $z=L$. For fully developed flows, changes in the kinetic energy flux and the work term will be negligible relative to changes in the enthalpy flux. Then, by neglecting the axial heat conduction term, the steady-sate energy balance becomes simply "rate of energy flow in = rate of energy flow out". Combining the energy balance equation with the conventional definition for the heat flow into the fluid, one may obtain Eq. (1).

$$h_{ln} = \frac{\dot{m}C_p\left(T_{b2}-T_{b1}\right)}{\pi DL\Delta T_{ln}} = \frac{\dot{m}C_p\left(T_{b2}-T_{b1}\right)}{\pi DL\left[\dfrac{\left(T_0-T_{b1}\right)-\left(T_0-T_{b2}\right)}{\ln\left(T_0-T_{b1}\right)-\ln\left(T_0-T_{b2}\right)}\right]} \tag{1}$$

in which h_{ln} (W.m^{-2}.ºC^{-1}) is the heat transfer coefficient based on the corresponding logarithmic mean temperature difference ΔT_{ln} (ºC), $\dot{m}$ (kg.s^{-1}) is the total mass flow rate, C_p is the fluid specific heat (J.kg^{-1}.ºC^{-1}), T_{b1} and T_{b2} (ºC) are respectively the inlet and outlet bulk temperatures of the fluid, D (m) is the tube internal diameter, L (m) is the tube length and T_0 (ºC) is the tube wall temperature. Therefore, the Nusselt numbers may be obtained from Eq. (2).

$$Nu_m = \frac{h_{ln}D}{k} \tag{2}$$

in which k is the thermal conductivity of the fluid ($W.m^{-1}.^{o}C^{-1}$). The dimensionless numbers Gz, Re_{mr} and Pr_g, given respectively by Eqs. (3), (4) and (5), may be used to correlate new empirical expressions for the Nusselt number.

$$Gz = \left(\frac{\dot{m}C_p}{kL}\right) \tag{3}$$

$$Re_{mr} = \left(\frac{\rho \overline{v}_z^{\,2-n} D^n}{8^{n-1}K}\right)\left(\frac{4n}{3n+1}\right)^n \tag{4}$$

$$Pr_g = \left(\frac{C_p K}{k}\right)\left(\frac{8\overline{v}_z}{D}\right)^{n-1}\left(\frac{3n+1}{4n}\right)^n \tag{5}$$

in which ρ is the fluid density ($kg.m^{-3}$) and $\overline{v}_z$ is the average axial flow velocity ($m.s^{-1}$).

All fitted functions were performed using Nonlinear Estimation Procedures from the software MatLab 6.0® (The MathWorks Inc., 2001) which solves nonlinear curve fitting problems in the least square sense. The adequacy of the fitted functions was evaluated by the correlation coefficient (R^2), the magnitude of the root mean square (RMS) error calculated according to Gabas et al. (2002), the chi-square values of fit (Chi^2), the sum of squares of the difference between data and fit values (SSR), and the absolute relative errors between observed and predicted values, as given by Telis-Romero et al. (2001).

RESULTS AND DISCUSSION

Heat Transfer in Circular Ducts

The heating experiments in circular sections permitted the evaluation of h_{ln} according to Eq. (1). Nusselt and Graetz numbers were obtained by Eqs. (2) and (3), respectively.

The experimental conditions can be summarized as follows:

- The flow was steady (the flows were sustained for ten minutes before the temperature readings were taken), laminar, which was guaranteed by the low fluid axial velocity ($0.14 \le \overline{v}_z \le 2.00$ and $2.0 \le Re_{mr} \le 271.3$) and fully established;
- The temperature profiles were in development; according to Prusa and Manglik (1994), the dimensionless thermal-entry length (z_{te}^*) for a Power-Law fluid with $n = 0.333$ is approximately $z_{te}^* = 4 \times 10^{-1}$, where $z_{te}^* = z/RPe$ and Pe is the Péclet number;
- Heat conduction in the axial direction was negligible, which is justified when $Re\,Pr >> 100$ for Newtonian fluids (Kays and Crawford 1993). The experimental conditions for the Péclet number were: $0.7 \times 10^4 \le Re_{mr}\,Pr_g \le 61.7 \times 10^4$;
- Heat produced by viscous dissipation could be neglected based on the Brinkman number ($Br << 1$); the experimental conditions were: $4.2 \times 10^{-4} \le Br \le 5.4 \times 10^{-2}$;
- Thermophysical properties of the fluid were almost constant between the tube entrance and the exit for each experimental test. The maximum variation of density in an experimental test was 8.3 $kg.m^{-3}$, with a mean variation for all the experiments of 1.8 $kg.m^{-3}$, while the maximum and mean variations of specific heat were 62.3 $kJ.kg^{-1}.^{o}C^{-1}$ and 11.8 $kJ.kg^{-1}.^{o}C^{-1}$, respectively. For thermal conductivity, the maximum variation in an experimental test was 0.014 $W.m^{-1}.^{o}C^{-1}$, and the mean variation for all experiments was 0.004 $W.m^{-1}.^{o}C^{-1}$;
- Natural convection effects could be neglected, which is justified by the nearly constant values of density between the tube inlet and outlet for all experimental tests;
- The flow behavior index presented a mean value for all the experiments of 0.855 and a standard deviation of 0.004, while the consistency index ranged from 0.10 to 2.3 Pa s^n;
- The ranges tested were: $28.5 \le Gz \le 14457.6$ and $31.3 \le L/D \le 184.9$.

Based on the conditions attained in the experiments, the correlation proposed by Metzner et al. (1957), given by Eq. (6), was compared with the experimental results. Metzner et al. (1957) presented the first theoretical analysis combined with an experimental study of the variables controlling heat transfer rates to non-Newtonian fluids; their experimental data covered the ranges: $0.3 \times 10^4 \le Gz \le 0.01 \times 10^4$ and $0.18 \le n \le 0.70$. Their expression for predicting average Nusselt numbers was obtained for a constant property, hydrodynamically fully developed, thermally developing, laminar flow through a circular tube, with negligible axial conduction and viscous dissipation. The tested fluids were aqueous CMC (carboxymethylcellulose) solutions and aqueous Carbopol solutions.

$$Nu_m = 1.75\,\delta^{1/3}\,Gz^{1/3}\left(\frac{K}{K_0}\right)^{0.14} \tag{6}$$

in which:

$$\delta = \frac{3n+1}{4n} \quad (7)$$

The Lévêque equation describing heat transfer to a Newtonian fluid was also compared with the experimental data, as well as other correlations for non-Newtonian fluids, such as those proposed by Pigford (1955), Gori (1978), Joshi and Bergles (1982) and Prusa and Manglik (1994). As observed by Metzner et al. (1957) and Christiansen and Craig (1962), predictions from the Lévêque equation deviate from the non-Newtonian behavior. As would be expected, the data lie considerably above the theoretical Newtonian curve with a RMS error value of 21.6 %. Introduction of the factor $\delta^{1/3}$ brings the data more nearly in line with the theoretical curve predicted by Pigford's expression, but there is still some scatter (RMS error = 12.1 %). The empirical correction factor $(K/K_0)^{0.14}$ to account for the distortion of the theoretical velocity profile adequately correlates the experimental results; in that case, the root mean square error was the smallest of all (RMS error = 8.9 %). Obviously, the presence of some experimental error was indicated in the work of Metzner et al. (1957), which points out some inaccuracy in the curve predicted by Eq (6). Also, in the present work there is some imprecision in the Nusselt predictions, since all fluid properties were estimated from empirical equations, which present some inherent deviations. Although some authors such as Christiansen and Craig (1962), Popovska and Wilkinson (1977) and Forrest and Forrest and Wilkilson (1973) had doubts about the accuracy of Eq. (6), all deviations are within experimental errors, and for design purposes Eq. (6) is perfectly acceptable. The expressions of Gori (1978), Joshi and Bergles (1982) and Prusa and Manglik (1994) provided similar results among themselves, with $\text{RMS error} \cong 15\%$, but evidently Eq. (6) displayed the best predictions.

In order to evaluate the performance of the system, experimental data obtained during flow of a CMC solution (1%, w/w) through the circular duct were used. Pipe dimensions, thermophysical properties and measured temperatures were substituted into Equations (2) and (3) to give the Graetz and Nusselt numbers, which were then correlated by Equation (6). These results are shown in Figure 2. The agreement between experimental and predicted values was very satisfactory, indicating the adequacy of the equipment and methodology used.

Eq. (6) has been widely used for engineering design purposes, and the good agreement between observed data and its predictions is clearly evident in Fig. 3 for liquid egg yolk. The experimental results obtained in this work were also submitted to nonlinear regression analysis, resulting in Eq. (8) with R^2 = 0.926 and SSR = 2569. Eq. (9), similar to Eq. (6), was also fitted to the experimental data and resulted in R^2 = 0.921 and SSR = 1098.

$$Nu_m = 2.827\, Gz^{0.329} \quad (8)$$

$$Nu_m = 1.819\, \delta^{0.33}\, Gz^{0.33} \left(\frac{K}{K_0}\right)^{0.14} \quad (9)$$

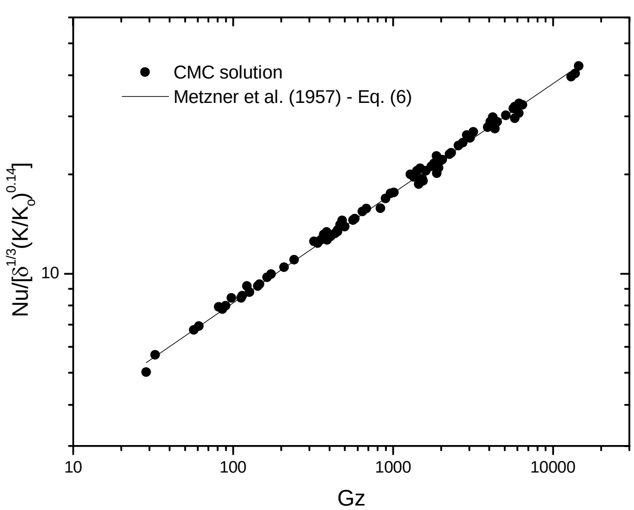


Figure 2: Average Nusselt numbers for CMC solution (1% w/w) for circular duct: experimental and predicted by Eq.(6) (Metzner et al., 1957).

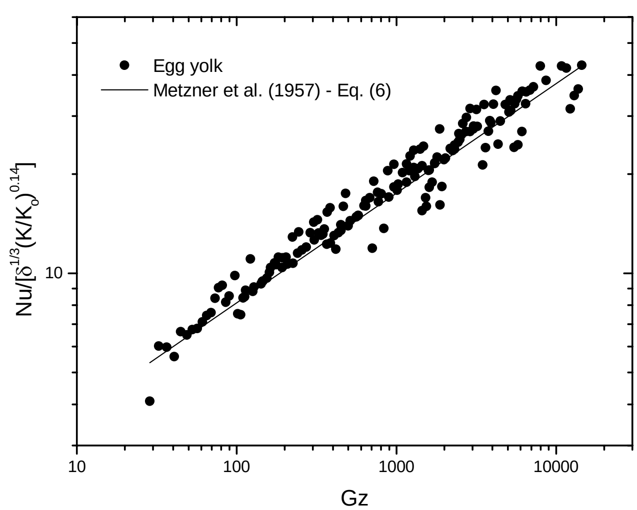


Figure 3: Average Nusselt numbers for egg yolk in circular duct: experimental and predicted by Eq.(6) (Metzner et al., 1957).

Heat Transfer in Annular Ducts

The heating experiments in the annular regions permitted evaluation of h_{ln}, Nusselt and Graetz according to Eqs. (1), (2) and (3), respectively, using the concept of the hydraulic diameter. The experimental conditions can be summarized as follows:

- The flow was steady, laminar ($0.14 \le \bar{v}_z \le 2.00$ and $42.8 \le Re_{mr} \le 2263.4$) and fully established;
- Conduction of heat in the axial direction was negligible ($Pe >> 100$); the experimental conditions for the Péclet number were: $2.1\times10^4 \le Re_{mr}\,Pr_g \le 4.5\times10^5$;
- Heat produced by viscous dissipation could be neglected ($Br << 1$); the experimental conditions were: $1.6\times10^{-3} \le Br \le 7.8\times10^{-2}$;
- The thermophysical properties of the fluid were almost constant between entrance and exit of the test section for each experimental test. The maximum variation between inlet and outlet bulk temperatures was 6.9ºC, with a mean variation of 2.8º for all experimental tests, which indicated small fluctuations of thermal properties;
- Natural convection effects could be neglected, which is justified by the nearly constant values of density between tube inlet and outlet for all experimental tests;
- The flow behavior index presented a mean value for all experiments of 0.855 and a standard deviation of 0.004, while the consistency index ranged from 0.10 to 2.3 Pa s^n;
- The ranges tested were: $130.6 \le Gz \le 34000$, $20.3 \le L/D_h \le 87.6$ and $0.188 \le \kappa\,(D_1/D_2) \le 0.50$.

An equation similar to that proposed by Metzner et al. (1957) for pseudoplastic fluids flowing in circular ducts (Eq. 6) was tested using the hydraulic diameter (D_h) instead of the pipe diameter D. The adjustment resulted in Eq. (10), but in spite of the R^2 value of 0.949, experimental data lay systematically above the prediction curve.

Still based on Metzner's correlation, another expression was proposed, in which an empirical factor was added to account for the effect of the radius ratio κ (D_1/D_2) of the annulus. The final proposed correlation for arbitrary values of κ ranging from 0.188 to 0.50 is given by Eq. (6), and was based on the fitting models given by Levenspield (1993) for flow of Newtonian fluids through an annulus. The statistical results were quite satisfactory: R^2 = 0.960 and SSR = 8142. The experimental results, along with predictions of Eq. (11), are presented in Fig. 4.

$$Nu_m = 1.48\,\delta^{1/3}\,Gz^{0.33}\left(\frac{K}{K_0}\right)^{0.14} \qquad (10)$$

$$Nu_m = 2.38\,\delta^{1/3}\,Gz^{0.33}\left(\frac{D_1}{D_2}\right)^{0.33}\left(\frac{K}{K_0}\right)^{0.14} \qquad (11)$$

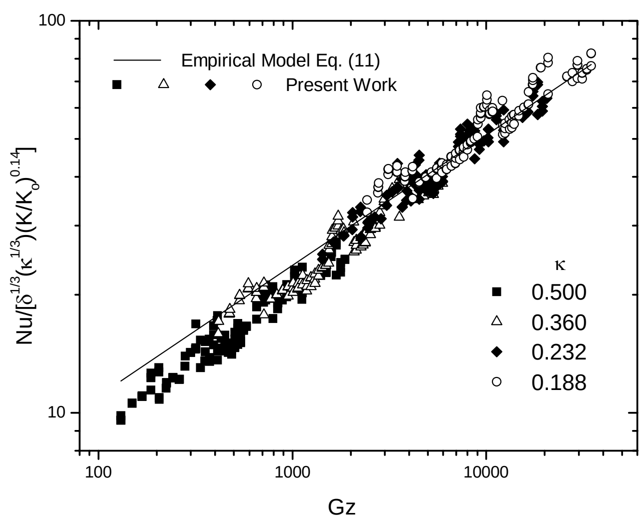


Figure 4: Average Nusselt numbers for egg yolk in annular duct: experimental and predicted by Eq. (11).

Little effort has been undertaken in collecting experimental data on heat transfer to Newtonian fluids in annuli for the laminar and turbulent regions. Some correlations can be found in Jakob (1949), which are generally given in terms of the aspect annulus ratio and the Graetz number, or the Reynolds and Prandtl numbers. To the authors' knowledge, just one correlation exists to determine the mean convective heat transfer coefficient for Power Law fluids flowing in concentric annular ducts, and very little experimental data are available. Tanaka and Mitsuishi (1975) obtained some experimental results for aqueous CMC solutions, aqueous PEO (polyethylene oxide) solutions and aqueous SPA (sodium polyacrylate) solutions flowing through annuli with κ = 0.354 and 0.746. They noticed that, since the Power Law fluid is generally highly viscous, the value of Gr/Re is small. Based on their numerical results, they proposed an expression for the Nusselt number, which was a function of κ, n, Re, Gz and Gr; their experimental results and the proposed equation agree within 15 %. In the experimental tests of Nouar et al. (2000) with CMC aqueous solutions, the variation range of the working conditions was: $200 \le Pr \le 1520$, $18 \le Re \le 131$ and $233 \le Gr \le 22,350$. The two cylinders of the test cell were heated by passing an electrical current through the tube wall, and their test section was covered by a thick polyurethane layer. Unfortunately, they did not express their results in terms of the average Nusselt number, and also did not provide a simple relationship for estimating it. They were focused on the effect of the variation of the rheological properties and that of the buoyancy force on the flow structure. They found that, in the case of forced convection coupled with a consistency variation with temperature, the flow structure is characterized by a displacement of fluid particles from the core region towards the heated wall. Then, the wall velocity gradient increases and the axial velocity in the central zone of the annular gap decreases to maintain the flow rate conservation; the deformation rate of the axial velocity profile decreases along the heating zone. As a matter of fact, in the boundary layer, the flow is quickly accelerated from the entrance, then the acceleration declines downstream. Therefore, they concluded that forced convection is the dominant mechanism near the entrance and the core flow is decelerated because of the decrease of the consistency index K as the temperature increases near the heated walls. In addition, they showed that the thermo-dependency effect of K is more characterized with increased shear-thinning of the fluid.

CONCLUSIONS

Heat transfer to laminar, hydrodynamically fully developed flow of a pseudoplastic fluid – egg yolk – in the thermal entrance of circular and annular ducts with constant wall temperatures had been experimentally investigated. The average convection heat transfer coefficients, as well as the average Nusselt numbers, were obtained by measuring inlet and outlet temperatures during heating of liquid egg yolk in the ducts test sections. For circular ducts, the experimental data were well correlated by empirical equations (Eqs. 8 and 9) and by the classic solution proposed by Metzner et al. (1957) for the thermal-entry and fully developed laminar flow of Power Law fluids. For annular regions, a simple empirical expression was developed based on the work of Metzner et al. (1957) and by adding an empirical factor to account for the effect of the radius ratio κ (D_1/D_2) of the annulus (Eq. 11). The new correlation, developed for design purposes of heat transfer devices, such as the triple-pipe or double-pipe heat exchangers, was obtained in the following ranges: $130.6 \le Gz \le 34000$, $42.8 \le Re_{mr} \le 2263.4$, $20.3 \le L/D_h \le 87.6$, $0.188 \le \kappa\ (D_1/D_2) \le 0.50$, $0.10 \le K \le 2.3$ and $n = 0.855 \pm 0.006$.

NOMENCLATURE

Br	Brinkman number	dimensionless, $Br = \dfrac{R^{(1-n)}\overline{v}_z{}^{(n+1)}K}{k(T_0 - T_{b1})}$
C_p	Specific heat	$J.kg^{-1}.^oC^{-1}$
$\dfrac{dv_z}{dr}$	Local shear rate	s^{-1}
D	Inside diameter of a circular tube	m
D_h	Hydraulic diameter	m
D_1	Inner diameter of an annulus	m
D_2	Outer diameter of an annulus	m
Gz	Graetz number	dimensionless
h_{ln}	average heat transfer coefficient based on the logarithmic mean temperature difference	$W.m^{-2}.^oC^{-1}$
k	Thermal conductivity	$W.m^{-1}.^oC^{-1}$
k_w	Thermal conductivity of pure water	$W.m^{-1}.^oC^{-1}$
K	Consistency index	$Pa.s^n$

L	Tube length	m
n	Flow behavior index	dimensionless
Nu_m	Average Nusselt number	dimensionless
Pe	Péclet number	dimensionless, $Pe = Re_{mr}\,Pr_g$
Pr_g	Prandtl number	dimensionless
R	Internal radius of a circular tube	m
R	Universal gas constant, R = 8.31451	J/mol·ºC
Re_{mr}	Metzner-Reed Reynolds number	dimensionless
T	Temperature	ºC or K
T_{b1}, T_{b2}	Inlet and outlet bulk temperatures of the fluid	ºC
T_0	Tube wall temperature	ºC
$\bar{v}_z$	Average axial flow velocity	$m.s^{-1}$
x_{ss}	Content of soluble solids	ºBrix
$\dot{m}$	Mass flow rate	$kg.s^{-1}$
z	Dimensional axial coordinate in a cylindrical system	m
z_{te}^*	Dimensionless thermal-entry length,	$z_{te}^* = z/RPe$

Greek Letters

δ	Ratio of the velocity gradients at the walls of a circular tube	
ρ_w	Logarithmic mean temperature difference	ºC
κ	Annulus aspect ratio	
ρ	Fluid density	$kg.m^{-3}$
ρ_w	Density of pure water	$kg.m^{-3}$
τ_{rz}	Local shear stress	Pa

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