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INCLUDING MODEL UNCERTAINTY IN THE MODEL PREDICTIVE CONTROL WITH OUTPUT FEEDBACK

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Abstract - This paper addresses the development of an efficient numerical output feedback robust model predictive controller for open-loop stable systems. Stability of the closed loop is guaranteed by using an infinite horizon predictive controller and a stable state observer. The performance and the computational burden of this approach are compared to a robust predictive controller from the literature. The case used for this study is based on an industrial gasoline debutanizer column.
Keywords: Predictive Control, Robust Stability, Infinite Horizon.

INTRODUCTION

Model predictive control (MPC) is the most popular advanced control technique in the process industry. For more details about the wide acceptance of MPC see the survey paper by Garcia et al. (1989). In industrial applications, production of a stable response is required despite changes in the operating conditions and model uncertainties. This issue is referred to as robust stability and has received considerable attention in academic research, as can be verified in the reviews by Mayne et al. (2000) and Morari and Lee (1999). From the point of view of industrial application, the review papers by Qin and Badgwell (1997, 2000) emphasize the lack of a practical industrial solution to the robust MPC problem.

In the robust MPC literature, the work of Badgwell (1997), which proposed extension of the nominal infinite horizon MPC (IHMPC) of Rawlings and Muske (1993) to the robust case, can be cited. Robustness is achieved by the inclusion of contracting constraints in the objective function corresponding to each model of the set of possible process models considered by the controller. Another contribution to the robust design of MPCs was made by Lee and Cooley (2000), who proposed a robust infinite horizon MPC for stable and integrating systems with model uncertainties on the input distribution matrix. In this case, the infinite prediction horizon cost becomes equal to the output error at the end of the control horizon. The paper by Kothare et al. (1996), which presented a robust MPC based on the LMI framework, can be also cited. An infinite prediction horizon is used, and the authors propose a method that minimizes an upper bound for the objective function.

At this point, it should be stressed that all these approaches can guarantee robust stability only for the regulator problem, since these authors assume that the state tends to zero when time tends to infinity. Here, a new robust infinite horizon MPC that was recently presented in the literature (Rodrigues and Odloak, 2001) is extended to the output feedback case. The controller is more general than the existing controllers qualified as robust, since

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it applies to both the regulator and the target tracking problems. The approach is based on a particular realization of the system state space model (Rodrigues, 2001; Tvrzská de Gouvêa and Odloak, 1997). This model realization allows the direct integration of the squared error of the system output so that the cost function of the controller can be defined in a more suitable form. Following this strategy, equality constraints, which ensure that the infinite horizon cost function is bounded, naturally arise. Model uncertainties are approximated by a finite set of disjunct linear models.

This paper is organized as follows: In Section 2 the state space model used in this paper is briefly introduced. In Section 3 the development of a modified nominal IHMPC is presented and in Section 4 these results are extended to the robust case. Simulation results are presented in Section 5, where the proposed controller is compared to a robust MPC presented in a previous paper. In Section 6 some concluding remarks are offered.

MODEL STRUCTURE

In this paper, it is assumed that the system is represented by the following linear discrete-time model:

$$[x]_{k+1} = A\,[x]_k + B\Delta u_k \quad (1)$$

$$y(k+t) = C(t)[x]_k \quad (2)$$

where $x \in \mathbb{R}^{nx}$ is the vector of states, $u \in \mathbb{R}^{nu}$ is the vector of inputs, $\Delta u_k = u_k - u_{k-1}$ is the process input increment, k is the present time step and $y \in \mathbb{R}^{ny}$ is the vector of outputs. Here the model representation designated output prediction oriented model (OPOM), which was proposed by Rodrigues (2001), is adopted. Using OPOM, the model matrices involved in eqs. (1) and (2) have the following structure:

$$[x] = \begin{bmatrix} x^s \\ x^d \end{bmatrix}, \qquad A = \begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix}, \qquad B = \begin{bmatrix} D^0 \\ D^d F N \end{bmatrix}, \quad (3)$$

$$C(t) = \begin{bmatrix} I & \Psi(t) \end{bmatrix}, \qquad nx = ny(1 + nu.na)$$

The state vector, *x* was partitioned into two auxiliary vectors, x^s and x^d. For stable systems, component $x^s \in \mathbb{R}^{ny}$ corresponds to the predicted value for the system output at steady state. Vector $x^d \in \mathbb{C}^{nd}$ is related to the dynamic modes of the system and *na* is the order of the respective transfer function model. Matrix D^0 contains the steady state gains of the process and matrix $\Psi(t)$ is shown below. More details on the OPOM formulation can be seen in Rodrigues (2001) and Rodrigues and Odloak (2002).

$$\Psi(t) = \begin{bmatrix} \Phi_1(t) & 0 & \cdots & 0 \\ 0 & \Phi_2(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_{ny}(t) \end{bmatrix} \quad (4)$$

$$\Phi_i(t) = \begin{bmatrix} e^{r_{i,1,1} t} \cdots e^{r_{i,1,na} t} \cdots \\ \\ r_{i,nu,1} t \cdots e^{r_{i,nu,na} t} \end{bmatrix} \qquad i = 1,\ 2, \cdots, ny$$

where

$$\Psi(t) \in \mathbb{C}^{ny \times nd}, \qquad \Phi_i \in \mathbb{C}^{nd}, \qquad r_{i=1,\ldots,nd}$$

are the poles of the system, and $nd = ny.nu.na$.

INFINITE HORIZON MPC (IHMPC)

An important feature of OPOM is that time appears continuously in the output prediction mapping. This peculiarity was used by Rodrigues (2001) to propose a modified infinite horizon MPC, whose main steps are described in the sequel.

Consider a modified expression for the objective function of MPC as given by (5):

$$J_{k,\infty} = \sum_{n=1}^{m} \int_{(n-1)T}^{nT} \{e_{k+t} + \delta_k\}^T Q \{e_{k+t} + \delta_k\} dt + \\ + \int_{mT}^{\infty} \{e_{k+t} + \delta_k\}^T Q \{e_{k+t} + \delta_k\} dt + \\ + \Delta u^T R_1 \Delta u + \delta_k^T S \delta_k \quad (5)$$

$e_{k+t} = y(k+t) - y^{sp}$, $y^{sp} \in \mathbb{R}^{ny}$ is the vector of output reference values; *m* is the control horizon; T is the length of the sampling time, and $Q \in \mathbb{R}^{ny \times ny}$,

$R \in \mathbb{R}^{nu \times nu}$ and $S \in \mathbb{R}^{ny \times ny}$ are positive definite weighting matrices. $\delta_k \in \mathbb{R}^{ny}$ is the vector of slack variables that was introduced into the IHMPC objective function to allow for processes with $ny > nu$. Inside the control horizon, the output error can be represented by the following expression:

$$e_{k+t} = [e^s]_k + \Psi(t)[x^d]_k + D_n^0 \Delta u + \Psi(t) W_n Z \Delta u \quad (6)$$

where

$(n-1)T \le t < nT$, $n \le m$

$[e^s]_k = [x^s]_k - y^{sp}$

$$D_n^0 = [\overbrace{D^0 \quad D^0 \quad \cdots \quad D^0}^{n} \quad 0 \quad \cdots \quad 0],$$

$D_n^0 \in \mathbb{R}^{ny \times m.nu}$

$W_n = [I \quad F^{-1} \quad \cdots \quad F^{-(n-1)} \quad 0 \quad \cdots \quad 0]$,

$W_n \in \mathbb{C}^{nd \times nd.m}$

$$Z = \begin{bmatrix} D^d N & 0 & \cdots & 0 \\ 0 & D^d N & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D^d N \end{bmatrix}, \quad Z \in \mathbb{C}^{m.nd \times m.nu},$$

$$\Delta u = \begin{bmatrix} \Delta u_k^T & \Delta u_{k+1}^T & \cdots & \Delta u_{k+m-1}^T \end{bmatrix}^T$$

Substituting (6) into the first term on the right-hand side of (5), one obtains

$$\begin{aligned} &\sum_{n=1}^{m} \int_{(n-1)T}^{nT} \{e_{k+t} + \delta_k\}^T Q \{e_{k+t} + \delta_k\} dt = \\ &\Delta u^T \sum_{n=1}^{m} \{D_n^{0T} Q D_n^0 T + 2 D_n^{0T} Q (G_1(n) - G_1(n-1)) W_n Z + Z^T W_n^T (G_2(n) - G_2(n-1)) W_n Z\} \Delta u \\ &+2 \sum_{n=1}^{m} \{[e^s + \delta]_k^T Q (D_n^0 T + (G_1(n) - G_1(n-1)) W_n Z)\} \Delta u \\ &+ \sum_{n=1}^{m} \{[x^d]_k^T [(G_1(n) - G_1(n-1))^T D_n^0 + (G_2(n) - G_2(n-1)) W_n Z]\} \Delta u \end{aligned} \quad (7)$$

where

$$G_1(n) = \int_0^{nT} \Psi(t) dt \text{ and}$$

$$G_2(n) = \int_0^{nT} \Psi(t)^T Q \Psi(t) dt.$$

When the time t is within the interval $mT \le t < \infty$, the error of the process output can be described by

$$[e_{k+t}]_k = [e^s]_k + \Psi(t)[x^d]_k + D_m^0 \Delta u + \Psi(t) W_m Z \Delta u \quad (8)$$

Substituting (8) into the infinite integral on the right-hand side of (5), one obtains:

$$\begin{aligned} &\int_m^{\infty} [e_{k+t} + \delta_k]^T Q [e_{k+t} + \delta_k] dt = \{[e^s]_k + \delta_k + D_m^0 \Delta u\}^T Q \{[e^s]_k + \delta_k + D_m^0 \Delta u\} \int_{mT}^{\infty} dt \\ &+2 \{[e^s]_k + \delta_k + D_m^0 \Delta u\}^T Q \int_{mT}^{\infty} \Psi(t) \{[x^d]_k + W_m Z \Delta u\} dt \\ &+ \int_{mT}^{\infty} \{x^d]_k + W_m Z \Delta u\}^T \Psi(t)^T Q \Psi(t) \{[x^d]_k + W_m Z \Delta u\} dt \end{aligned} \quad (9)$$

For stable systems $\lim_{t\to\infty}\Psi(t)=0$ and consequently the second and third terms on the right-hand side of (9) are bounded. However, the first term on the right-hand side of (9) will be bounded only if the equality below holds:

$$[e^s]_k+\delta_k+D_m^0\Delta u=0 \tag{10}$$

Introducing (10) into (9), results in

$$\int_m^{\infty}[e_{k+t}+\delta_k]^T Q[e_{k+t}+\delta_k]dt=$$

$$=\{[x^d]_k+W_m Z\Delta u\}^T \tag{11}$$

$$[G(\infty)-G(m)]\{[x^d]_k+W_m Z\Delta u\}$$

ROBUST INFINITE HORIZON MPC (RIHMPC)

In this section, the modified infinite horizon MPC presented in the previous section is extended to the case of model uncertainty. When the stabilizing control law takes into account model uncertainties, the resulting controller is designated robust. It is well known that a single linear model can not accurately represent the actual plant under different operating conditions. Recently, the multimodel representation of uncertain processes has been introduced in the robust MPC literature (see Badgwell, 1997 and Rodrigues, 2001). This model mismatch representation considers a finite set of disjunct linear systems (Ω), which are obtained at different points of the operating domain, or better:

$$(A,B)\in\Omega \mid \Omega:=\{(A_1,B_1)\ \cdots\ (A_L,B_L)\} \tag{12}$$

which means that the matrices of the model described by (1) and (2) are not exactly known, but belong to Ω.

Considering this kind of uncertainty and the developments presented in Section 3, Rodrigues (2001) proposed a robust infinite horizon MPC for the case where all state variables of the process can be measured (state feedback). However, during actual plant operation only a subset of the model states, the outputs, is measured. One way to extend the referred robust IHMPC to the output feedback case is based on the separation principle (Levine, 1996). Briefly, the separation principle consists in splitting the feedback control design into two steps. In the first step, a full-state feedback controller is designed assuming that all the model states can be measured, and the control sequence is calculated using these model states. In the second step, an observer is introduced to estimate the state of the system based upon the system outputs. This framework was employed here to design the robust output feedback MPC. In the simulations presented here the following sequence is adopted:

a) One model of Ω (the most probable) is assumed to be the true plant. The output of this model is designated $[y_p]_k$.

b) The states of the remaining models are estimated as follows:

$$[x_i]_{k+1|k+1}=A_i[x_i]_{k|k}+B_i\Delta u_k+K_F\{[y_p]_k-$$

$$-C_i(A_i[x_i]_{k|k}+B_i\Delta u_k)\}=(I-K_FC_i)A_i[x_i]_{k|k}+ \tag{13}$$

$$+(I-K_FC_i)B_i\Delta u_k+K_F[y_p]_k \qquad i=1,\ldots,L-1$$

c) The observer used here has a static gain given by:

$$K_F=[I\quad 0]^T,\ K_F\in\mathbb{R}^{nd\times ny} \tag{14}$$

This observer is based on the assumption that the difference between model output and plant measurement is produced by a step disturbance in the system output. This assumption is adopted by most MPC packages but is restricted to stable systems, which is the case considered here.

Therefore, assuming that the uncertain process belongs to Ω, one estimates the states of all the linear models lying in Ω using (13) and (14). Substituting (7) and (11) into (5), one can write the following optimization problem for RIHMPC:

Problem P1)

$$\min_{\Delta u,\delta_i,\cdots,\delta_L,\gamma}\ \gamma$$

subject to

$$\begin{bmatrix} I & \sqrt{H_i}Y_i \\ Y_i^T\sqrt{H_i} & \gamma-2c_{f,i}^T Y_i \end{bmatrix}>0,\qquad i=1,\ldots,L \tag{15}$$

$$\Delta u_{k+j} \in U, \; j \geq 0 \tag{16}$$

$$[e_i^s]_k + [\delta_i]_k + D_{m,i}^0 \Delta u = 0, \; i = 1, \ldots, L \tag{17}$$

where

$$Y_i^T = \begin{bmatrix} \Delta u^T & [\delta_i]_k^T \end{bmatrix},$$

$$H_i = \begin{bmatrix} H_{i_{11,m}} + H_{i_{11,\infty}} + R_1 & H_{i_{12}} \\ H_{i_{12}}^T & mQT + S \end{bmatrix}$$

$$c_{fi}^T = \begin{bmatrix} c_{fi_m}^T + c_{fi_\infty}^T & c_{fi_\delta}^T \end{bmatrix}$$

$$H_{i_{11,m}} = \sum_{n=1}^{m} \{D_{n,i}^{0\;T} Q D_{n,i}^0 T + 2 D_{n,i}^{0\;T} Q [G_{1,i}(n) - G_{1,i}(n-1)] W_{n,i} Z_i\} + \sum_{n=1}^{m} \{Z_i^T W_{n,i}^T [G_{2,i}(n) - G_{2,i}(n-1)] W_{n,i} Z_i\}$$

$$H_{i_{11,\infty}} = Z_i^T W_{m,i}^T [G_{2,i}(\infty) - G_{2,i}(m)] W_{m,i} Z_i$$

$$H_{i_{12}} = \sum_{n=1}^{m} \{D_{n,i}^{0\;T} QT + Z_i^T W_{n,i}^T [G_{1,i}(n) - G_{1,i}(n-1)]^T Q\}$$

$$c_{fi_m}^T = \sum_{n=1}^{m} \{[e_i^s]_k^T Q [D_{n,i}^0 T + (G_{1,i}(n) - G_{1,i}(n-1)) W_{n,i} Z_i]\} + \sum_{n=1}^{m} \{[x_i^d]_k^T [G_{1,i}(n) - G_{1,i}(n-1)]^T Q D_{n,i}^0 + [G_{2,i}(n) - G_{2,i}(n-1)] W_{n,i} Z_i\}$$

$$c_{fi_\infty}^T = [x_i^d]_k^T [G_{2,i}(\infty) - G_{2,i}(m)] W_{m,i} Z_i,$$

$$c_{fi_\delta}^T = m [e_i^s]_k^T QT + [x_i^d]_k^T \sum_{n=1}^{m} [G_{1,i}(n) - G_{1,i}(n-1)]^T Q, \qquad R_1 = \mathrm{diag}(R \;\; \cdots \;\; R)$$

U defines the set of feasible solutions related to the input constraints. Problem P1 can be solved by the available methods of linear matrix inequalities (LMI). The theorem below summarizes the robust stability of the RIHMPC with output feedback defined by Problem P1).

Theorem:

Consider an open-loop stable process where the true plant model belongs to Ω and whose states are estimated by a stable observer. If there is a feasible solution to Problem P1), then the resulting control law stabilizes all the plants belonging to Ω.

Proof:

The proof for the state feedback case can be found in Rodrigues and Odloak (2002). It remains to be proved that when a stable state observer is used the resulting closed loop remains stable. It is easy to verify that the observer gain defined by (14) corresponds to $|\lambda(I - K_F C_i) A_i| < 1, \; i = 1, \ldots, L$. Thus, as a consequence of the separation principle, when one uses IHMPC, which stabilizes all the models belonging to set Ω when the states are perfectly known, and a stable state observer, the resulting closed-loop will also be stable.

COMPARING RIHMPC TO A ROBUST MPC FROM THE LITERATURE

Rodrigues and Odloak (2000) proposed a finite horizon robust output feedback MPC, which was named RSMPC. The approach is based on the parameterization of the control moves by an output feedback scheme similar to the conventional linear quadratic regulator. In this way, it was possible to use the quadratic Lyapunov stability condition that was included in MPC as an additional constraint. In their MPC, these authors considered a polytopic representation of the uncertain models. The multimodel version of the Rodrigues and Odloak (2000) robust predictive controller is described below:

Problem P2)

$$\min_{\gamma, K_m, P_1, \ldots, P_L} \gamma$$

subject to

$$\begin{bmatrix} \left(\bar{B}^T \bar{Q} \bar{B} + \bar{R}\right)^{-1} & K_m [e_k]_k \\ [e_k]_k^T K_m^T & \begin{matrix} \gamma + [e_k]_k^T A^T \bar{Q} \bar{B} K_m [e_k]_k + \\ +[e_k]_k^T K_m^T \bar{B}^T \bar{Q} A [e_k]_k \end{matrix} \end{bmatrix} > 0 \quad (18)$$

$$\begin{bmatrix} P_i & P_i (A_i + B_i N_s K_m C) \\ (A_i + B_i N_s K_m C)^T P_i & P_i \end{bmatrix} > 0, \quad (19)$$

$i = 1, \ldots, L$

$$\gamma > 0, \; P_i = P_i^T > 0, \; i = 1, \ldots, L \quad (20)$$

where

$$\bar{B} = \begin{bmatrix} S_2 & S_1 & 0 & 0 \\ S_3 & S_2 & \ddots & \vdots \\ \vdots & \vdots & \vdots & S_1 \\ S_{n_p} & S_{n_p-1} & \cdots & \vdots \\ S_{n_e} & S_{n_e-1} & \cdots & S_{n_e-m+1} \\ D^0 & D^0 & \cdots & D^0 \\ D^d FN & D^d FN & \cdots & D^d FN \end{bmatrix},$$

$$N_s = \begin{bmatrix} I_{nu} & 0 \end{bmatrix}, \qquad N_s \in \Re^{[nu]x[nu.m]}$$

S_i is the step response coefficient matrix at sampling step i

K_m is the feedback gain of the controller

Rodrigues and Odloak (2000) proposed an iterative algorithm based upon LMI tools to solve Problem P2). It is important to observe that matrices A and $\bar{B}$ of RSMPC are related to the augmented system (parameters + outputs at prediction steps). Consequently, the dimension of the state *x* of this controller is different from the dimension of the state considered by RIHMPC.

SIMULATION RESULTS

The process simulated in this work is a debutanizer column of an oil refinery, and it was borrowed from Rodrigues and Odloak (2000). This column produces LPG (liquefied petroleum gas) as the top stream and stabilized gasoline as the bottom stream. The controlled outputs are the concentration of $C5^+$ in the top stream (y_1) and the Reid vapor pressure of the bottom stream (y_2). The manipulated inputs are the top reflux flow rate (u_1) and the reboiler heat duty (u_2). The set of uncertain models, Ω, was built by linear models identified at three different operating points of the column. These models can be seen in Table 1. The most probable model is assumed to be $G_1(s)$.

RIHMPC was simulated using $G_1(s)$ as the true plant, which together with the other two models constitutes the set Ω. The tuning parameters of RIHMPC are

$T = 1$; $m = 2$; $Q = \text{diag}(1,1)$;

$R = \text{diag}(10^{-2}, 10^{-2})$; $S = \text{diag}(100,100)$;

$u_{j,max} = 15$; $u_{j,min} = -21$ and $\left|\Delta u_{j,max}\right| = 5$.

RSMPC was simulated using the same set of models as RIHMPC and the following tuning parameters:

$T = 1$; $m = 2$; $Q = \text{diag}(1,1)$;

$R = \text{diag}(10^{-2}, 10^{-2})$; $u_{j,max} = 15$; $u_{j,min} = -21$;

$\left|\Delta u_{j,max}\right| = 5$, n_p =3; n_e = 75; ε = 0.1

where n_p is the number of equally spaced system output prediction steps, n_e is the extra prediction step included in the computation of output error and ε is the convergence parameter of RSMPC.

Figure 1 shows the results of RIHMPC and RSMPC for the models and tuning parameters described above. The case of output tracking where the reference values of the two outputs were simultaneously increased is considered. It can be observed that for controlled output y_1 ($C5^+$ in the LPG stream), RIHMPC has a better performance than RSMPC. For y_2 the performance of RIHMPC is quite similar to that of RSMPC. The ratio between the computational time expended to run one sample step and the sampling time length (CPU Time/*T*) was 0.0022 for RIHMPC and 10.4208 for RSMPC at a COMPAQ XP1000 workstation (512 MB RAM).

Table 1: Models of the debutanizer column.

$G_1(s)$:	$G_2(s)$:	$G_3(s)$
$\begin{bmatrix} \frac{-0.2623}{60s^2+59.2s+1} & \frac{0.1368}{1164s^2+99.7s+1} \\ \frac{0.1242}{218.7s^2+16.2s+1} & \frac{-0.1351}{70s^2+20s+1} \end{bmatrix}$	$\begin{bmatrix} \frac{-0.3544}{218.6s^2+59.2s+1} & \frac{0.2044}{1150.2s^2+93.86s+1} \\ \frac{0.0685}{100.2s^2+11.32s+1} & \frac{-0.1256}{20s^2+15s+1} \end{bmatrix}$	$\begin{bmatrix} \frac{-0.2790}{59.77s^2+99.61s+1} & \frac{0.050}{499.8s^2+73.77s+1} \\ \frac{0.1950}{220.1s^2+18.93s+1} & \frac{-0.1722}{29.74s^2+20.71s+1} \end{bmatrix}$

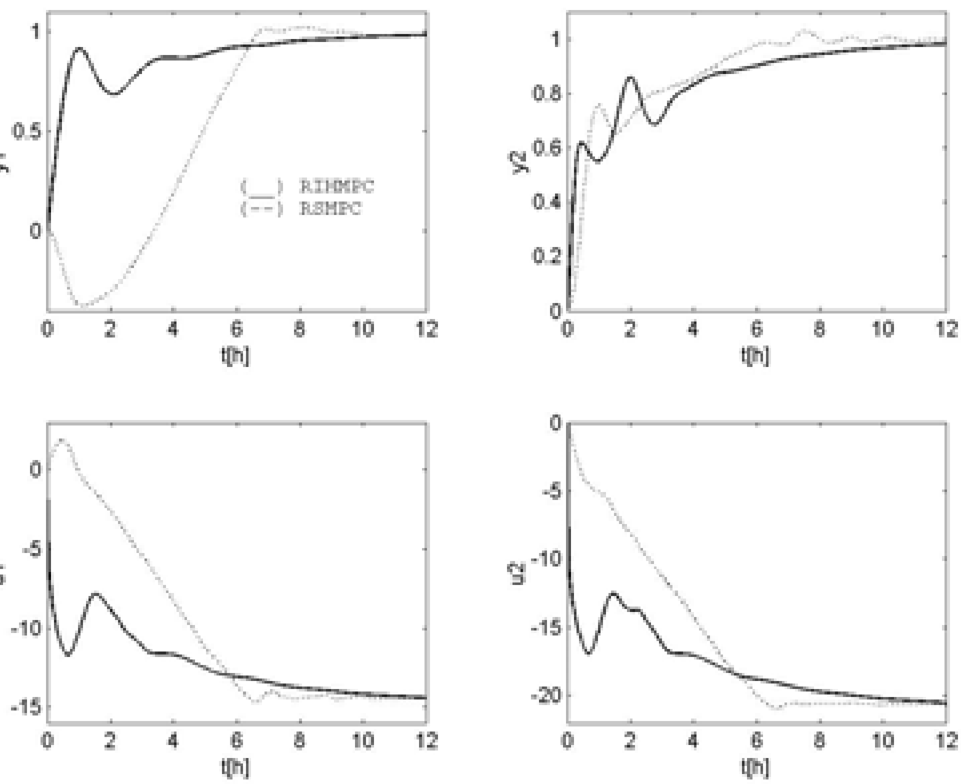

Figure 1: Input and output profiles for RIHMPC and RSMPC for the debutanizer column.

CONCLUSION

In this paper an extension of a highly numerical efficient robust infinite horizon MPC was developed for the output feedback case. A case study using an industrial process was carried out in order to compare the proposed controller to another robust MPC presented earlier in the MPC literature. The simulation results showed that RIHMPC has a very low computational burden compared to RSMPC. Therefore, it can be concluded that RIHMPC is appealing for industrial applications. Although RIHMPC is a controller where a min-max problem is solved, and consequently the worst case performance is considered, its closed loop performance is better then the performance of RSMPC whose control law is based on the most probable system model. Consequently, inclusion of the Lyapunov stability condition in the MPC problem can make the controller more conservative than the controller based on the min-max approach. The main advantage of RSMPC over RIHMPC is that Problem P2) may be convex in the model matrices if the Lyapunov matrix P_i is the same for all the models lying in Ω. In this case, stability is guaranteed for any plant whose model is a convex combination of the models of Ω. This is not true for RIHMPC, since Problem P1) is not convex and stability is only guaranteed for the plants lying in Ω.

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