

RELATIONSHIP OF LAMBDA AND OVERSHOOT OF STEP RESPONSE FOR A DIRECT SYNTHESIS PI CONTROLLER

L. W. Tan, R. K. Raja Ahmad, M. N. Ibrahim and F. S. Taip*

Department of Process and Food Engineering, Department of Electrical and Electronic Engineering, Faculty of Engineering, Phone: + 603-89466357, Fax: + 603-89464440, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia.
E-mail: saleena@eng.upm.edu.my

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Abstract - The direct synthesis (DS) tuning method is a model-based method for a feedback controller. The main principle of this method is to obtain the controller settings based on a predetermined desired closed loop response. The main advantage is that there is only one parameter to be adjusted, which is lambda (λ), the speed of the desired closed loop response. There are several guidelines available for selecting λ in order to ensure that the closed loop step response matches the desired response. In this paper, a guideline that relates λ and overshoot is proposed and it worked well over a wide range of R varying from 0.05 to 2. For a fair comparison of DS-tuned controllers with different λ guidelines, both performance and robustness for a unit step change in the set point are considered. It was found that the DS-tuned controller with this proposed guideline performed better and the gain margin (GM) and phase margin (PM) lie between $2 \leq GM \leq 5$ and $30^\circ \leq PM \leq 75^\circ$, respectively. Besides, its overshoot changed less with a $\pm 25\%$ process model mismatch, except for τ mismatch.

Keywords: Direct synthesis; Lambda; Overshoot; PI controller.

INTRODUCTION

PI and PID controllers were introduced by Ziegler and Nichols in 1942. Both controllers have been the heart of control engineering practice for seven decades. In industrial processes such as chemical processes, food processes, manufacturing processes and so on, more than 95% of the control tools are PI and PID controls (Åström and Hägglund, 1995; Dufour, 2006; Shamsuzzoha and Skogestad, 2010). They are adequate to be used in blending (Chua *et al.*, 2009), spray drying (Tan *et al.*, 2009), pH control (Valarmathi *et al.*, 2009), biochemical (Rodrigues and Maciel Filho, 1997) and other manufacturing processes.

However, 90% of PID controllers do not use derivative action because it is not simple and is time-consuming to optimize three controller settings (Majhi,

2005; Skogestad, 2003). Tuning is the engineering adjustment of the controller settings to achieve satisfactory control. Most of the tuning methods were proposed for first order plus time delay (FOPTD) systems, as they represented the behavior of a wide range of process (Madhuranthakam *et al.*, 2008). The tuning methods include closed-loop methods and model-based control methods. Closed loop methods do not require a process model and usually perform in a closed loop system (Abdul Majid *et al.*, 2009). Model-based control methods are based on the process model with the performance criteria established before the control system is installed (Haley and Mulvaney, 1995).

In some processes, it is desirable to have high overshoot and return rapidly to the steady state. However, high overshoot is a problem in several processes. For example, overshoot is not allowed in

*To whom correspondence should be addressed

plastic glove manufacturing to prevent plastic film wrinkle (Basilio and Matos, 2002). Several tuning methods focused on controller parameter adjustment in order to reduce the excessive overshoot to a given overshoot of a closed loop system. The Chien-Hrones-Reswick (CHR) method (Åström and Hägglund, 1995) was designed to reduce the overshoot to zero and 20% overshoot. Basilio and Matos (2002) proposed a tuning rule for a systematic adjustment of controller gain to produce a closed loop response without overshoot. Skogestad (2003) suggested a direct synthesis (DS) method with λ equal to a time delay in order to produce a response with 5% overshoot (Martin *et al.*, 1976). Abbas (1997) developed controller tuning relations relating the overshoot and model parameters by modifying the direct synthesis tuning method. In his work, a first-order Padé approximation for the time delay term was considered.

The DS method is one of the tuning methods for a feedback controller. The main principle of this method is to obtain the controller settings based on a predetermined closed loop response. The performance requirements of the output response can be incorporated directly through specification of the closed loop response. The main advantage of the DS method over other tuning methods is due to the fact that only one parameter, lambda (λ), needs to be adjusted (Coughanowr and LeBlanc, 2009; Foley *et al.*, 2005). The output response tuned by the DS method may be overdamped or underdamped, depending on the choice of λ . There are several guidelines available for selecting λ in order to ensure that the closed loop step response matches the desired response. Tan *et al.* (2006) mentioned that λ equal to 0.25 of the time delay had a better performance but was less robust. Smith and Corropio (2006) reported that λ should be set to 2/3 of the time delay approximately to minimize the integral absolute of error (IAE) of PI controller response. As mentioned, λ equal to the time delay gives a response with 5% overshoot (Martin *et al.*, 1976; Smith and Corropio, 2006). However, there is no λ guideline that lets users select the amount of overshoot to be produced in a step response.

In this work, a DS guideline that relates λ and overshoot (OS) and the ratio of time delay to time constant (R) is developed for a PI controller. The proposed guideline is more useful because it can produce the specified overshoot as selected by the user. The plant process with a first order process plus time delay (FOPTD) model is considered. For a fair comparison of DS-tuned controllers with the proposed λ guideline and other existing λ guidelines,

both performance and robustness for a unit step change in the set point are considered.

PROPOSED WORK

Direct Synthesis Tuning Method

For the feedback controller shown in Figure 1, the closed loop transfer function can be expressed by Equation (1). $\frac{Y}{Y_{sp}}$ is the closed loop transfer function, G_c is the transfer function of the controller and G_p represents the process transfer function.

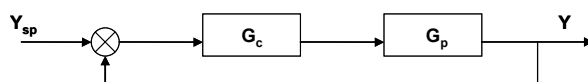


Figure 1: Closed loop feedback control system

$$\frac{Y}{Y_{sp}} = \frac{G_c G_p}{1 + G_c G_p} \quad (1)$$

A practical design equation can be obtained by replacing the unknown actual process, G_p , by the model \tilde{G}_p and $\frac{Y}{Y_{sp}}$ is replaced by the desired closed

loop transfer function, $\left(\frac{Y}{Y_{sp}}\right)_d$. The controller

performance strongly depends on the specification of the desired closed loop transfer function. Ideally, a perfect control is achieved if the desired closed loop transfer equals to 1, but this ideal situation cannot be achieved by feedback control. The desired closed loop transfer function for FOPTD is shown as Equation (2).

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{e^{-\theta s}}{\lambda s + 1} \quad (2)$$

Then the controller strategy is of the form:

$$G_c = \frac{1}{\tilde{G}_p} \left(\frac{\left(\frac{Y}{Y_{sp}}\right)_d}{1 - \left(\frac{Y}{Y_{sp}}\right)_d} \right) = \frac{1}{\tilde{G}_p} \frac{e^{-\theta s}}{\lambda s + 1 - e^{-\theta s}} \quad (3)$$

and let $\tilde{G}_p = \frac{K_p e^{-\theta s}}{\tau s + 1}$,

$$G_c = \frac{\tau s + 1}{K_p e^{-\theta s}} \cdot \frac{e^{-\theta s}}{\lambda s + 1 - e^{-\theta s}} = \frac{\tau s + 1}{K_p (\lambda s + 1 - e^{-\theta s})} \quad (4)$$

If the Taylor series expansion $e^{-\theta s} \approx 1 - \theta s$ is applied, a PI control with the settings shown in Equation (5) is derived.

$$G_c = \frac{\tau s + 1}{K_p (\lambda + \theta) s} = \frac{\tau}{K_p (\lambda + \theta)} \left(1 + \frac{1}{\tau s} \right) \equiv K_c \left(1 + \frac{1}{\tau_I s} \right) \quad (5)$$

Equation (5) shows that only the controller gain, K_c , depends on λ , the speed of the closed loop response. It is important to develop a λ guideline based on the process parameters that optimizes the closed loop response. Thus, an expression was developed for λ as a function of both the ratio of time delay to time constant, R , and overshoot, OS.

Simulations and Formula Formation

Simulations were carried out using SIMULINK in MATLAB software to find the relationship between lambda, λ , and overshoot of the step response, OS. Two process gains ($K_p=1$ and $K_p=2$), two time delays ($\theta=1$ and $\theta=2$) and twenty one values of the ratio of time delay to time constant ($R=0.05$ to 2) were explored. Each R had four groups of data, which were $K_p=1, \theta=1$; $K_p=2, \theta=1$; $K_p=1, \theta=2$; and $K_p=2, \theta=2$, and were run at five λ values of $0.2\theta, 0.4\theta, 0.6\theta, 0.8\theta$ and θ . Hence, in total, 420 runs were obtained. After the data were collected, a general correlation of OS and λ in terms of process parameters was obtained by using the Curve Fitting Toolbox in MATLAB software.

Simulation Studies and Analysis

In order to demonstrate the performance and robustness of the proposed λ guideline and existed λ guidelines such as 0.5τ (general, rule of thumb), 0.25θ (Tan *et al.*, 2006), $2/3\theta$ (Smith and Corropio, 2006) and θ (Skogestad, 2003), three FOPTD processes with different R values mentioned by Madhuranthakam *et al.* (2008) were involved. The effects of the uncertainty in the model were evaluated by applying a $\pm 25\%$ change from the nominal values. The performance was compared in terms of the overshoot, rise time, settling

time and integral of error measures (IAE and ISE). Gain margin (GM) and phase margin (PM) are two well-known measures for maintaining the robustness of a control system. The ranges of GM and PM in a well tuned controller ranged from $2 \leq GM \leq 5$ and $30^\circ \leq PM \leq 75^\circ$, respectively (Åström and Hägglund, 1995).

Model Uncertainties

The effects of model uncertainties were analyzed by applying a $\pm 25\%$ change in K_p, τ and θ .

RESULTS AND DISCUSSION

Correlation of Lambda (λ) and Overshoot (OS) with Process Parameters

From the simulations, it was found that the closed loop response gave similar OS values for the same λ , although the process gain or time constant was varied. Besides, λ is related to the ratio of time delay to time constant, R , and time delay, θ , but has no clear relation with process gain, K_p . As λ decreased, the PI controller had larger controller gain, K_c , and integral gain, K_I . Therefore, this resulted in a more oscillatory response (large overshoot), as expected. Data from the simulations were used to obtain a general correlation of λ and OS with the process parameters by using Curve Fitting Toolbox in MATLAB software. As mentioned, λ is related to R and θ and an exponential relationship between OS and $\lambda / R\theta$ was obtained. Figure 2 shows a graph of OS versus $\lambda / R\theta$ for $R=1$ and Equation (6) shows the exponential relationship between OS and $\lambda / R\theta$.

$$OS = a \left(\frac{\lambda}{R\theta} \right)^b + c \quad (6)$$

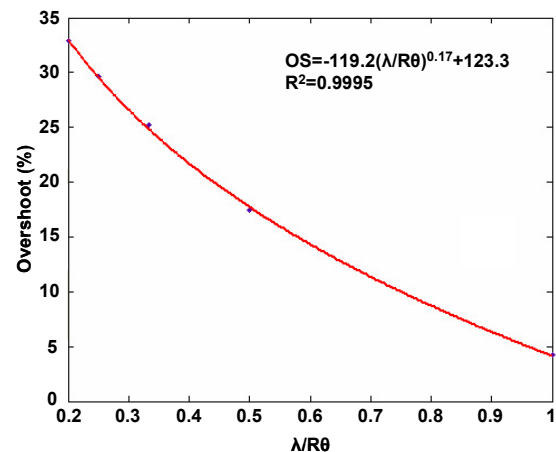


Figure 2: A graph of OS versus $\lambda / R\theta$ for $R=1$

Table 1 provides the values of the parameters a, b and c according to the goodness of fit curve. The coefficient of determination (R^2) is a measure of the degree of fit. The sum of square error (SSE), adjusted R^2 , and the square root of mean square error (RMSE) can be used to compare the goodness of fit of the curve. Table 1 shows that all curves obtained had a high R^2 and a small RMSE. In order to define λ , rearranging Equation (6) gives Equation (7) (limited to $0.05 \leq R \leq 2$).

$$\lambda = R\theta \left(\frac{OS - c}{a} \right)^{\frac{1}{b}} \quad (7)$$

Table 2 shows the parameters a, b and c grouped according to R. There were two formulas derived for the parameters a and c for $0.05 \leq R < 1$ and $1 \leq R \leq 2$. Parameter b was 0.18, the value obtained from the

average after accumulation of numerous data. The accuracy of the proposed λ guideline (Equation (7)) was tested. It was found that overshoot can be specified from 3% to 10% instead of 1% and 2%. The main reason is that it is difficult to obtain the simulated overshoot if the overshoot is too small. If the use of this equation is limited to $4\% \leq OS \leq 10\%$, then the percentage error in the specified value of OS is less than $\pm 10\%$.

Simulation Results and Analysis

The proposed λ formula with the chosen specification of 5% OS was compared with other conventional λ formulas such as 0.5τ , θ , $2/3\theta$ and 0.25θ using a PI controller. The aim of the simulation studies was to test whether the proposed λ guideline could reduce the excessive overshoot to 5% with satisfactory performance and robustness.

Table 1: Parameters a, b and c and the goodness of fit of the curves.

R	Parameter			The goodness of fit curve			
	a	b	c	R^2	SSE	Adjusted R^2	RMSE
0.05	-65.2	0.18	117.5	0.9994	0.2593	0.9990	0.3601
0.1	-75.1	0.18	118.6	0.9996	0.2352	0.9991	0.3429
0.2	-89.4	0.18	122.6	0.9995	0.2483	0.9991	0.3524
0.3	-91.8	0.18	118.6	0.9994	0.3094	0.9988	0.3933
0.4	-96.8	0.18	118.6	0.9997	0.1756	0.9994	0.2963
0.5	-100.8	0.18	118.5	0.9994	0.3109	0.9988	0.3943
0.6	-104.7	0.18	119.0	0.9993	0.3489	0.9987	0.4045
0.7	-110.7	0.18	122.0	0.9996	0.2214	0.9992	0.3327
0.8	-113.9	0.17	122.6	0.9995	0.2489	0.9991	0.3528
0.9	-115.5	0.18	121.8	0.9996	0.2222	0.9992	0.3333
1.0	-119.2	0.18	123.3	0.9995	0.2876	0.9989	0.3792
1.1	-121.2	0.18	123.3	0.9995	0.2905	0.9989	0.3811
1.2	-122.5	0.18	122.6	0.9997	0.1389	0.9995	0.2635
1.3	-124.4	0.18	123.0	0.9995	0.2904	0.9989	0.3811
1.4	-122.2	0.18	118.9	0.9996	0.2358	0.9991	0.3434
1.5	-120.6	0.19	116.0	0.9995	0.2381	0.9991	0.3450
1.6	-124.2	0.18	118.4	0.9994	0.2962	0.9989	0.3849
1.7	-124.1	0.19	116.6	0.9994	0.2976	0.9989	0.3858
1.8	-122.1	0.19	113.2	0.9993	0.3558	0.9987	0.4218
1.9	-123.8	0.19	113.6	0.9993	0.3480	0.9987	0.4171
2.0	-124.7	0.19	113.2	0.9993	0.3569	0.9987	0.4224

Table 2: Parameters a, b and c for different ranges of R

For $0.05 \leq R < 1$	
$a = 57.61R^6 - 385.1R^5 + 989.4R^4 - 1229R^3 + 780.9R^2 - 280.1R - 53.66$, $R^2=0.9949$
$b = 0.18$, $R^2=0.9990$
$c = 32.27R^6 + 242.8R^5 - 594.9R^4 + 672.9R^3 - 351.2R^2 + 76.57R + 114.5$, $R^2=0.8770$
For $1 \leq R \leq 2$	
$a = 57.61R^6 - 385.1R^5 + 989.4R^4 - 1229R^3 + 780.9R^2 - 280.1R - 53.66 - 0.467R^{4.036}$, $R^2=0.9990$
$b = 0.18$, $R^2=0.9990$
$c = 32.27R^6 + 242.8R^5 - 594.9R^4 + 672.9R^3 - 351.2R^2 + 76.57R + 114.5$, $R^2=0.8770$

Case Study 1

A FOPTD process with R of 0.2 was considered as shown in Equation (8). The PI controller parameters and the performance characteristics are given in Table 3.

$$G_p = \frac{e^{-s}}{5s+1}, R=0.2 \quad (8)$$

Figure 3 shows the closed loop responses for the G_{PI} process using different λ . λ is a user-specified parameter that corresponds to the speed of the closed loop response. As expected, larger λ exhibited a more robust and stable response. The response obtained using a DS-tuned controller with $\lambda=0.5\tau$ (largest λ value), which is plotted in Figure 3 is critically damped with no overshoot because the damping ratio, ζ is equal to 1. ζ is an important parameter to determine the nature of the response. However, its response is too slow and 13 minutes is required for it to first reach the

set point because the rise time increases with increasing ζ (Coughanowr and LeBlanc, 2009). Due to slow response, the largest IAE and ISE values were produced and resulted in the worst performance. Table 4 clearly shows that its gain margin (GM) and phase margin (PM) values are over the specification; therefore, it was proven not to be a well tuned controller.

In contrast, decreasing λ tends to give a large K_c and speeds up the response until it becomes oscillatory. Table 3 shows that the DS-tuned controller with $\lambda=0.25\theta$ (the smallest λ value) implies $\zeta=0.35$ and the fastest response (the lowest rise time). However, this PI controller was tuned by minimization of ISE and resulted in an oscillatory closed loop response (the highest overshoot). Furthermore, it is the worst tuning method because the GM value was less than the specified value, in spite of the PM value of 44.16° . Therefore, this controller is unable to withstand greater change in the system.

Table 3: Case study 1-PI controller parameters and performance analysis

λ guideline	λ value	Tuning parameters		Performance characteristics				
		K_c	K_I	OS (%)	t_r (min)	t_s (min)	IAE	ISE
0.5 τ	2.500	1.429	0.286	0.0	12.97	8.05	3.500	3.240
Proposed	0.968	2.540	0.508	4.7	3.65	3.30	2.161	2.487
θ	1.000	2.500	0.500	4.1	3.75	3.37	2.173	2.502
2/3 θ	0.667	3.000	0.600	11.8	2.93	5.22	2.108	2.349
0.25 θ	0.250	4.000	0.800	30.0	2.29	6.81	2.296	2.184

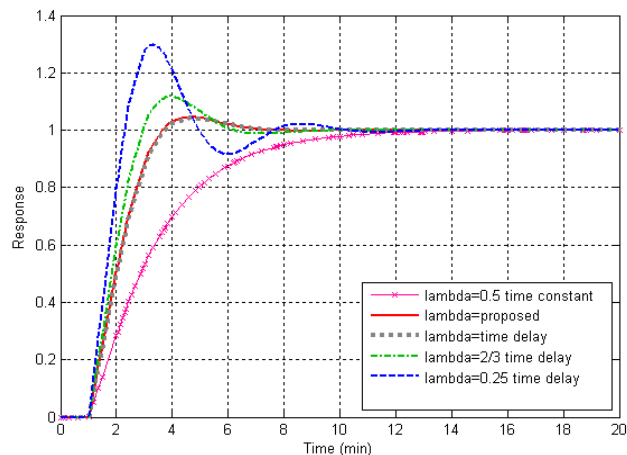


Figure 3: Responses for G_{PI} with different values of λ

Table 4: Case study 1-Robustness analysis

λ guideline	GM	PM
0.5 τ	5.50	73.63
Proposed	3.09	60.89
θ	3.14	61.35
2/3 θ	2.62	55.62
0.25 θ	1.96	44.16

Smith and Corropio (2006) mentioned that $\lambda=2/3\theta$ approximately minimizes the IAE value. It is clearly shown in Table 3 that the DS-tuned controller with $\lambda=2/3\theta$ resulted in minimization of IAE. The DS-tuned controllers with the proposed λ and $\lambda=\theta$ were capable of producing the response with the specified overshoot. Martin *et al.* (1976) recommended $\lambda=\theta$ in order to produce a response having 5% OS; however, 4.1% OS was produced in this case study. In order to make a fair comparison between the proposed λ guideline and $\lambda=\theta$, the specified overshoot for the proposed λ guideline is 5% overshoot. Analyzing Table 3, the performance of the proposed λ guideline was seen to be very similar to $\lambda=\theta$, but it resulted in 4.7% OS, which was closer to the specified overshoot. In order to maximize performance as well as ensure robustness, the closed loop response of the proposed λ guideline with a PM of 60.89° and GM of 3.09 guarantees both performance and robustness.

Case Study 2

The following FOPTD transfer function shown in Equation (9) was considered when to the ratio of time delay to time constant, R, equals to 1. Table 5

shows the numerical values of the PI controller parameters and performance characteristics, whereas the robustness properties are listed in Table 6. The closed loop responses for the G_{P2} process using different λ are illustrated in Figure 4.

$$G_{P2} = \frac{4e^{-10s}}{10s+1}, R=1.0 \quad (9)$$

As R increased to unity, a sluggish response was obtained using $\lambda=\theta$ and the proposed λ guideline (largest λ value). They gave the lowest overshoot and the longest rise time of about 36.6-37.7 minutes to achieve the set point. From Table 5, it can clearly be seen that the proposed λ and $\lambda=\theta$ could achieve 5.4% and 5.8% OS respectively. This indicates that the proposed λ guideline has the capability to reduce the excessive overshoot to 5% overshoot compared with $\lambda=\theta$. Overall, the performance of the proposed λ guideline is superior to that of $\lambda=\theta$. In terms of gain margin (GM), both settings provided robust design with a GM of about 2.96 and 3.14, which is suggested by Skogestad (2006).

Table 5: Case study 2-PI controller parameters and performance analysis

λ guideline	λ value	Tuning parameters		Performance characteristics				
		K_c	K_I	OS (%)	t_r (min)	t_s (min)	IAE	ISE
0.5 τ	5.000	0.167	0.017	17.3	26.08	46.62	21.270	16.500
Proposed	9.675	0.127	0.013	5.4	36.60	33.00	21.380	17.780
θ	10.000	0.125	0.013	5.8	37.69	33.46	21.480	17.890
2/3 θ	6.667	0.150	0.015	11.3	29.30	44.46	20.800	16.820
0.25 θ	2.500	0.200	0.020	29.8	22.84	66.60	22.460	16.410

Table 6: Case study 2-Robustness analysis

λ guideline	GM	PM
0.5 τ	2.36	51.80
Proposed	2.96	59.58
θ	3.14	61.35
2/3 θ	2.62	55.62
0.25 θ	1.96	44.16

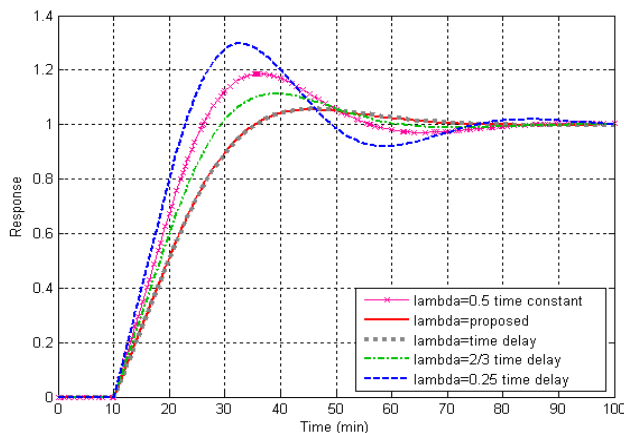


Figure 4: Responses for G_{P2} with different values of λ

In case study 1, a critically damped response was obtained using a DS-tuned controller with $\lambda=0.5\tau$. However, in Figure 4, the DS with a $\lambda=0.5\tau$ setting gave an underdamped response with ζ of about 0.49. Analyzing the PM and GM values in Table 6, the values of the robustness measures were similar to those of case study 1, except for the DS-tuned controller with $\lambda=0.5\tau$, because it has no relationship with R. If the minimum ISE is the priority of the user, $\lambda=0.25\theta$ is a wise choice. However, its minimization may result in an unsatisfactory response and oscillation over a long time period. As summarized in Table 5, the weakness of this guideline was the long settling time, 30% more than that of other λ guidelines because the ISE criterion weights all errors equally, independently of time (Krohling and Rey, 2001). Due to its oscillation before reaching the steady state, the highest IAE value resulted. Furthermore, it is the worst tuning method and unable to withstand greater changes in the system due to the GM is less than 2. From Table 5, $\lambda=2/3\theta$ would be the most suitable λ guideline to optimize the minimization of IAE.

Case Study 3

Equation (10) is the FOPTD transfer function model to be considered. The ratio of time delay to time constant, R is equal to 2. Table 7 shows the numerical values of the PI controller parameters.

Figure 5 illustrates the closed loop responses for the G_{P3} process using different λ formulas and their performance characteristics are presented in Table 7.

$$G_{P3} = \frac{e^{-14s}}{7s+1}, R=2.0 \quad (10)$$

As the R value is increased to 2, the dynamic behavior becomes slower and takes a longer time to reach the set point and settle. All DS-tuned controllers could provide constant values for the overshoot, IAE and ISE in these three case studies except the DS-tuned controller with $\lambda=0.5\tau$. The guideline with $\lambda=0$ showed a sluggish response with the lowest overshoot, as expected. Thus, a longer rise time was taken to reach the set point and the settling time was significantly reduced (Table 7). Figure 5 shows that the response of a DS-tuned controller with $\lambda=0.5\tau$ became underdamped and its damping ratio, ζ , was reduced to 0.35 for as $R=2$. $\lambda=0.25\theta$ and $\lambda=0.5\tau$ had the same smallest λ value. Therefore, their oscillatory responses gave the highest overshoot (29.5%) and took the longest time to settle (about 93 minutes), although they are fast to reach the set point. Table 7 clearly shows that the proposed λ guideline and $\lambda=0$ could achieve 4.0% overshoot and 3.8% overshoot, respectively. Besides, other performance criteria such as rise time, settling time, IAE and ISE were also seen to be very similar to $\lambda=0$.

Table 7: Case study 3-PI controller parameters and performance analysis

λ guideline	λ value	Tuning parameters		Performance characteristics				
		K_c	K_I	OS (%)	t_r (min)	t_s (min)	IAE	ISE
0.5 τ	3.500	0.400	0.057	29.5	32.00	92.55	30.860	22.490
Proposed	13.647	0.253	0.036	4.0	51.51	46.43	29.820	24.490
θ	14.000	0.250	0.036	3.8	51.81	47.25	29.940	24.580
2/3 θ	9.333	0.300	0.043	11.3	40.97	71.76	28.920	23.120
0.25 θ	3.500	0.400	0.057	29.5	32.00	92.55	30.860	22.490

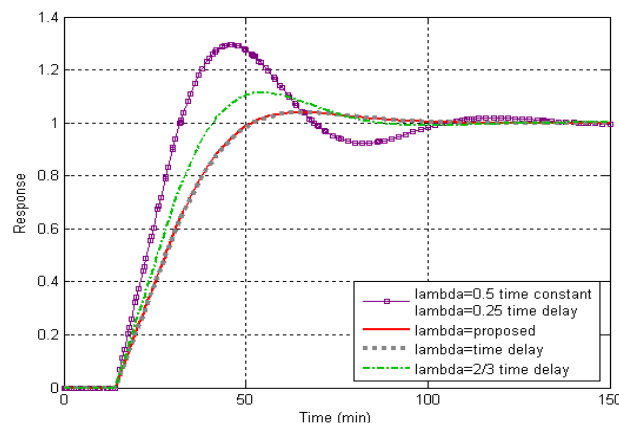


Figure 5: Responses for G_{P3} with different values of λ

The robustness measures of the DS with the proposed λ guideline setting are compared with four other λ guidelines in Table 8. Similar to the performance characteristics, all controllers could provide identical GM and PM values, excluding the controller with $\lambda=0.5\tau$. The results show that the DS-tuned controllers with $\lambda=0.5\tau$ and $\lambda=0.25\theta$ are the worst controllers because they are not robust ($GM < 2$ and $PM < 45^\circ$) and their performances are also not satisfactory. These controllers cannot withstand greater changes in the system. Other controllers are considered to be well tuned controllers because their GM and PM values fall in the suggested range.

Table 8: Case study 3-Robustness analysis

λ guideline	GM	PM
0.5τ	1.96	44.16
Proposed	3.10	60.98
θ	3.14	61.35
$2/3\theta$	2.62	55.62
0.25θ	1.96	44.16

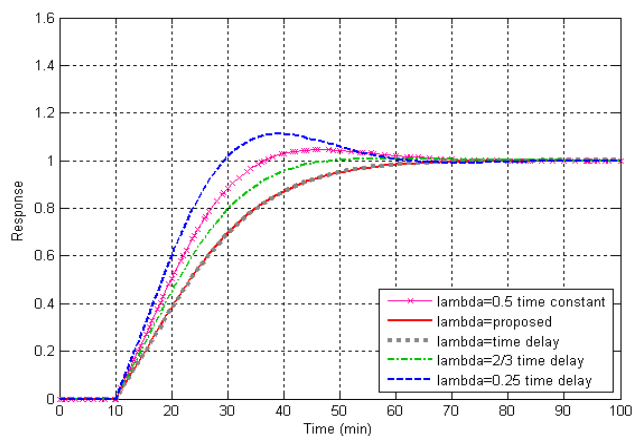
Model Uncertainties

Model uncertainty is important because errors in the model parameters lead to poorer control performance and potentially instability in the response. In this case study, $\pm 25\%$ uncertainty in each nominal model parameter of the G_{P2} process was calculated.

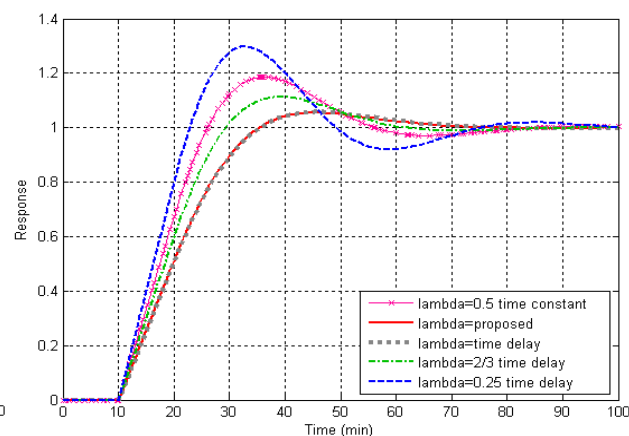
Figures 6, 7 and 8 illustrate the closed loop responses for the G_{P2} process after mismatch in K_p , τ

and θ , respectively. When -25% K_p and θ uncertainties were considered, all responses became slower and less oscillatory compared to when there is no mismatch in K_p and θ , respectively, which are plotted in Figures 6 and 8. Among these five λ guidelines, the proposed λ guideline and $\lambda=\theta$ exhibited critically damping and the nominal overshoot was reduced from 5.4%-5.8% overshoot to zero. In contrast, the nominal overshoots for the other three guidelines were largely reduced to about 11%-35% overshoot. With $+25\%$ changes in K_p and θ , a stability problem arises and the response of the system becomes oscillatory as expected (Saravanakumar and Wahidabanu, 2009; Tan *et al.*, 2009). In this case, the overshoot was increased to about 14%-20%. By comparison, the proposed λ guideline and $\lambda=\theta$ only increased to about 10-11% overshoot. When $\pm 25\%$ uncertainty in τ is considered (Figure 7), the closed loop responses of the DS tuned by $\lambda=0.5\tau$, $\lambda=2/3\theta$ and $\lambda=0.25\theta$ were similar to the nominal response. In term of overshoot, the overshoot with $\pm 25\%$ τ was similar to the nominal overshoot. However, the overshoot for the proposed λ guideline and $\lambda=\theta$ settings was reduced to zero at -25% τ mismatch and increased to 10% overshoot at $+25\%$ τ mismatch.

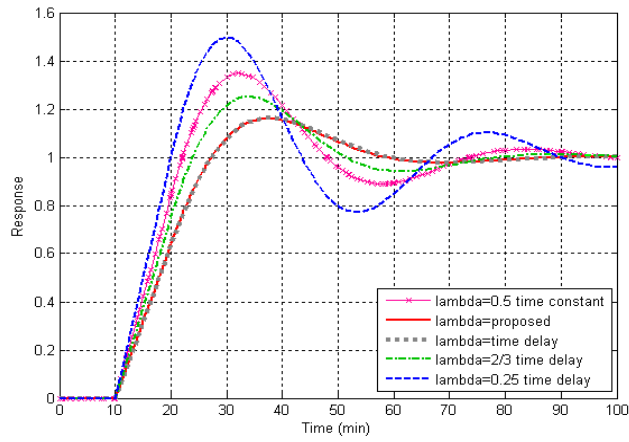
The process model mismatch simulations showed that the DS tuned controllers with the proposed λ guideline and $\lambda=\theta$ gave better, oscillation-free responses. Besides, their overshoots were not too different from the nominal overshoot.



(a)

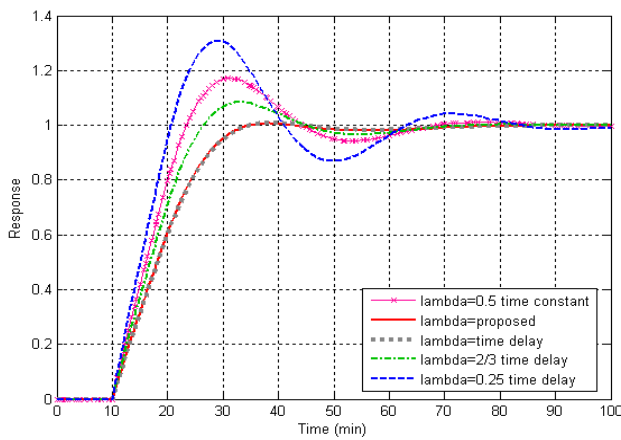


(b)

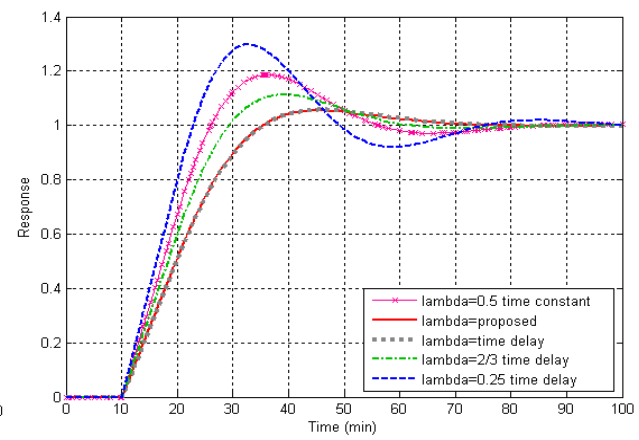


(c)

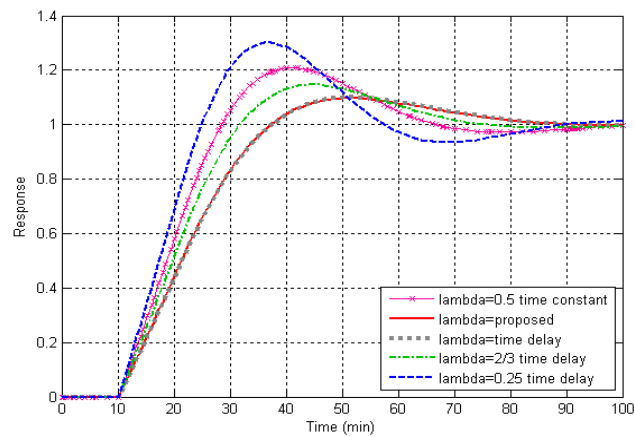
Figure 6: Process gain, K_p with (a) -25%, (b) nominal and (c) +25% variation, respectively



(a)



(b)



(c)

Figure 7: Time constant, τ with (a) -25%, (b) nominal and (c) +25% variation, respectively

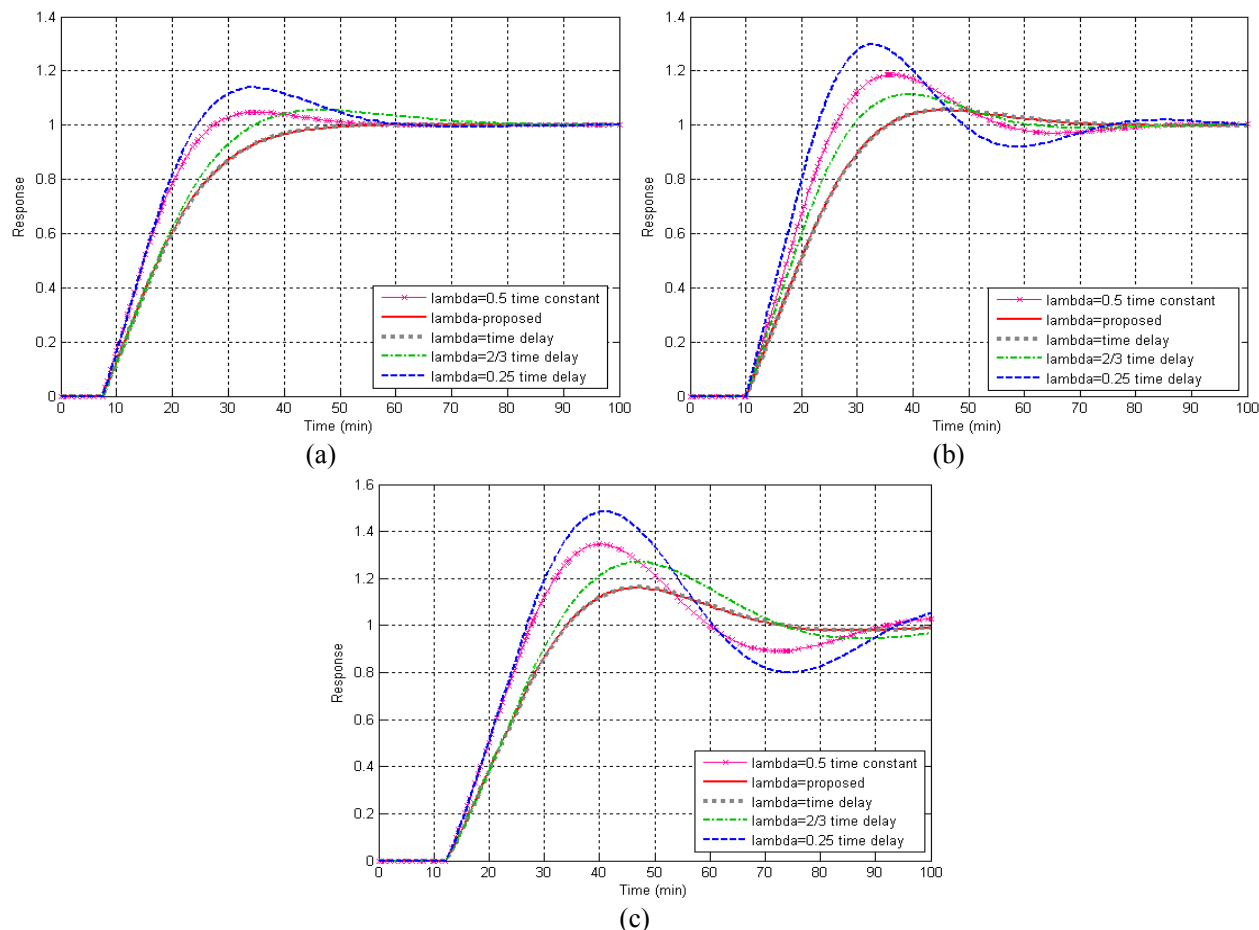


Figure 8: Time delay, θ with (a) -25%, (b) nominal and (c) +25% variation, respectively

CONCLUSIONS

The direct synthesis method is a model-based tuning method for a feedback controller. An expression that relates λ to overshoot of the process response (OS), and the ratio of time delay to time constant (R) was developed. The λ guideline is suggested to reduce excessive overshoot to $4\% \leq OS \leq 10\%$. The proposed guideline worked well over a wide range of R varying from 0.05 to 2, as proven in the case studies. Besides, the overshoot changed less with $\pm 25\%$ process model mismatch except for τ mismatch.

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NOMENCLATURE

G_c	Transfer function of controller
G_p	Process transfer function
$\frac{Y}{Y_{sp}}$	Closed loop transfer function
FOPTD	First order plus time delay
GM	Gain margin
IAE	Integral of the absolute value of the error
ISE	Integral of square of the error
K_c	Controller gain
K_I	Integral gain
K_p	Process gain
θ	Time delay
T	Time constant
R	The ratio of time delay to time constant
λ	Lambda
OS	Overshoot
PI	Proportional-integral

PID	Proportional-integral-derivative
PM	Phase margin
t_r	Rise time
t_s	Settling time

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