The Alpha-Cluster Bands in ^{94}Mo

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The ^{94}Mo nucleus is treated as an α-cluster interacting with an inert core through a phenomenological local potential. The properties of the ground state band of the $\alpha+^{90}$Zr system, such as the energy levels, intercluster rms radii and $B(E2)$ transition strengths were calculated. These results are compared with previous references and available experimental data. Some predictions were made concerning the negative parity band and the excited positive parity band.

The α-cluster structure is an important feature for light nuclei. The α-cluster model has been successful in reproducing the behaviour of the energy spacings, electromagnetic properties, α-widths and α-nucleus elastic scattering data around the double shell closures such as ^{16}O and ^{40}Ca [1–3]. The search for α-cluster states in heavier nuclei is the natural sequence of investigations. Recent studies [4–6] indicate the persistence of α clustering in the $A = 90$ mass region.

According to the α-cluster model, the total nucleus is viewed as an α-particle orbiting an inert core. In the case where both constituents have spin zero, no complications arise from the spin-orbit coupling. The ^{90}Zr medium-heavy nucleus presents a closed shell for neutrons and filled subshells for protons up to $2p_{1/2}$ and may be considered approximately as an inert core. In this way the proposed model is employed to the ^{94}Mo nucleus, which is regarded as an $\alpha+^{90}$Zr system. The interaction was described through a local phenomenological potential $V(r) = V_C(r) + V_N(r)$ containing the nuclear and Coulomb terms. The Coulomb potential $V_C(r)$ is taken to be that of a point α-particle interacting with an uniformly charged spherical core of radius $R = 5.793$ fm. The intercluster nuclear potential

$$V_N(r) = -V_0\left\{\frac{b}{1+\exp\left(\frac{r-R}{a}\right)} + \frac{1-b}{\left[1+\exp\left(\frac{r-R}{3a}\right)\right]^3}\right\}$$

proposed by Buck, Merchant and Perez [4] was used, where $V_0 = 220$ MeV, $a = 0.65$ fm, $b = 0.3$ and R is the radius used in the Coulomb potential. The parameters V_0, a and b have been fitted to reproduce satisfactorily the experimental excitation energies of the ground state bands of ^{20}Ne, ^{44}Ti, ^{94}Mo and ^{212}Po, as well as the experimental α-decay half-lives for several even-even heavy nuclei, while R is fitted for ^{94}Mo. The form of $V_N(r)$ has been adopted because of its similarity to the real parts of the optical potentials determined from α-^{16}O and α-^{40}Ca elastic scattering [4].

The α-^{90}Zr system can be solved numerically to obtain the properties of the bound, quasibound and resonant states. The energy eigenvalues yield the levels of the spectrum and the associated wave functions are used to calculate other nuclear properties. The nucleons of the α-cluster must lie in shell-model orbitals above those already occupied by the core nucleons, according to the Pauli principle. This restriction is defined by the quantum number

$$G = 2N + L\,,$$

where N is the number of internal nodes in the radial wave function and L is the orbital angular momentum. G is a global quantum number which identifies a band of levels. In the case of the α-^{90}Zr system we have $G \geq 16$, where $G = 16$ corresponds to the ground state band. This value is obtained from the Wildermuth condition [7] and yields an appropriate description of the α-^{90}Zr ground state band [4, 5].

TABLE I: Theoretical and experimental values for the $B(E2)$ transition rates and theoretical values for the intercluster rms radii concerning the α-^{90}Zr ground state band ($G = 16$). The experimental values are from Ref. [11].

J^π	$\sqrt{\langle r^2\rangle}$ (fm)	$B(E2)_{theor.}$ (e^2 fm^4)	$B(E2)_{exp.}$ (e^2 fm^4)
0^+	5.13		
2^+	5.12	199	392±6
4^+	5.08	272	660±102
6^+	5.02	274	
8^+	4.95	250	
10^+	4.86	211	

The energy levels of the ground state band (Fig. 1) and the respective radial wave functions were obtained solving directly the Schrödinger equation for the system. These functions were used to determine the associated $B(E2)$ transition rates and the intercluster rms radii for this band (Table I). The calculated ground state band shows a quasirotational spectrum and it is in total agreement with the theoretical values of Ref. [4], where the spectrum is obtained by using the Bohr-Sommerfeld quantization rule. The calculated energies give a satisfactory description of the experimental spectrum, if one takes into account that the parameters of this local potential are not L dependent or energy dependent.

The rms intercluster distance is seen to decrease from 5.13 fm for the 0^+ state to 4.86 fm for the 10^+ state. This antistretching effect also appears in calculations for the α-^{16}O [1] and α-^{40}Ca [2, 3] cluster systems. A comparison of the calculated rms radii with the sum of the experimental radii [10] for α (1.674 fm) and ^{90}Zr (4.244 fm) suggests that the ground state band has a compact α-cluster structure, in agreement

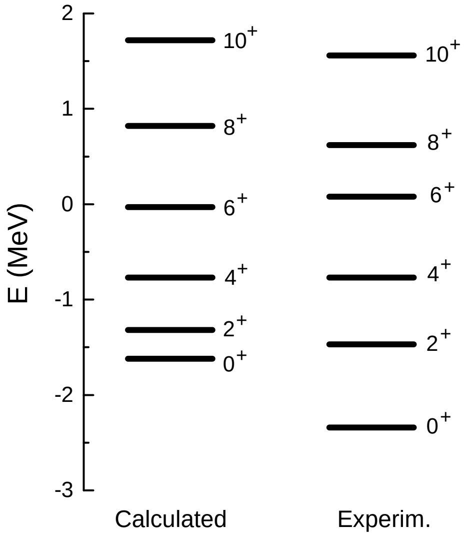

FIG. 1: Comparison of the calculated and measured α-^{90}Zr energies for the ground state band ($G = 16$). The values are given with respect to the theoretical α-^{90}Zr threshold. The experimental values are from Ref. [9].

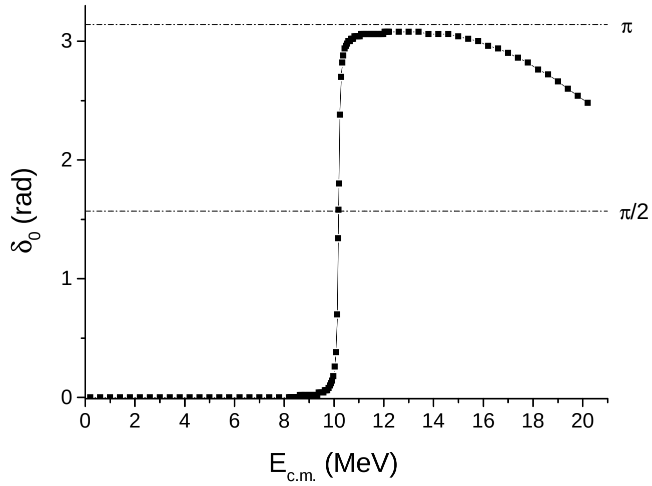

FIG. 2: Scattering phase shifts calculated with $L = 0$ for the α-^{90}Zr system. The resonance energy for which $\delta_0 = \pi/2$ corresponds to the 0^+ state of the $G = 18$ band.

with calculations of Ohkubo [5]. The calculated $B(E2)$ transition strengths have the same magnitude of the experimental values. These results may be considered satisfactory, once the $B(E2)$ values are calculated without any effective charge.

The degree of shell-model-like character in the ground state band was estimated through the expansion of the corresponding radial wave functions in the harmonic oscillator basis with the parameter $\nu = \mu\omega/2\hbar$, where μ is the reduced mass of the system and ω is given by $\hbar\omega = 41A^{-1/3}$ MeV. This expansion shows a large overlap between these radial wave functions and the respective harmonic oscillator wave functions with similar quantum numbers G and L (the overlap increases from 89.1% for the 0^+ state to 95.0% for the 10^+ state). Such result indicates that the ground band states present a shell-model trend.

The intercluster potential $V(r)$ has also been used to make some predictions about higher lying bands which are above the α+^{90}Zr threshold. The functions $\delta_L(E)$ for $L = 0$, 1, 3 and 5 were calculated, where δ_L is the scattering phase shift for a given L (see Figs. 2 and 3). The total α-width Γ_α can be extracted from the functions $\delta_L(E)$ by [3, 8]

$$\Gamma_\alpha = \frac{2}{(d\delta_L/dE)|_{E=E_R}},$$

where E_R is the resonance energy for which the phase shift crosses $\pi/2$ up to a constant multiple of π. The vicinity of each resonance was located previously by using the bound state approximation and the resonance energies were determined through the functions $\delta_L(E)$. In the numerical calculation of these functions, an energy step $\Delta E = 0.4$ MeV was applied for energies far from the resonances; however, for energies in the vicinity of each resonance, ΔE must be reduced in orders of magnitude to provide an accurate value for $d\delta_L/dE$ in E_R.

The calculated resonance energies and total α-widths are

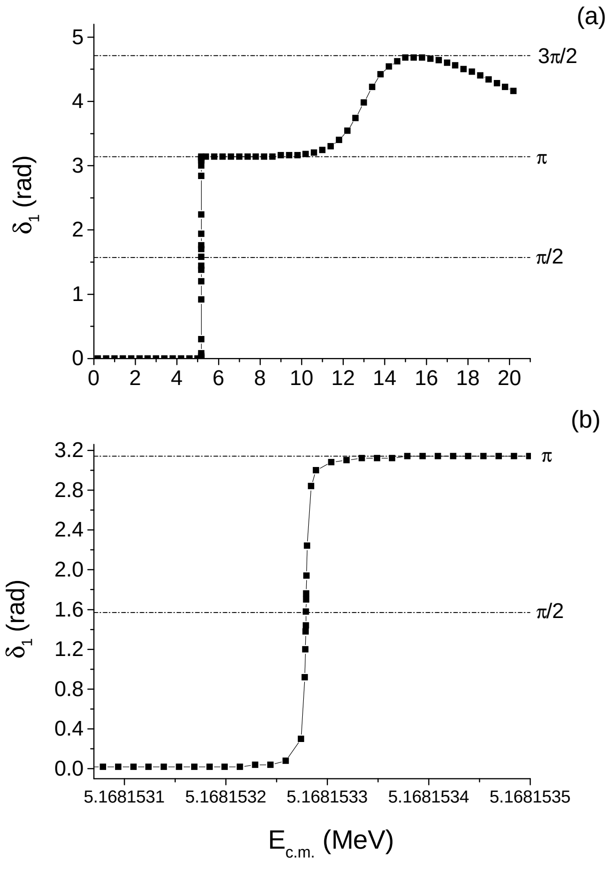

FIG. 3: Scattering phase shifts calculated with $L = 1$ for the α-^{90}Zr system. The graph (a) shows the phase shifts calculated between 0 and 21 MeV. The graph (b) shows the phase shifts calculated in the vicinity of the resonance energy for which $\delta_1 = \pi/2$, corresponding to the 1^- state of the $G = 17$ band.

TABLE II: Theoretical resonance energies and respective total α-widths for some states of higher lying bands of the α-^{90}Zr system. The energies are given with respect to the theoretical α+^{90}Zr threshold.

J^{π}	G	E_R (MeV)	Γ_{α} (10^{-3} eV)
1^-	17	5.17	3.1
3^-	17	5.66	13.3
5^-	17	6.45	53
0^+	18	10.18	8.7×10^7

given in Table II. The values refer to the first members of the negative parity band (1^-, 3^- and 5^-) and the first member of the excited positive parity band (0^+). The very small widths of the states of the negative parity band indicate that the properties of such states may be investigated within the bound state approximation. Therefore, it is suggested that the adequate treatment for the $G = 17$ band is analogous to the treatment given for the negative parity band of the α-^{40}Ca system [3]. The level 0^+ of the $G = 18$ band lies at about 1.5 MeV below the Coulomb barrier, resulting in a larger width (87 keV). This indicates that the $G = 18$ band presents a typical resonance feature. Higher spin members of the $G = 17$ and $G = 18$ bands are expected above the calculated levels, as in lighter α-core systems.

The present calculations confirm that the α-cluster model is still an important tool for the spectroscopic analysis of ^{94}Mo. The procedures described in this work may provide other predictions concerning the higher spin states of the $G = 16$, 17 and 18 bands. More experimental data are necessary to verify our calculated energy levels for the negative parity band and the excited positive parity band.

Acknowledgments

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