Dynamical properties of a mutualism system in the presence of noise and time delay

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The normalized correlation function $C(s)$ and the associated relaxation time T_c of the mutualism system in the presence of noise and time delay are investigated. The effects of noise and time delay on $C(s)$ and T_c for a mutualism system are discussed. Based on the numerical computation, it is found that: (i) The noise intensity D slows down the the fluctuation decay of species density firstly and then enhances it. (ii) The time delay τ slows down the fluctuation decay of species density while the mean nterspecies interaction intensity J speeds up the fluctuation decay of species density.

Keywords: time delay; relaxation time; correlation function; mutualism system

I. INTRODUCTION

The behavior of a nonlinear system under the influence of noise have been widely studied from both theoretical and experimental points of view [1]. Particulary, noise-induced phase transition [2], noise-enhanced stability [3], resonant activation [4], noise-induced nonequilibrium transport [5], noise-induced multistability [6], and stochastic resonance [7, 8] have been intensively investigated in a large variety of physical, and biological systems. However, these investigations neglect the effects induced by time delays. In practice, in many physical as well as biological systems, time delays always exist and play a significant role in the dynamics, such as, biophysiological controls [9], and signal transmissions in biological and artificial neuronal networks[10], and laser dynamics in optical cavities [11, 12] etc. Meanwhile, it appears that the combination of noise and time delay is ubiquitous in nature and often change fundamentally dynamics of the system [13–16]. Recently Nie and Mei [17] have studied the effects of noise and time delay in a classical Lotka-Volterra model of the mutualism system, and found that the combination of noise and time delay completely suppressed the population explosion of the mutualism system. Yet in order to characterize further the dynamics properties of the mutualism system with noise and time delay, the correlation function and the associated relaxation time of species density should be considerd.

In the present paper, the normalized correlation function $C(s)$ and the associated relaxation time T_c of the mutualism system with noise and time delay are investigated. In section 2, the analytical expressions of $C(s)$ and T_c of the mutualism system with noise and time are obtained. The effects of noise and time delay on $C(s)$ and T_c are discussed. Finally, a conclusion is given in Section 3.

II. STATIONARY PROBABILITY DISTRIBUTION, RELAXATION TIME AND CORRELATION FUNCTION

In order to study the effects of noise and time delay on the relaxation time and correlation function of the mutualism system. First, the effects of noise and time delay on the one-species mutualism system is presented. Then the effects of noise and time delay on the multispecies mutualism system is analyzed.

II.1. The one-species mutualism system

Consider a classical Lotka-Volterra model introduced by Vito Volterra for the description of struggle for existence among species [18, 19]. In the deterministic case, the differential equation describing the one-species mutualism system reads

$$\frac{dx(t)}{dt} = x(t)[r + ax(t)], \tag{1}$$

where x is the species density, r is the growth rate, and a represents the intraspecies interaction parameter. We consider effects due to some environmental perturbations, such as temperature, and climate, etc. These factors will give rise to a fluctuation of intraspecies interaction parameter. We can rewrite the intraspecies interaction parameter a in Eq. (1) as $a + \sqrt{2D}\eta(t)$, where $\eta(t)$ is the Gaussian white noise defined as $\langle\eta(t)\rangle = 0$ and $\langle\eta(t)\eta(t')\rangle = \delta(t-t')$, in which D is the noise intensity. If a time delay is also introduced into the system. Then Eq. (1) can be rewritten as

$$\frac{dx(t)}{dt} = x(t)[r + ax(t-\tau)] + \sqrt{2D}x^2(x)\eta(t), \tag{2}$$

where τ is the delay time of the system. The delay Fokker-Plank equation corresponding to Eq. (2) is [20, 21]

$$\begin{aligned}\frac{\partial P(x,t)}{\partial t} =& -\frac{\partial}{\partial x}\int \left[x(r + ax_\tau) - 2Dx^3\right] P(x,t;x_\tau,t-\tau)dx_\tau \\ &+ D\int \frac{\partial^2}{\partial x^2} x^4 P(x,t;x_\tau,t-\tau)dx_\tau,\end{aligned} \tag{3}$$

in which $P(x,t;x_\tau,t-\tau)$ is the joint probability density. If the small time delay approximation and first order approximation

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are employed [20–23], the stationary probability distribution (SPD) corresponding to Eq. (3) can be obtained [17, 24]

$$P_{st}(x) = Nx^{a^2\tau-2}\exp\left\{-\frac{1}{D}\left[\frac{r}{2x^2}+\frac{a}{x}+a\tau\left(\frac{r}{x}+2Dx\right)\right]\right\}, \quad (4)$$

here N are the normalization constant. Then the expectation values of the nth power of the species density x are given by

$$\langle x^n\rangle_{st} = \frac{\int_{-\infty}^{+\infty} x^n P_{st}(x)dx}{\int_{-\infty}^{+\infty} P_{st}(x)dx}. \quad (5)$$

In this paper, we are interested in the stationary correlation function and associated relaxation time, which are used to describe the fluctuation decay of dynamical variable x. The normalized correlation function is [25, 26]

$$C(s) = \exp\left[-\frac{D\langle x^4\rangle_{st}}{\langle x^2\rangle_{st}-\langle x\rangle_{st}^2}s\right], \quad (6)$$

and the associated relaxation time is

$$T_c = \frac{\langle x^2\rangle_{st}-\langle x\rangle_{st}^2}{D\langle x^4\rangle_{st}}. \quad (7)$$

By virtue of the expressions of the normalized correlation function (6) and the associated relaxation time (7). The effects of noise and time delay on $C(s)$ and T_c can be studied by the numerical computation, respectively. T_c as a function of D for various values of τ is plotted in Fig. 1. From figure 1, T_c increases firstly and then decreases with D increasing, showing a single-peak structure. While for a fixed D value, as the value of τ increases, the peak of T_c becomes higher and the position of the peak does not change. $C(s)$ versus the decay time s are shown in Figs. 2 and 3 for various values of τ and for various values of D, respectively. $C(s)$ decreases exponentially as the decay time s increases. It is seen that the value of $C(s)$ increases as τ increases (see Fig. 2), while for a fixed τ value, the value of $C(s)$ increases as D increases from 0.01 to 0.05 (see Fig. 3a) and decreases as D increases from 0.2 to 1 (see Fig. 3b).

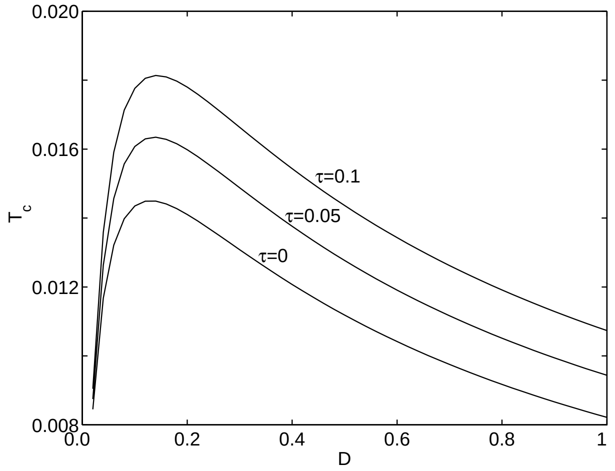

FIG. 1: The relaxation time T_c vs D for $r = 1.5$ and $a = 1$. τ takes 0, 0.05, and 0.1, respectively

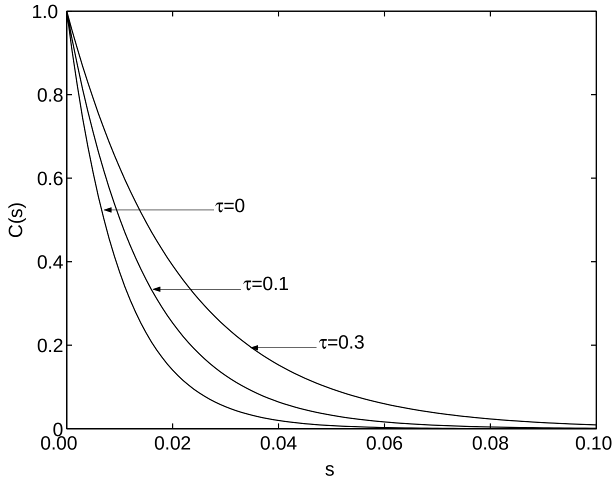

FIG. 2: The correlation function $C(s)$ vs s for $D = 0.05$. τ takes 0, 0.1, and 0.3, respectively. The other parameter values are the same as those in Fig . 1.

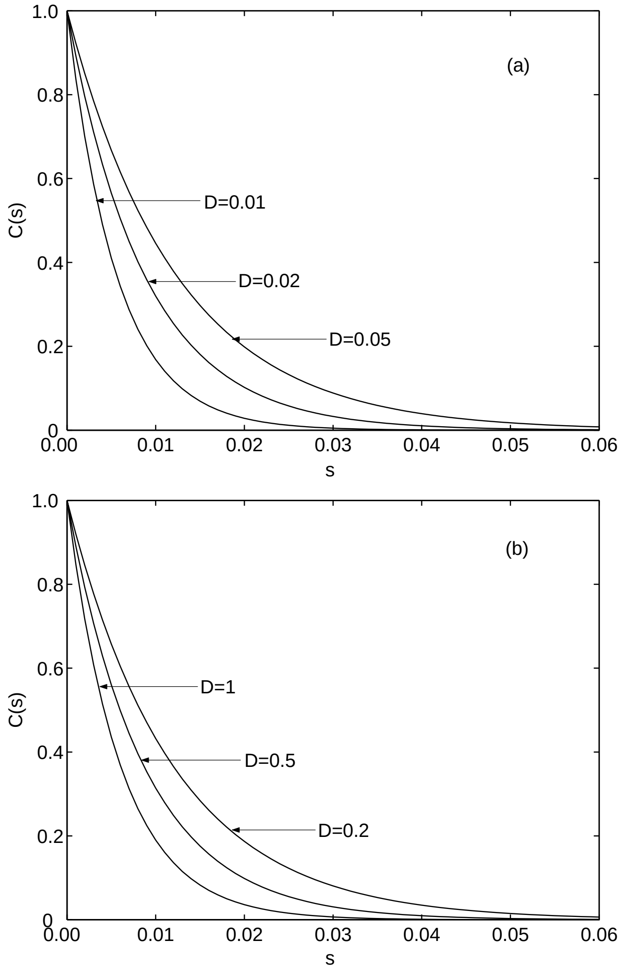

FIG. 3: The correlation function $C(s)$ vs s for $\tau = 0.05$. (a) D takes 0.01, 0.02, and 0.05, respectively. (b) D takes 0.2, 0.5, and 1, respectively. The other parameter values are the same as those in Fig . 1.

II.2. The multispecies mutualism system

Consider the multispecies mutualism system subjected to noise and time delay. It can be described by the following differential equation

$$\frac{dx_i(t)}{dt} = x_i(t)\left[r + ax_i(t) + \sum_{j\neq i} J_{ij}x_j(t-\tau)\right] + \sqrt{2D}x_i^2(x)\eta_i(t), \quad i,j = 1,....,M, \tag{8}$$

in which x_i is the i-th species density, M is the total number of species, J_{ij} represents the interspecies interaction intensity between the i-th species and the j-th one, and $J_{ij} > 0$. $\eta_i(t)$ is the Gaussian white noise by environments on the i-th species, and define as $\langle\eta_i(t)\rangle = 0$ and $\langle\eta_i(t)\eta_j(t')\rangle = \delta_{ij}\delta(t-t')$. As $M \to \infty$, making use of the mean-field approximation to the system, we obtain

$$\sum_{j\neq i} J_{ij}x_j(t-\tau) = \frac{J}{M}\sum_j x_j(t-\tau) = J\langle x\rangle_{st}, \tag{9}$$

where J is the mean nterspecies interaction intensity, and $\langle x\rangle_{st}$ is the stationary average value of x. Thus Eq. (8) can be rewritten as

$$\frac{dx(t)}{dt} = x(t)[r + J\langle x\rangle_{st} + ax(t-\tau)] + \sqrt{2D}x^2(x)\eta(t), \tag{10}$$

with

$$\langle\eta(t)\rangle = 0, \quad \langle\eta(t)\eta(t')\rangle = \delta(t-t'). \tag{11}$$

In the condition of small time delay approximation and first order approximation, the SPD of x can be obtained [17, 24]

$$P_{st}(x) = N'x^{a^2\tau-2} \exp\left\{-\frac{1}{D}\left[\frac{r+J\langle x\rangle_{st}}{2x^2} + \frac{a}{x} + a\tau\left(\frac{r+J\langle x\rangle_{st}}{x} + 2Dx\right)\right]\right\}, \tag{12}$$

where N' are the normalization constant. The expectation values of the nth power of the species density x are

$$\langle x^n\rangle_{st} = \frac{\int_{-\infty}^{+\infty} x^n P_{st}(x)dx}{\int_{-\infty}^{+\infty} P_{st}(x)dx}. \tag{13}$$

The stationary normalized correlation function and the associated relaxation time can be respectively expressed as [25, 26]

$$C(s) = \exp\left[-\frac{D\langle x^4\rangle_{st}}{\langle x^2\rangle_{st} - \langle x\rangle_{st}^2}s\right], \tag{14}$$

and

$$T_c = \frac{\langle x^2\rangle_{st} - \langle x\rangle_{st}^2}{D\langle x^4\rangle_{st}}. \tag{15}$$

Making use of the expressions of the normalized correlation function (14) and the associated relaxation time (15). The results are shown in Figs. 4 and 5. Figure 4 displays the effects of noise and time delay on T_c of the multispecies mutualism system. Compared with the one-species mutualism system with the noise and the time delay, the multispecies mutualism system exhibits its own peculiarities: i) the fluctuation decay of its species density is faster than that without interspecies interaction ($J = 0$), which can be seen from the comparison of the T_c heights between the dashed line and the solid line of $\tau = 0.01$ in Fig. 4; ii) the peak position of the T_c is shifted to the larger value of D with τ increasing, but for the one-species mutualism system it does not change. From figure 5, the larger J is, the smaller the value of $C(s)$. J speeds up the the fluctuation decay of species density.

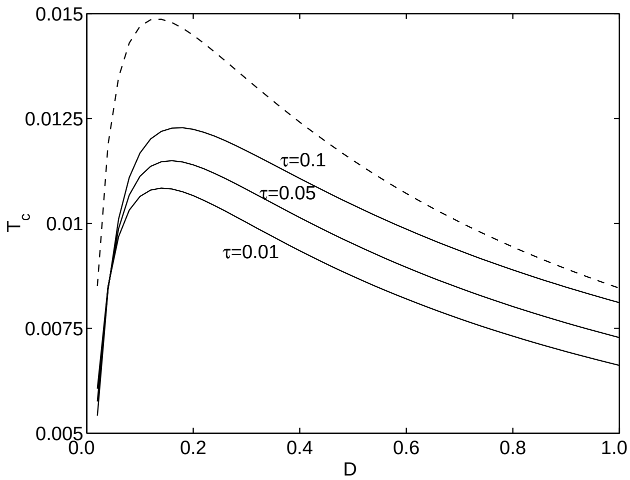

FIG. 4: The relaxation time T_c vs D for $J = 2$. τ takes 0.01, 0.05, and 0.1, respectively. The dashed line represents $J = 0, \tau = 0.01$. The other parameter values are the same as those in Fig. 1.

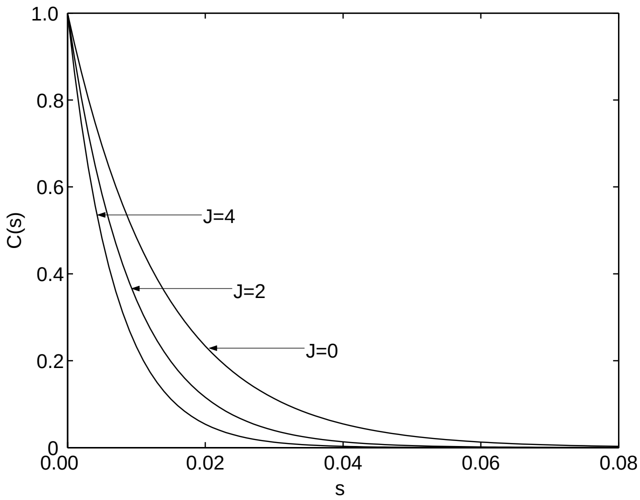

FIG. 5: The correlation function $C(s)$ vs s for $D = 0.05, \tau = 0.05$. J takes 0, 2, and 4, respectively. The other parameter values are the same as those in Fig . 1.

III. CONCLUSIONS

In this paper, we have studied the effects of noise and time delay on the relaxation time and correlation function of the mutualism system subjected to noise and time delay. The results of the numerical computation show that noise and time delay play important roles in the mutualism system. The noise intensity D slows down the the fluctuation decay of species density firstly and then enhances it. The time delay τ decreases the fluctuations and slows down the fluctuation decay of species density, on the contrary, the mean nterspecies interaction intensity J enhances the fluctuations and speeds up the fluctuation decay of species density.

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