Generalized z-Scaling and pp Collisions at RHIC *

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New generalization of the z-scaling in inclusive particle production is proposed. The scaling variable z is expressed in terms of the momentum fractions x_1 and x_2 of the incoming protons. Explicit dependence of z on the momentum fractions y_a and y_b of the scattered and recoil constituents carried by the inclusive particle and recoil object is included. The scaling function $\psi(z)$ for charged and identified hadrons produced in proton-proton collisions is constructed. The scheme allows unique description of data on inclusive cross sections of charged hadrons, pions, kaons, antiprotons and lambdas produced at RHIC energies. The obtained results suggest that the z-scaling may be used as a tool for searching for new physics phenomena of particle production in high transverse momentum and high multiplicity region at proton-proton colliders RHIC and LHC.

Keywords: Proton-proton interactions; High energy; Inclusive spectra; Scaling; Multiplicity

I. INTRODUCTION

Search for new physics in collisions of hadrons and nuclei at sufficiently high energies, high transverse momenta and high multiplicities is one of the main goals of investigations at Relativistic Heavy Ion Collider (RHIC) at BNL and Large Hadron Collider (LHC) at CERN. Experimental data on particle production can provide constraints for different theoretical models. Processes with high transverse momenta of produced particles are used for a precise test of perturbative QCD. The multiple particle production is most suitable for verification of non-perturbative QCD and investigation of phase transitions in non-Abelian theories.

Various regularities of particle production were found to reflect general principles in interacting systems. One of the most basic principles is the self-similarity of hadron production valid both in soft and hard physics. Other general principles are locality and fractality which can be applied to hard processes at small scales. The locality of hadronic interactions means that they can be described in terms of the interactions of their constituents. Fractality in hard processes is a specific feature connected with sub-structure of the constituents. This includes the self-similarity over a wide scale range. Fractality of soft processes concerning the multi-particle production was investigated by many authors (A.Bialas, I.Dremin, E.DeWolf and others). Fractality in inclusive reactions with high-p_T particles was considered for the first time in the framework of the z-scaling [1]. The approach is based on principles of locality, self-similarity and fractality. It takes into account the fractal structure of the colliding objects, the interaction of their constituents and particle formation. The scaling function $\psi(z)$ and the variable z are constructed using the experimentally measured inclusive cross section $Ed^3\sigma/dp^3$ and the multiplicity density $dN/d\eta$.

The original version [1] of the z-scaling was generalized for various multiplicities of produced particles [2]. It represents a regularity in both soft and hard regime in proton-(anti)proton collisions over a wide range of initial energies and multiplicities of the produced particles. However, the generalization of the scaling for various multiplicities was obtained at the expense of the angular independence of the scaling function.

In this paper we present new generalization [3] of z-scaling and show that independence of the scaling function $\psi(z)$ on the collision energy $\sqrt{s}$, multiplicity density $dN/d\eta$ and the angle θ can be restored simultaneously, if two momentum fractions y_a and y_b for the scattered constituent and its recoil are introduced in the final state, respectively.

II. NEW GENERALIZATION OF z-SCALING

We consider a collision of extended objets (hadrons and nuclei) at sufficiently high energies as an ensemble of individual interactions of their constituents. Interacting constituents carry the fractions x_1 and x_2 of the momenta P_1 and P_2 of the incoming objects. The sub-process underlying the reaction $M_1 + M_2 \to m_1 + X$ is considered to be a binary collision

$$(x_1M_1) + (x_2M_2) \to (m_1/y_a) + (x_1M_1 + x_2M_2 + m_2/y_b) \quad (1)$$

of the constituents (x_1M_1) and (x_2M_2) resulting in the scattered (m_1/y_a) and recoil $(x_1M_1 + x_2M_2 + m_2/y_b)$ constituents in the final state. The inclusive particle with the mass m_1 and the 4-momentum p carries the fraction y_a of the 4-momentum of the scattered constituent. Its counterpart (m_2), produced in accordance with internal conservation laws (baryon number, isospin, strangeness...), carries the 4-momentum fraction y_b of the produced recoil. The binary sub-process is subject to the condition

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = (x_1M_1 + x_2M_2 + m_2/y_b)^2. \quad (2)$$

The above equation is an expression of the locality of the hadron interaction at a constituent level. It represents a kinematic constraint on the fractions x_1, x_2, y_a and y_b.

The self-similarity of hadron interactions reflects a property that hadron constituents and their interactions are similar. This is connected with dropping of certain dimensional quantities out of the description of physical phenomena. The self-similar solutions are constructed in terms of the self-similarity

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parameters. We search for a scaling function

$$\psi(z) = \frac{1}{N\sigma_{in}}\frac{d\sigma}{dz} \tag{3}$$

depending on a single self-similarity variable z. Here σ_{in} is an inelastic cross section of the inclusive reaction and N is an average particle multiplicity.

In the generalized approach, the variable z is defined as

$$z = \frac{s_\perp^{1/2}}{(dN/d\eta|_0)^c} \cdot \Omega^{-1} \tag{4}$$

where

$$\Omega(x_1,x_2,y_a,y_b) = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}. \tag{5}$$

Here $s_\perp^{1/2}$ is transverse kinetic energy of the constituent sub-process consumed on production of the inclusive particle (m_1) and its counterpart (m_2). The scale of the variable z is given by the average multiplicity density $dN/d\eta|_0$ of charged particles produced in the central region of collision at pseudorapidity $\eta = 0$ raised to the power c. The parameters δ_1 and δ_2 have relation to the fractal structure of the colliding objects (hadrons or nuclei) at small scales. The parameters ε_a and ε_b characterize the fragmentation process in the final state. For the inelastic proton-proton collisions we have $\delta_1 = \delta_2 \equiv \delta$. We also assume that the fragmentation of the scattered and recoil constituents is governed by the same values of $\varepsilon_a = \varepsilon_b \equiv \varepsilon$.

The variable z has character of a fractal measure with a typical divergent property in terms of the resolution,

$$z = z(\Omega) \to \infty \qquad \text{if} \qquad \Omega^{-1} \to \infty. \tag{6}$$

The divergent factor Ω^{-1} describes a resolution at which the underlying constituent sub-process can be singled out of the inclusive reaction. The quantity $\Omega(x_1,x_2,y_a,y_b)$ is a relative number of parton configurations containing the constituents which carry the fractions x_1 and x_2 of the incoming momenta P_1 and P_2 and which fragment to the inclusive particle (m_1) and its counterpart (m_2) with the corresponding momentum fractions y_a and y_b. The momentum fractions x_1, x_2, y_a and y_b are determined form a principle of a minimal resolution of the fractal measure z. The principle states that the resolution Ω^{-1} should be minimal with respect to all binary sub-processes (1) satisfying the condition (2).

There is a relation between the variable z and the entropy

$$S = \ln W \tag{7}$$

of the system via number of the configurations which can contribute to the production of the inclusive particle with the momentum p. Relative number of the configurations W which include the constituent sub-process (1) characterized by the momentum fractions x_1,x_2,y_a and y_b is given by

$$W = (dN/d\eta|_0)^c \cdot \Omega. \tag{8}$$

The variable z is the ratio

$$z = \frac{s_\perp^{1/2}}{W} \tag{9}$$

of the transverse kinetic energy $s_\perp^{1/2}$ and maximal number of the configurations W. Using equations (7) and (8), we get

$$S = c\ln[dN/d\eta|_0] + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}]. \tag{10}$$

Exploiting analogy with the thermodynamical formula

$$S = c\ln T + R\ln V + const., \tag{11}$$

we interpret the parameter c as a "heat capacity" of the produced medium. The multiplicity density $dN/d\eta|_0$ of charged particles in the central region characterizes "temperature" of the colliding system. The second term in (10) depends on the volume in the space of the momentum fractions. This analogy emphasizes once more the interpretation of the parameters δ_1, δ_2, ε_a and ε_b as fractal dimensions. Let us note that the entropy (7) is determined up to an arbitrary constant $\ln W_0$. This degree of freedom is related to the transformation $z \to W_0 \cdot z$ and $\psi \to W_0^{-1} \cdot \psi$. In such a way the scaling variable and the scaling function are determined up to an arbitrary multiplicative constant.

Based on the definition (3), the scaling function $\psi(z)$ can be expressed as follows

$$\psi(z) = -\frac{\pi s A_1 A_2}{(dN/d\eta)\sigma_{in}} J^{-1} E\frac{d^3\sigma}{dp^3}, \tag{12}$$

where s is the square of the center-of-mass energy of the corresponding NN system, A_1 and A_2 are atomic numbers and J is the corresponding Jacobian (see [3]). The function $\psi(z)$ is normalized as follows

$$\int_0^\infty \psi(z)dz = 1. \tag{13}$$

The above relation allows us to interpret the function $\psi(z)$ as a probability density to produce an inclusive particle with the corresponding value of the variable z.

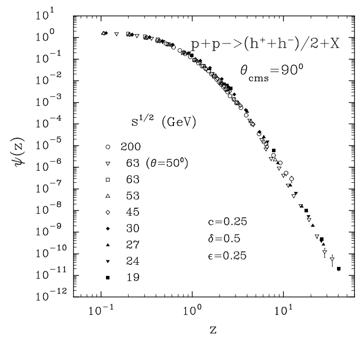

FIG. 1: The z-presentation of transverse momentum spectra of the charged hadrons produced in pp collisions at $\sqrt{s} = 19 - 200$ GeV. Experimental data are taken from Refs. [4–6, 8].

III. z-SCALING IN pp COLLISIONS AT RHIC

We have analyzed experimental data [4]-[15] on inclusive cross sections of hadrons $(h^\pm, \pi^-, K^-, \bar{p}, K_S^0, \Lambda)$ produced in

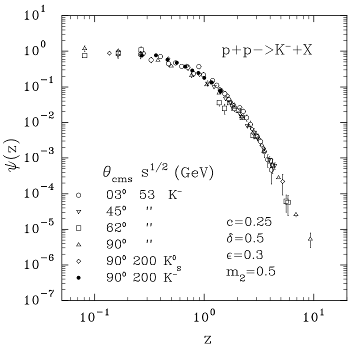

FIG. 2: The z-presentation of transverse momentum spectra of the K^--mesons produced in pp collisions for different angles at $\sqrt{s} = 53$ and 200 GeV. Experimental data are taken from Refs. [5, 10–12].

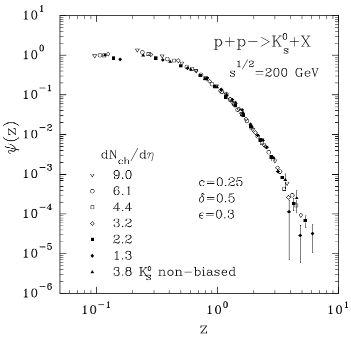

FIG. 3: The z-presentation of transverse momentum spectra of the K_S^0-mesons produced in pp collisions for different multiplicity densities at $\sqrt{s} = 200$ GeV. The K_S^0-meson spectrum for non-biased pp collisions at $\sqrt{s} = 200$ GeV is shown by black triangles. Experimental data are taken from Ref. [15].

proton-proton collisions at RHIC and compared them with ISR and FNAL data in z presentation.

As established in our previous analysis [1], z presentation of data demonstrate energy and angular independence of the scaling function $\psi(z)$ for different types of hadrons over a wide range of the center-of-mass energy $\sqrt{s}$ and the transverse momentum p_T. New analysis of data in the generalized z presentation [3] is illustrated in Figs. 1-3. The proposed generalization of the scaling variable allows angular and energy independence of $\psi(z)$ together with its multiplicity independence as well. It was established that the angular, energy and multiplicity independence of the scaling function for charged and identified hadrons can be obtained for constant values of the parameters $\delta = 0.5$ and $c = 0.25$. The value of ε increases with mass of the produced hadron.

IV. CONCLUSIONS

New generalization of z-scaling for the inclusive particle production in proton-proton collisions was suggested. The scaling variable z is a function of the multiplicity density $dN/d\eta|_0$ of charged particles in the central region of collision. The variable z depends on the parameters c, δ and ε, interpreted as a specific heat of the produced medium, a fractal dimension of the proton and a fractal dimension of the fragmentation process, respectively.

Results of analysis of experimental data on inclusive cross sections of hadrons ($h^\pm$, π^-, K^-, K_S^0, $\bar{p}$ and Λ) measured in proton-proton collisions at FNAL, ISR and RHIC are compatible with each other in the generalized z presentation.

The energy, angular and multiplicity independence of the scaling function was established. The scaling function $\psi(z)$ manifests two regimes of the particle production. The hard regime is characterized by the power law $\psi(z) \sim z^{-\beta}$ for large z. The soft processes correspond to the behavior of $\psi(z)$ for small z. Based on the results of the performed analysis we conclude that z-scaling in proton-proton collisions is a regularity which reflects the self-similarity, locality and fractality of the hadron interaction at the constituent level. It concerns the structure of the colliding objects, interactions of their constituents and fragmentation process. The obtained results can be exploited to search for and study of new physics phenomena in the particle production over a wide range of collision energies, high transverse momenta and large multiplicities in proton-proton and nucleus-nucleus interactions at RHIC and LHC.

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