Optimal paths for minimizing lost available work during heat transfer processes with a generalized heat transfer law

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A common of finite-time heat transfer processes between high- and low-temperature sides with a generalized heat transfer law [$q \propto (\Delta(T^n))^m$] are studied in this paper. The optimal heating and cooling configurations for minimizing lost available work are derived for the fixed initial and final temperatures of the working fluid of the system (low-temperature side). Optimal paths are compared with the common strategies of constant heat flux, constant source (reservoir) temperature and the minimum entropy generation operation by numerical examples. The condition corresponding to the minimum lost available work strategy is that corresponding to a constant rate of lost available work, not only valid for Newton's heat transfer law [$q \propto \Delta T$] but also valid for the generalized convective heat transfer law [$q \propto (\Delta T)^m$]. The obtained results are more general and can provide some theoretical guidelines for the designs and operations of practical heat exchangers.

Keywords: Finite time thermodynamics; Lost available work; Optimal control; Heat transfer process; Heat transfer law

1. INTRODUCTION

Since the mid 1970s, finite-time thermodynamics [1-10] has been applied to optimize the performance of various thermodynamic systems and processes. Heat exchanger is used in modern industry widely. With optimizing the performance of the heat exchanger, the energy utilization efficiency could be improved and the total volume and weight of the heat transfer equipment could be reduced, so the heat exchanger is always one of the main research subjects in finite time thermodynamics. Bejan [11] first analyzed the least combined entropy production induced by the heat transfer and the fluid viscosity as the objective function to optimize the geometry of heat transfer tubes and to find optimum parameters for heat exchangers. Lineskin and Tsirlin [12], Andresen and Gordon [13] showed that the counter-flow heat exchanger could represent the optimal solution for minimizing entropy generation, in which heat transfer between high- and low-temperature sides obeyed Newton's heat transfer law [$q \propto (\Delta T)$]. Badescu [14] further showed that the counter-flow heat exchanger could also represent the optimal solution for minimizing lost available work. In general, heat transfer is not necessarily Newton's heat transfer law and also obeys other laws. Heat transfer laws not only have significant influences on the performance of the given thermodynamic process [15-19], but also have influences on the optimal configurations of thermodynamic process for the given optimization objectives [20-23]. Nummedal and Kjelstrup [24] considered the reciprocal temperature difference $\Delta(1/T)$ as the driving force for heat transfer in irreversible thermodynamics (i.e. the linear phenomenological heat transfer law [$q \propto \Delta(T^{-1})$]), and optimized the heat transfer process for minimizing entropy generation. The results show that the reciprocal temperature difference $\Delta(1/T)$ is a constant, i.e. the principle of equipartition of forces (EoF). Based on Ref. [24], Johannessen *et al.* [25] further considered that the heat transfer coefficient is related to the local temperature changes, and showed that the local entropy generation rate is constant, i.e. equipartition of entropy production (EoEP). For a common class of finite-time heat transfer processes between high- and low-temperature sides with generalized radiative heat transfer law [$q \propto \Delta(T^n)$], Andresen and Gordon [26] derived optimal heating and cooling strategies for minimizing entropy generation. Optimal paths are compared with the common strategies of constant heat flux and constant source (reservoir) temperature operation. For a system of uniform temperature in contact with a thermal bath, in which heat transfer between high- and low-temperature sides obeys generalized radiative heat transfer law, Badescu [27] derived optimal heating and cooling strategies for minimizing lost available work. Optimal paths are also compared with the conventional strategies of constant heat-flux, constant source (reservoir) temperature operation and the optimal strategies for minimizing entropy generation. Chen *et al.* [28] and Li *et al.* [29,30] investigated the optimal performances of irreversible Carnot heat engine, refrigerator and endoreversible Carnot heat pump with a generalized heat transfer law [$q \propto (\Delta(T^n))^m$], which included the results with Newton's heat transfer law, the linear phenomenological heat transfer law, the radiative heat transfer law [$q \propto \Delta(T^4)$], the Dulong-Petit heat transfer law [$q \propto (\Delta T)^{1.25}$][31], the generalized convective transfer law [$q \propto (\Delta T)^m$] and the generalized radiative transfer law. One of aims of finite time thermodynamics is to pursue generalized rules and results. This paper will extend the previous work [12,13,14,26,27] by using the generalized heat transfer law [$q \propto (\Delta T^n)^m$], in the heat transfer processes between high- and low-temperature sides, to find the optimal heating or cooling strategies for minimizing lost available work. The obtained results are more general and can provide some theoretical guidelines for the designs of practical exchangers.

2. HEAT TRANSFER PROCESS MODEL

The heat transfer process model to be considered in this paper is illustrated in Fig. 1. The following assumptions are made for this model. The only non-negligible thermal resistance is at the heat transfer interface between system and external reservoir, where there is a known thermal conductance k. The practical control variable is the reservoir temperature $T_1(t)$and the fixed duration of the heat transfer process is τ. Both the initial temperature $T_2(0)$ and the final temperature $T_2(\tau)$ of the system are given. For specificity and clarity of

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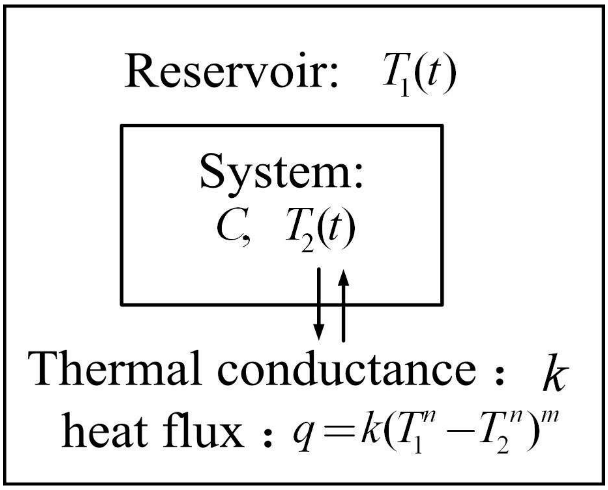

FIG. 1: Schematic of one-node thermal model for a system exchanging heat with a variable-temperature reservoir

presentation, only the case $T_1(t) > T_2(t)$ is explicitly considered here (i.e., the system is heated). System cooling involves a change of sign for the heat flux q. Consider that the heat transfer between the system and the reservoir follows a generalized law, including generalized convective heat transfer law and generalized radiative heat transfer law, $[q \propto (\Delta T^n)^m]$[28-30]. Then

$$q = k(T_1^n - T_2^n)^m \tag{1}$$

As the existence of heat resistance, the entropy generation rate associated with the heat flux q is denoted $\dot{S}$ and is given by:

$$\dot{S} = q(1/T_2 - 1/T_1) \tag{2}$$

The total entropy generation for the duration τ of the heat transfer process is:

$$S = k\int_0^{\tau}(T_1^n - T_2^n)^m(T_2^{-1} - T_1^{-1})dt \tag{3}$$

According to Ref. [27], the lost available work rate is given by:

$$\dot{W}_l = T_1\dot{S} = k\,(T_1^n - T_2^n)^m\left(T_1T_2^{-1} - 1\right) \tag{4}$$

The total lost available work W_l is obtained by integrating Eq. (4) during the heating process:

$$W_l = k\int_0^{\tau}(T_1^n - T_2^n)^m(T_1T_2^{-1} - 1)dt \tag{5}$$

In terms of the first law of thermodynamics, the change of system temperature $T_2(t)$ is governed by

$$q = k(T_1^n - T_2^n)^m = C\dot{T}_2 \tag{6}$$

where C is the heat capacity of the working fluid in the system and $\dot{T}_2 = dT_2/dt$.

3. OPTIMAL CONFIGURATION

In a fixed timeτ, the system must be heated from a known initial temperature $T_2(0)$ to a known final temperature $T_2(\tau)$. Our problem now is to determine the optimal configuration of the reservoir temperature $T_1(t)$ for minimizing lost available work subjected to the constraint of Eq. (6). Apparently, it is an optimal control problem. Correspondingly, the modified Lagrangian L with a time-dependent Lagrange multiplier $\lambda(t)$ is given by:

$$L = k(T_1^n - T_2^n)^m(T_1T_2^{-1} - 1) - \lambda(t)[k(T_1^n - T_2^n)^m - C\dot{T}_2] \tag{7}$$

The independent variables are T_1, T_2 , and $\dot{T}_2$ (and in principle $\dot{T}_1$, which does not appear in L). The Euler-Lagrange equations to determine the optimal strategy are then

$$\frac{\partial L}{\partial T_2} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{T}_2}\right) = 0, \quad \frac{\partial L}{\partial T_1} = 0 \tag{8}$$

Substituting Eq. (7) into Eq. (8) yields:

$$\dot{\lambda} = (T_1^{1-n}T_2^{n-2} - T_1T_2^{-2})\dot{T}_2 \tag{9}$$

$$\lambda = (mn+1)T_1/(mnT_2) - (T_2/T_1)^{n-1}/(mn) - 1 \tag{10}$$

Differentiating Eq. (10) with respect to time t yields:

$$\dot{\lambda} = \left[\left(1+\frac{1}{mn}\right)\frac{1}{T_2} + \frac{(n-1)T_2^{n-1}}{mnT_1^n}\right]\dot{T}_1 - \left[\left(1+\frac{1}{mn}\right)\frac{T_1}{T_2^2} + \frac{(n-1)}{mn}\frac{T_2^{n-2}}{T_1^{n-1}}\right]\dot{T}_2 \tag{11}$$

Eliminating $\dot{\lambda}$ by using Eqs. (9) and (11) yields:

$$\frac{dT_1}{dT_2} = \frac{(mn+n-1)T_1^{1-n}T_2^{n-1} + T_1T_2^{-1}}{mn+1+(n-1)T_1^{-n}T_2^n} \tag{12}$$

The detailed analytical expressions of the optimal paths will be shown here in dimensionless form. This will make the approach more flexible, increases the generality of the results and allows easy implementation. The following dimensionless variables and constants are defined:

$$\begin{array}{lll} \omega = t/\tau, & z = T_2(t)/T_2(0), & u = T_1(t)/T_2(t) \\ y = zu = T_1(t)/T_2(0) & z_f = T_2(\tau)/T_2(0), & A = k\tau[T_2(0)]^{mn-1}/C \end{array} \tag{13}$$

where z is the dimensionless system temperature, u is the ratio of the reservoir temperature to the system temperature, y is the dimensionless reservoir temperature, z_f is the final dimensionless system temperature. A is a special constant, which depends on the parameters k, τ, $T_2(0)$, m and n. The following relationships exist for the independent and dependent dimensionless variables:

$$0 \le \omega \le 1, \;\; 0 \le z \le z_f, \;\; z \le u \le y \tag{14}$$

By using the notion Eq. (13), one can define the dimensionless entropy generation rate $\dot{\tilde{S}}$ and the entropy generation $\tilde{S}$, respectively:

$$\dot{\tilde{S}} = \frac{\dot{S}}{k\,[T_2(0)]^{mn-1}} = \frac{z^{mn-1}}{u}\,(u^n-1)^m\,(u-1) \tag{15}$$

$$\tilde{S} = \frac{\tilde{S}}{k\,[T_2(0)]^{mn-1}\tau} = \int_0^1 \frac{z^{mn-1}}{u}\,(u^n-1)^m\,(u-1)\,d\omega \tag{16}$$

Here, Eqs. (2) and (3) are used. Similarly, the dimensionless rate of lost available work $\tilde{\dot{W}}_l$ and the dimensionless lost available work $\tilde{W}_l$ are defined, respectively:

$$\tilde{\dot{W}}_l = \frac{\dot{W}_l}{k\,[T_2(0)]^{mn}} = z^{mn}\,(u^n-1)^m\,(u-1) \tag{17}$$

$$\tilde{W}_l = \frac{W_l}{k\,[T_2(0)]^{mn}\,\tau} = \int_0^1 z^{mn}\,(u^n-1)^m\,(u-1)\,d\omega \tag{18}$$

Here, Eqs. (4), (5) and (13) are used. By using the notation Eq. (13), the new forms of Eqs. (6) and (12) are, respectively,

$$dz/d\omega = Az^{mn}\,(u^n-1)^m \tag{19}$$

$$\frac{(mn+1)\,u^n+(n-1)}{u\,(u^n-1)}du = -mn\frac{dz}{z} \tag{20}$$

Eq. (19) gives through integration:

$$z^{mn}\,(u^n-1)^{m+1} - C_W u^{n-1} = 0 \tag{21}$$

where C_W is an integration constant, which is determined by $T_2(0)$ and z_f. From Eqs. (20) and (21) one obtains:

$$\frac{du}{d\omega} = \frac{-mnAC_W^{(mn-1)/(mn)}\,u^{(mn^2-n+1)/(mn)}\,(u^n-1)^{(m+1)/(mn)}}{(mn+1)u^n+n-1} \tag{22}$$

The optimal heating paths are obtained by solving Eqs. (21) and (22). Inputs are the values of m, n, A and z_f.

4. DISCUSSIONS

4.1. Effects of heat transfer laws

When $m=1$, the heat transfer law becomes the generalized radiative heat transfer law, Eqs. (21) and (22) become

$$z^n\,(u^n-1)^2 - C_W u^{n-1} = 0 \tag{23}$$

$$\frac{du}{d\omega} = \frac{-nAC_W^{(n-1)/n}\,u^{(n^2-n+1)/n}\,(u^n-1)^{2/n}}{(n+1)u^n+n-1} \tag{24}$$

They are the same results as those obtained in Ref. [27]. If $n=1$ further, Equations (23) and (24) are the results of the heat transfer process with Newton's heat transfer law [14, 27]. If $n=-1$ further, Equations (23) and (24) are the results of the heat transfer process with the linear phenomenological heat transfer law [27]. If $n=4$ further, Equations (23) and (24) are the results of the heat transfer process with the radiative heat transfer law [27].

When $n=1$, the heat transfer law becomes the generalized convective heat transfer law. Eqs. (21) and (22) take the form

$$z^m\,(u-1)^{m+1} = C_W \tag{25}$$

$$\frac{du}{d\omega} = \frac{-mAC_W^{(m-1)/m}\,(u-1)^{(m+1)/m}}{m+1} \tag{26}$$

If $m=1$ further, Equations (25) and (26) are the results of the heat transfer process with Newton's heat transfer law [14, 27]. If $m=1.25$ further, Equations (14) and (15) are the results of the heat transfer process with the Dulong-Petit heat transfer law, which become:

$$z(\omega) = \left[1+(z_f^{5/9}-1)\omega\right]^{9/5} \tag{27}$$

$$u(\omega) = 1+\left[\frac{4A}{9(z_f^{5/9}-1)}\right]^{9/20}\left[\frac{4A\omega}{9}+\frac{4A}{9(z_f^{5/9}-1)}\right]^{-5/4} \tag{28}$$

Substituting Eqs. (27) and (28) into Eqs. (17) and (18) yields:

$$\tilde{\dot{W}}_l = \left[9(z_f^{5/9}-1)\big/(4A)\right]^{9/5}, \ \ \tilde{W}_l = \left[9(z_f^{5/9}-1)\big/(4A)\right]^{9/5} \tag{29}$$

From Eq. (29), it can be concluded that the condition corresponding to the minimum lost available work strategy with the Dulong-Petit heat transfer law is that corresponding to a constant rate of lost available work.

When $m=1.25$ and $n=4$, the heat transfer law becomes a complex heat transfer law $q \propto (\Delta(T^4))^{1.25}$. There is no analytical solution for this case, so a numerical approach is necessary.

Salamon *et al.* [32] proved that for any linear finite-time process, the strategy that minimizes entropy generation was the one that corresponds to a constant rate of entropy generation. Andresen and Gordon [26] further showed that only for the specific cases of Newton's and linear phenomenological transfer laws was the observation of Salamon *et al* valid. Meanwhile, they proved that for general non-linear problems, a constant rate of entropy generation was not the optimal strategy. Badescu [27] showed that the optimum rate of lost available work is constant over time in the case of the minimum lost available work strategy with Newton's heat transfer law.

When $n=1$, Eq. (17) becomes

$$\tilde{\dot{W}}_l = z^m\,(u-1)^{m+1} \tag{30}$$

Evidently, the right side of Eq. (30) is the same as the left side of Eq. (25). Hence, the condition corresponding to the minimum lost available work strategy is that corresponding to a constant rate of lost available work, not only valid for Newton's heat transfer law but also valid for the generalized convective heat transfer law, in which the conclusions in Refs. [14, 27] are included.

4.2. Other heating and cooling strategies

Besides the optimal strategy for minimizing lost available work, there are other cases, such as the optimal strategies for minimizing entropy generation, the strategies of constant heat flux and constant reservoir temperature operation and so on.

4.2.1. Optimal strategies for minimizing entropy generation

According to Ref. [33], the optimal strategies for minimizing entropy generation are:

$$y^n - C_S y^{(n+1)/(m+1)} - z^n = 0 \tag{31}$$

$$\frac{dy}{d\omega} = \frac{nC_S^m A y^{m(n+1)/(m+1)}\left(y^n - C_S y^{(n+1)/(m+1)}\right)^{(n-1)/n}}{ny^{n-1} - C_S(n+1)y^{(n-m)/(m+1)}/(m+1)} \tag{32}$$

where C_S is an integration constant, which is determined by $T_2(0)$ and z_f.

4.2.2. Constant reservoir temperature operation

When $y = const$, the unknown y is determined by the following implicit equation:

$$\int_1^{z_f} 1/(y^n - z^n)^m dz = A \tag{33}$$

Once y is found from Eq. (33), $z(\omega)$ is obtained by substituting y into the following equation:

$$\int_1^{z(w)} \frac{1}{(y^n - z^n)^m} dz = A\omega \tag{34}$$

Finally, The value of $u(\omega)$ is obtained by $u(\omega) = y/z(\omega)$. When $m = 1$ and n is equal to 1, -1, and 4, respectively, the results with Newton's, the linear phenomenological and radiative heat transfer laws are obtained in Refs. [26, 27]. When $m = 1.25$ and $n = 1$, the result with the Dulong-Petit heat transfer law is obtained. In this case, Eqs. (33) and (34) become

$$(y - z_f)^{-1/4} - (y-1)^{-1/4} = A/4 \tag{35}$$

$$z(\omega) = y - \left[A\omega/4 + (y-1)^{-1/4}\right]^{-4} \tag{36}$$

When $m = 1.25$ and $n = 4$, the result with the complex heat transfer law $[q \propto (\Delta(T^4))^{1.25}]$ is obtained. In this case, Eqs. (33) and (34) become

$$\frac{z_f}{(y^4 - z_f^4)^{1/4}} - \frac{1}{(y^4-1)^{1/4}} - Ay^4 = 0 \tag{37}$$

$$\frac{z(\omega)}{(y^4 - z(\omega)^4)^{1/4}} - \frac{1}{(y^4-1)^{1/4}} - Ay^4\omega = 0 \tag{38}$$

4.2.3. Constant heat flux operation

When $q = const$, from Eqs. (1) and (13), one can obtain

$$z(\omega) = 1 + (z_f - 1)\omega \tag{39}$$

$$u(\omega) = \left\{1 + \left\{\frac{z_f - 1}{A\left[1 + (z_f - 1)\omega\right]^{mn}}\right\}^{1/m}\right\}^{1/n} \tag{40}$$

When $m = 1$ and n is equal to 1, -1, and 4, respectively, the results with Newton's, the linear phenomenological and radiative heat transfer laws are obtained in Refs. [26, 27]. When $m = 1.25$ and $n = 1$, the result with the Dulong-Petit heat transfer law is obtained. In this case, Eq. (40) becomes

$$u(\omega) = 1 + \left[(z_f - 1)/A\right]^{4/5}\left[1 + (z_f - 1)\omega\right] \tag{41}$$

When $m = 1.25$ and $n = 4$, the result with the complex heat transfer law $[q \propto (\Delta(T^4))^{1.25}]$ is obtained. In this case, Eq. (40) becomes

$$u(\omega) = \left\{1 + \left\{\frac{z_f - 1}{A\left[1 + (z_f - 1)\omega\right]^5}\right\}^{4/5}\right\}^{1/4} \tag{42}$$

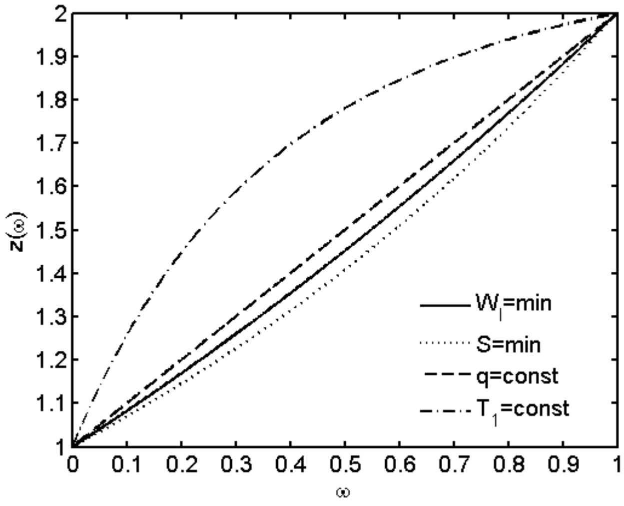

FIG. 2: Dependence of the dimensionless system temperature $z(\omega)$ on the dimensionless time ω in case of the Dulong-Petit heat transfer law

5. NUMERICAL EXAMPLES

For the convenience of analysis, W_l = mindenotes the optimal strategy for minimizing lost available work, S = mindenotes the optimal strategy for minimizing entropy generation, $q = const$ denotes the strategy of constant heat flux operation, and $T_1 = const$ denotes the strategy of constant reservoir temperature operation.

5.1. Numerical example for the Dulong-Petit heat transfer law

In this case, $A = 2.5$ and $z_f = 2$ are set. Fig. 2 shows dependence of the dimensionless system temperature $z(\omega)$ on the dimensionless time ω in case of the Dulong-Petit heat transfer law. The most abrupt variation of $z(\omega)$ at the starting time corresponds to the strategy of $T_1 = const$. It is associated with the highest rate of entropy generation. The other three strategies show an almost linear increase of system temperature over time. Fig. 3 shows dependence of the ratio of the reservoir temperature to the system temperature $u(\omega)$ on the dimensionless time ω in case of the Dulong-Petit heat transfer law. The strategies of $S = \min$ and $W_l = $ minshow a slight function $u(\omega)$ decreasing over time. Both the final reservoir temperature and the final system temperature for the strategies of $q = const$ are equal to those for the strategies of $W_l = \min$, but the initial reservoir temperature for the strategies of $q = const$ is larger than that for the strategies of $W_l = \min$, i.e. the temperature difference between the reservoir and the system for the strategies of $q = const$ is larger than that for the strategies of $W_l = \min$. Hence the entropy generation for the strategies of $q = const$ is larger than that for the strategies of $W_l = \min$. The temperature difference between the reservoir and the system for the strategies of $T_1 = const$ during the heat transfer processes is the largest among various strategies, so the strategies of $T_1 = const$ leads to the largest entropy generation [26, 27, 33].

Fig. 4 shows dependence of the dimensionless rate of lost

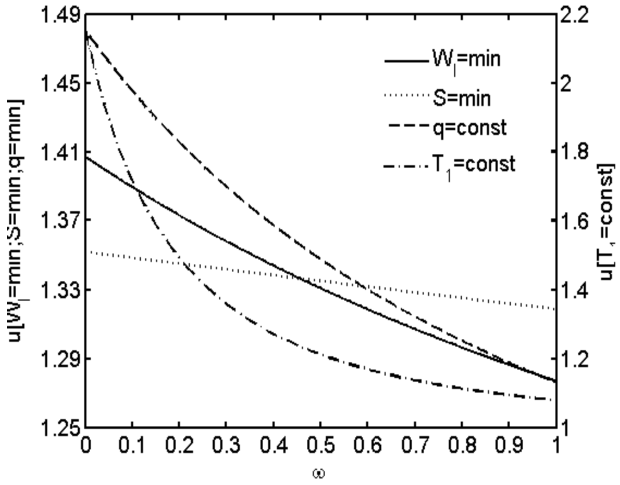

FIG. 3: Dependence of the ratio of the reservoir temperature to the system temperature $u(\omega)$ on the dimensionless time ω in case of the Dulong-Petit heat transfer law

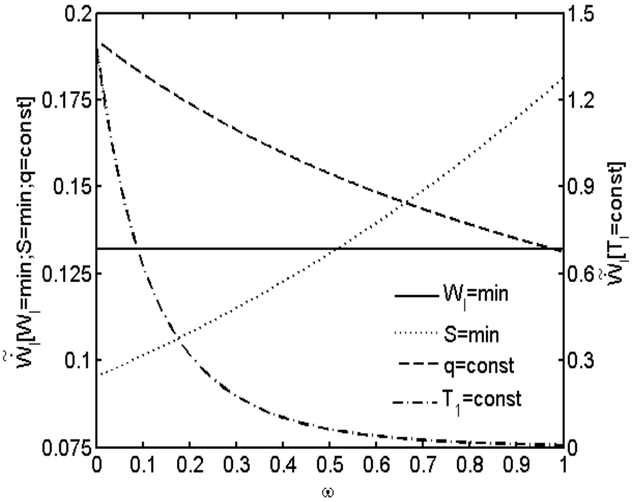

FIG. 4: Dependence of the dimensionless rate of lost available work $\tilde{\dot{W}}_l$ on the dimensionless time ω in case of the Dulong-Petit heat transfer law

available work $\tilde{\dot{W}}_l$ on the dimensionless time ω in case of the Dulong-Petit heat transfer law. From Fig. 4, it can be seen that the profiles of $\tilde{\dot{W}}_l$ for various heating strategies are different. The strategy of $q = const$ shows a function $\tilde{\dot{W}}_l$ decreasing over time, while the strategy of $S = \min$ shows a function $\tilde{\dot{W}}_l$ increasing over time. The strategy of $T_1 = const$ shows a fast function $\tilde{\dot{W}}_l$ decreasing at the beginning of the heat transfer process, but $\tilde{\dot{W}}_l$ varies little at the end of the heat transfer process. Also from Fig. 4, the rate of lost available work $\tilde{\dot{W}}_l$ for the strategy of $W_l = \min$ is constant in the case of the Dulong-Petit heat transfer law, which is the same as the result with Newton's heat transfer law [27]. As is discussed in section 4.1, both Newton's and the Dulong-Petit heat transfer laws belong to special cases of the generalized convective heat transfer law, so the condition corresponding to the minimum lost available work strategy is that corresponding to a constant rate of lost available work, not only valid for Newton's heat transfer law but also valid for the Dulong-Petit heat transfer law. Fig. 5 shows dependence of the dimensionless lost available work $\tilde{W}_l$ on the dimensionless system final temperature z_f in case of the Dulong-Petit heat transfer law. The strategies of $W_l = \min$, $S = \min$ and $q = const$ show similar results of the lost available work $\tilde{W}_l$, i.e. all of the values of $\tilde{W}_l$ increases with the increase of z_f. Also from Fig. 5, it can be seen that keeping a constant reservoir temperature is the worst strategy as far as the lost available work is concerned.

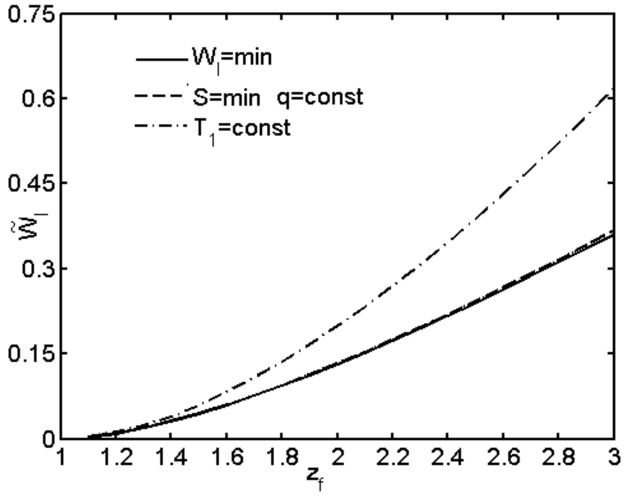

FIG. 5: Dependence of the dimensionless lost available work $\tilde{W}_l$ on the dimensionless system final temperature z_f in case of the Dulong-Petit heat transfer law

5.2. Numerical example for the heat transfer law $q \propto (\Delta(T^4))^{1.25}$

In this case, $A = 0.02$ and $z_f = 2$ are set. Fig. 6 shows dependence of the dimensionless system temperature $z(\omega)$ on the dimensionless time ω in case of the heat transfer law $q \propto (\Delta(T^4))^{1.25}$. The highest dimensionless system temperature $z(\omega)$ during the whole heat transfer process corresponds to the strategy of $T_1 = const$, the dimensionless system temperature $z(\omega)$ for the strategy of $q = const$ lies between that for the strategy of $T_1 = const$ and that for the strategy of $S = \min$, the lowest dimensionless system temperature $z(\omega)$ corresponds to the strategy of $W_l = \min$. Fig. 7 shows dependence of the ratio of the reservoir temperature to the system temperature $u(\omega)$ on the dimensionless time ω in case of the heat transfer law $q \propto (\Delta(T^4))^{1.25}$. At the beginning of heat transfer process, the temperature ratio $u(\omega)$ for the strategy of $W_l = \min$ is larger than that for the other strategies, while at the end of heat transfer process, the temperature ratio $u(\omega)$ for the strategy of $W_l = \min$ is smaller than those for the other strategies. Also from Fig. 7, the variation of the reservoir temperature for the strategy of $S = \min$ is very similar to that for the strategy of $W_l = \min$. Fig. 8 shows dependence of the dimensionless rate of lost available work $\tilde{\dot{W}}_l$ on the dimensionless time ω in case of the heat transfer law $q \propto (\Delta(T^4))^{1.25}$. Compared to the

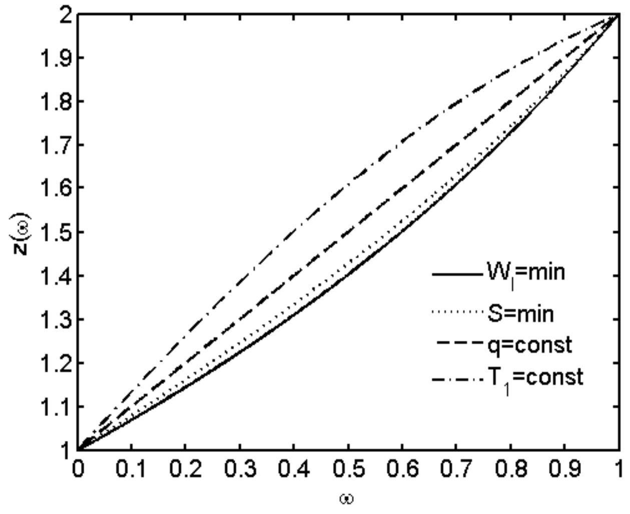

FIG. 6: Dependence of the dimensionless system temperature $z(\omega)$ on the dimensionless time ω in case of the heat transfer law $q \propto (\Delta(T^4))^{1.25}$

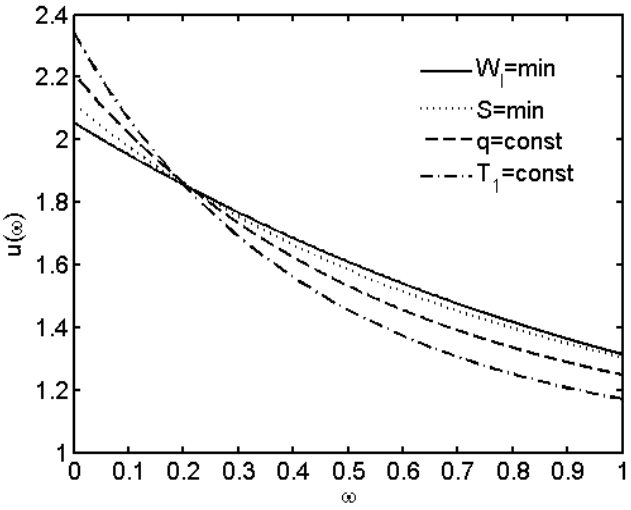

FIG. 7: Dependence of the ratio of the reservoir temperature to the system temperature $u(\omega)$ on the dimensionless time ω in case of the heat transfer law $q \propto (\Delta(T^4))^{1.25}$

results with the Dulong-Petit heat transfer law, the differences of the time variations of $\tilde{W}_l$ among the strategies of $q = const$, $S = \min$ and $T_1 = const$ are smaller. The strategy of $W_l = \min$ shows a slow and almost linear time variation of $\tilde{W}_l$, i.e. at the beginning of heat transfer process, the rate of lost available work $\tilde{W}_l$ for the strategy of $W_l = \min$ is smaller than those for the other strategies, while at the end of heat transfer process, the rate of lost available work $\tilde{W}_l$ for the strategy of $W_l = \min$ is larger than those for the other strategies.

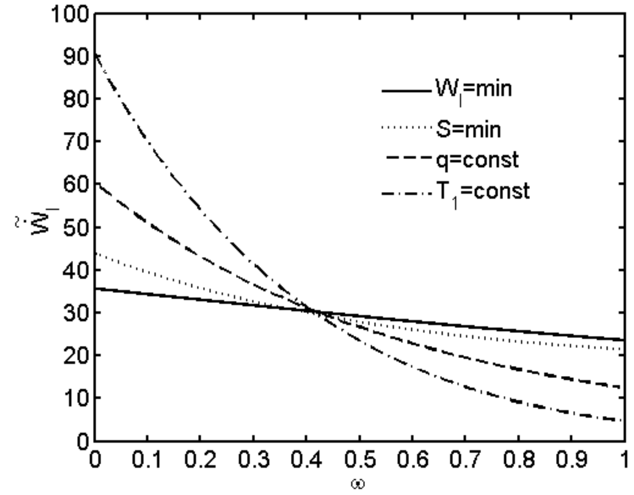

FIG. 8: Dependence of the dimensionless rate of lost available work $\tilde{W}_l$ on the dimensionless time ω in case of the heat transfer law $q \propto (\Delta(T^4))^{1.25}$

5.3. Performance comparisons for special heat transfer laws

According to Ref. [27], the parameters in Newton's, the linear phenomenological and radiative heat transfer laws are used. $A = 10$ and $z_f = 2$ are set for Newton's heat transfer law. $A = -10$ and $z_f = 2$are set for the linear phenomenological heat transfer law, $A = 2.5$and$z_f = 2$ are set for the Dulong-Petit heat transfer law, $A = 0.025$ and $z_f = 2$ are set for the radiative heat transfer law, and $A = 0.02$ and $z_f = 2$ are set for the heat transfer law $q \propto (\Delta(T^4))^{1.25}$. Fig. 9 shows dependence of the dimensionless system temperature $z(\omega)$ on the dimensionless time ω in case of the minimum lost available work with the special heat transfer laws. The results show that differences between the optimal variation of the system temperature with Newton's heat transfer law and that with the Dulong-Petit heat transfer law are so small that could be negligible. The time variation of the dimensionless system temperature $z(\omega)$ with the Dulong-Petit heat transfer law is very similar to that with Newton's heat transfer law, while the time variation of the dimensionless system temperature $z(\omega)$ with the radiative heat transfer law is very similar to that with the heat transfer law $q \propto (\Delta(T^4))^{1.25}$. It's evident that the heat transfer laws have effects on the optimal configuration of the system temperature. Fig. 10 shows dependence of the dimensionless reservoir temperature $y(\omega)$ on the dimensionless time ω in case of the minimum lost available work with the special heat transfer laws. From Fig. 10, the optimal configuration of the system temperature for Newton's heat transfer law are almost the same as that for the Dulong-Petit heat transfer law, but the optimal configurations of the reservoir temperature for them are different. The reservoir temperature for the Dulong-Petit heat transfer law over the time is smaller than that for Newton's heat transfer law. The variation of the optimal reservoir temperature for Newton's heat transfer law is very similar to that for the Dulong-Petit heat transfer law, while the variation of the optimal reservoir temperature for the radiative heat transfer law is very similar to that for the heat transfer law $q \propto (\Delta(T^4))^{1.25}$.

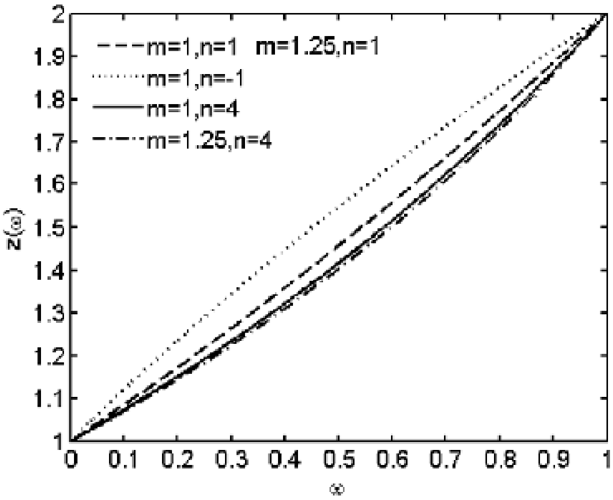

FIG. 9: Dependence of the dimensionless system temperature $z(\omega)$ on the dimensionless time ω in case of the minimum lost available work with the special heat transfer laws

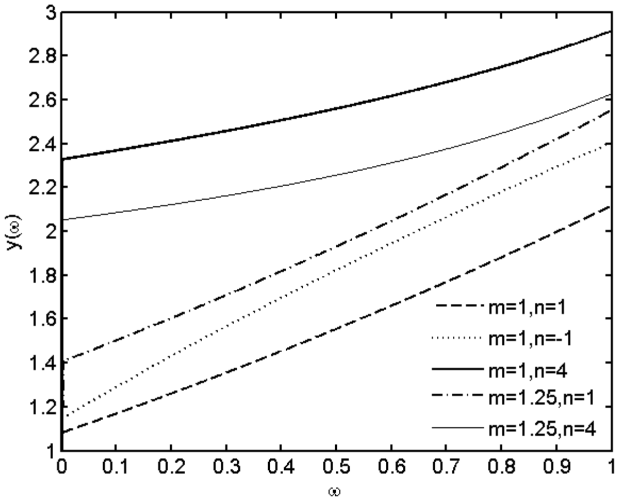

FIG. 10: Dependence of the dimensionless reservoir temperature $y(\omega)$ on the dimensionless time ω in case of the minimum lost available work with the special heat transfer laws

Besides, the variation of the optimal reservoir temperature for the linear phenomenological heat transfer law is different from those for the other heat transfer laws. This shows that the heat transfer laws also have effects on the optimal configuration of the reservoir temperature.

6. CONCLUSION

A common of finite-time heat transfer processes between high- and low-temperature sides with a generalized heat transfer law $q \propto (\Delta(T^n))^m$ is studied in this paper. The optimal heating and cooling configurations for minimizing lost available work are derived for the fixed initial and final temperatures of the low-temperature side working fluid of the system, the obtained results are general, including those results with special heat transfer laws. A more general conclusion is formed: the condition corresponding to the minimum lost available work strategy is that corresponding to a constant rate of lost available work, not only valid for Newton's heat transfer law but also valid for generalized convective heat transfer law $q \propto (\Delta T)^m$, in which the results in Refs. [14, 27] are included. Numerical examples for minimizing lost available work with the Dulong-Petit heat transfer law and the heat transfer law $q \propto (\Delta(T^4))^{1.25}$ are provided, and the results are also compared with optimal strategy for minimizing entropy generation, and the common strategies of constant heat flux and constant reservoir temperature operation. Numerical calculations show that the constant reservoir temperature strategy leads to the largest lost available work, the strategies of minimum entropy generation and constant heat flux produce less lost available work. Performance comparisons for special heat transfer laws are also provided, and the results show that the heat transfer laws have effects on the optimal time path of heat transfer processes. As has been stated in Refs. [14, 27], optimizing the thermodynamic systems for minimizing lost available work is different from that for minimizing entropy generation when choosing different reference states. Certainly, the two methods are equivalent when the universe environment is chosen to be the reference state. Choosing between the two optimum criteria (i.e. minimum entropy generation and minimum lost available work) depends on the particular implementation of the heating/cooling process. The method based on the entropy generation minimization could be used, for example, in the case of a chemical factory that delivers various products and secondary utilities as flows of heat and power. The entropy generation should be seen in this case as a common measure for the cost of production of all these outputs of different nature, allowing an overall optimization. The method based on lost available work minimization could be used for example during the design of some power plants or in those cases where the main interest is in delivering a maximum output power. The obtained results in this paper are general and can provide some theoretical guidelines for the designs of practical heat exchangers.

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