Ground State Masses and Binding Energies of the Nucleon, Hyperon and Heavy Baryons in a Light-Front Model

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The ground state masses and binding energies of the nucleon, Λ^0, Λ_c^+, Λ_b^0 are studied within a constituent quark QCD-inspired light-front model. The light-front Faddeev equations for the Qqq composite spin 1/2 baryons, are derived and solved numerically. The experimental data for the masses are qualitatively described by a flavor independent effective interaction.

I Introduction

Modelling the light-front hadron wave-function is a challenge as long as the exact wave-function from Quantum Cromodynamics is not yet avaliable. The wave-function contains physical information complementary to the spectrum. At scales below 1 GeV, the wave-function is described by effective degrees of freedom, and the lowest Fock-state component of the light-front hadron wave function is composed by the minimal number of constituent quarks necessary to give the quantum numbers. Within this general framework, a light-front QCD-inspired model was recently applied to the pion and other mesons [1, 2]. A reasonably description of the pion structure as well as the masses of the vector and pseudo-scalar mesons was found. This model, without confinement, describes the vector meson as weakly bound system of constituent quarks. The spin does not play a dynamical role besides justifying the contact term coming from the hyperfine interaction. In this way the one-gluon-exchange interaction is simplified to two components only: a contact term and a Coulomb-type potential. The contact term is essential to collapse the constituent quark-antiquark system to form the pion, while the vector meson is dominated by the Coulomb-type potential. The contact interaction brings to the model the physical scale of the pion mass which determines the masses of the other pseudo-scalar and vector mesons [2].

Here, we follow closely the work of Ref. [3] and revise the extension of the concepts coming from the effective QCD-model, applied to mesons [1, 2], to study the spin 1/2 low-lying states of the nucleon, Λ^0, Λ_c^+ and Λ_b^0 [3]. We use a flavor independent effective interaction between the constituent quarks, a property necessary to describe the masses of these baryons [3]. The three-quark relativistic dynamics of the Qqq system is formulated within the light-front framework in a truncated Fock-space [4] which is stable under kinematical boost transformations [5] and yields a wave-function covariant under kinematical boosts [6, 7]. We use only the contact interaction, which in this case provides to the model the physical scale of the mass of the nucleon ground state, while the spin is averaged out. The binding energy of the baryon is calculated using three-quark Faddeev equations [3] as a function of the mass of one of the constituent quarks (Q), while the bare strength of the effective contact interaction and the mass of the quark q are kept constant. This light-front model with a contact force [4] has been applied to the proton and described its mass, charge radius and electric form factor up to 2(GeV/c)^2 [8]. Recently, it was also applied to study the dissolution of the nucleon at finite temperature and baryonic density [9].

This work is organized as follows. In Sec.II, we present the coupled integral equations for the Faddeev components of the vertex of the three-quark light-front bound-state wave function from a flavor independent contact interaction derived in Ref. [3]. In Sec. III, we present the numerical results for the masses and binding energies of the nucleon, Λ^0, Λ_c^+ and Λ_b^0, obtained from the numerical solution of light-front integral equations. A key point in this section is the assignement of the constituent quark masses which are done using the experimental values of the vector meson ground states masses. In Sec. IV, we present our summary.

II Light-front model

The light-front hyperplane is defined by the time $x^+ = x^0 + x^3 = 0$ and the position coordinates on the hypersurface are defined by $x^- = x^0 + x^3$ and $\vec{x}_\perp = (x^1, x^2)$ [6, 7]. The coordinate x^+ is the light-front time and the momentum, $k^- = k^0 - k^3$, corresponds to the light-front energy. The momentum coordinates k^+ and $\vec{k}_\perp$, are the kinematical momenta canonically conjugated to x^- and $\vec{x}_\perp$, respectively. The dynamics of the model is defined by a two-body contact interaction between the constituent quarks at equal light-front times [4, 8, 3]. Considering the contact interaction between the quarks, the baryon-Qqq vertex is described by two terms written as $v_\alpha(\vec{q}_\perp, y)$ in the baryon rest frame, with $\alpha = q$ or Q, where $y = q^+/M_B$ is the Bjorken momentum fraction and M_B is the baryon mass. The separable structure of the contact interaction implies that the vertex function depends only on the kinematical variables of the spectator quark in the process of the interaction of the other two. As we are averaging over spin, only two independent vertex functions are necessary to describe the baryon wave function, i.e., v_q and v_Q.

The coupled Faddeev-Bethe-Salpeter equations for a heavy-light-light three quark system (Qqq) in the light-front are derived in the ladder approximation [3], and are a generalization of the Weinberg equation [10] to three particle systems. Their diagrammatical representation are given in Figs. 1 and 2. The first light-front equation in which the quark Q is the spectator while the pair qq interacts is represented in Fig. 1, from where one reads:

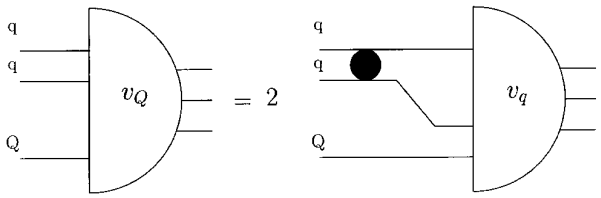

Figure 1. Diagrammatic representation of Eq.1. The black bubble represents the two-quark scattering amplitude.

$$v_Q(\vec{q}_\perp, y) = -\frac{2i}{(2\pi)^3}\tau_{qq}\left(M_{qq}^2\right)\int_0^{1-y}\frac{dx}{x\,(1-x-y)}\int d^2k_\perp \times \frac{\theta\left(x-\frac{m_q^2}{M_B^2}\right)\theta\left(k_\perp^{\max}(m_q)-k_\perp\right)v_q(\vec{k}_\perp,x)}{M_B^2-\frac{q_\perp^2+m_Q^2}{y}-\frac{k_\perp^2+m_q^2}{x}-\frac{(P_B-q-k)_\perp^2+m_q^2}{1-x-y}}. \quad (1)$$

The second light-front equation, which is represented diagrammatically in Fig. 2, the quark pair Qq interacts while the quark q is the spectator. The second equation is given by:

$$v_q(\vec{q}_\perp, y) = -\frac{i}{(2\pi)^3}\tau_{Qq}\left(M_{Qq}^2\right)\int_0^{1-y}\frac{dx}{x\,(1-x-y)}\int d^2k_\perp \times \left[\frac{\theta\left(x-\frac{m_q^2}{M_B^2}\right)\theta\left(k_\perp^{\max}(m_q)-k_\perp\right)v_q(\vec{k}_\perp,x)}{M_B^2-\frac{q_\perp^2+m_q^2}{y}-\frac{k_\perp^2+m_q^2}{x}-\frac{(P_B-q-k)_\perp^2+m_Q^2}{1-x-y}} + \frac{\theta\left(x-\frac{m_Q^2}{M_B^2}\right)\theta\left(k_\perp^{\max}(m_Q)-k_\perp\right)v_Q(\vec{k}_\perp,x)}{M_B^2-\frac{q_\perp^2+m_q^2}{y}-\frac{k_\perp^2+m_Q^2}{x}-\frac{(P_B-q-k)_\perp^2+m_q^2}{1-x-y}}\right]. \quad (2)$$

The maximum value for $k_\perp$ is chosen to keep the mass squared of the qq or Qq subsystem real, i.e., $M_{qq}^2 \geq 0$ and $M_{Qq}^2 \geq 0$, respectively. These constraints in the spectator quark phase-space come through the theta functions in the integrations of Eqs. (1) and (2). For $M_{Qq}^2 \geq 0$ one has $k_\perp < k_\perp^{\max}(m_q) = \sqrt{(1-x)(M_B^2 x - m_q^2)}$, and $x \geq (m_q/M_B)^2$. For $M_{qq}^2 \geq 0$ one has $k_\perp < k_\perp^{\max}(m_Q) = \sqrt{(1-x)(M_B^2 x - m_Q^2)}$, and $x \geq (m_Q/M_B)^2$. For equal particles, Eq.(1), reduces to the one derived in Ref. [4]. The baryon four-momentum is given by P_B, the light and heavy quark masses are m_q and m_Q, respectively. The masses of the virtual two-quark subsystems are $M_{qq}^2 = (P_B - q)^2$ and $M_{Qq}^2 = (P_B - q)^2$ due to the conservation of the total four-momentum.

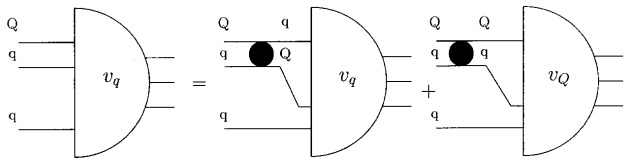

Figure 2. Diagrammatic representation of Eq. (2). The black bubble represents the two-quark scattering amplitude.

The two-quark scattering amplitudes $\tau_{qq}\left(M_{qq}^2\right)$ and $\tau_{Qq}\left(M_{Qq}^2\right)$ are the solutions of the Bethe-Salpeter equations in the ladder approximation for a contact interaction between the quarks [4, 11]. In this approximation the scattering amplitude, which is the infinite sum of the powers of the product of the "bubble"-diagram with the bare interaction strength, is given by the geometrical series:

$$\tau_{\alpha q}\left(M_{\alpha q}^2\right) = \frac{1}{i\lambda^{-1} - \mathcal{B}_{\alpha q}\left(M_{\alpha q}^2\right)}\,, \qquad (3)$$

where $\alpha = q$ or Q and λ is the bare interaction strength, and

$$\begin{aligned}\mathcal{B}_{\alpha q}\left(M_{\alpha q}^2\right) &= \int \frac{d^4k}{(2\pi)^4}\frac{i}{(k^2 - m_q^2 + i\varepsilon)} \\ &\times \frac{i}{((P_{\alpha q} - k)^2 - m_\alpha^2 + i\varepsilon)}\,, \end{aligned} \qquad (4)$$

in which $P_{\alpha q}$ is the total four-momentum of the quark pair and $P_{\alpha q}^2 = M_{\alpha q}^2$.

The four-dimensional integration of the function $\mathcal{B}_{\alpha q}\left(M_{\alpha q}^2\right)$ is the "bubble"-diagram is performed in light-front variables. First, the virtual propagation of the intermediate quarks is projected at equal light-front times [4, 12], by analytical integration over k^- in the momentum loop. Then, using the frame in which $\vec{P}_{\alpha q\perp}$ is zero and introducing the invariant quantity $x = \frac{k^+}{P_{\alpha q}^+}$, one obtains:

$$\begin{aligned}\mathcal{B}_{\alpha q}\left(M_{\alpha q}^2\right) &= \frac{i}{2\left(2\pi\right)^3}\int \frac{dx d^2k_\perp}{x\left(1-x\right)} \\ &\times \frac{\theta(1-x)\theta(x)}{M_{\alpha q}^2 - \frac{k_\perp^2 + \left(m_\alpha^2 - m_q^2\right)x + m_q^2}{x(1-x)}}. \end{aligned} \qquad (5)$$

We suppose that the light-quark pair system has a bound state, which allows to define the two-quark scattering amplitude. This physical condition has been used in Ref. [8]. Therefore, the bound state pole of the light-quark scattering amplitude, $\tau_{qq}(M_{qq}^2)$ is found when M_{qq} is equal to the mass of the bound qq pair, M_d which demands that $i\lambda^{-1} = \mathcal{B}_{qq}\left(M_d^2\right)$. This is suficient to render finite the scattering amplitudes τ_{qq} and τ_{Qq}. Using that the bare strength of the effective contact interaction between the constituent quarks q and Q does not depend on flavor, the final equation for the two-quark scattering amplitude is:

$$\tau_{\alpha q}\left(M_{\alpha q}^2\right) = \frac{1}{\mathcal{B}_{qq}\left(M_d^2\right) - \mathcal{B}_{\alpha q}\left(M_{\alpha q}^2\right)}\,. \qquad (6)$$

The log-type divergence of $\tau_{\alpha q}$ is removed by the subtraction in Eq.(6).

III Results

The solution of the coupled integral equations (1) and (2) for a relativistic system of three constituent quarks with a pairwise zero range interaction, gives the baryon mass as a function of the quark mass m_Q. The physical inputs of the model are the constituent quark mass m_q and the diquark bound state mass. The result for the mass of the ground state baryon (M_B) alllows to calculate the binding energy, defined by $B_B = 2m_q + m_Q - M_B$. However, the quarks are in fact confined in the hadron and to compare the model with data, one has to define an experimental quantity which could be compared to the model binding energy.

For that purpose, we use that the low-lying vector mesons are weakly bound systems of constituent quarks while the pseudo-scalars are more strongly bound within the same model [2]. Therefore, we suppose that the masses of the constituent quarks can be derived directly from the vector meson masses as:

$$\begin{aligned} m_u &= \frac{1}{2}M_\rho = 0.384\ \text{GeV} \\ m_s &= M_{K^*} - \frac{1}{2}M_\rho = 0.508\ \text{GeV} \\ m_c &= M_{D^*} - \frac{1}{2}M_\rho = 1.623\ \text{GeV} \\ m_b &= M_{B^*} - \frac{1}{2}M_\rho = 4.941\ \text{GeV}, \end{aligned} \qquad (7)$$

where the masses of the mesons are taken from Ref. [13].

Using the constituent quark masses from Eq. (7) and the experimental values of the baryon masses [13], we can attribute a binding energy to the low-lying spin 1/2 baryons, as given below:

$$\begin{aligned} B_p^{exp} &= \frac{3}{2}M_\rho - M_p = 0.214\ \text{GeV} \\ B_{\Lambda^0}^{exp} &= M_{K^*} + \frac{1}{2}M_\rho - M_{\Lambda^0} = 0.161\ \text{GeV} \\ B_{\Lambda_c^+}^{exp} &= M_{D^*} + \frac{1}{2}M_\rho - M_{\Lambda_c^+} = 0.106\ \text{GeV} \\ B_{\Lambda_b^0}^{exp} &= M_{B^*} + \frac{1}{2}M_\rho - M_{\Lambda_b^0} = 0.085\ \text{GeV}\,. \end{aligned} \qquad (8)$$

In Fig. 3, we plot the binding energies of the low-lying pseudo-scalar mesons, defined as $B_M = M_v - M_{ps}$, where M_v and M_{ps} are the masses of the vector and pseudoscalar low-lying mesons respectively, against the mass of the corresponding pseudo-scalar meson. Also in the figure is shown the values of the binding energies of the spin 1/2 baryons (N, Λ^0, Λ_c^+ and Λ_b^0) from Eq. (8) as a function of the corresponding baryon mass. The systematic behaviour of the defined binding energy for the hadron is qualitative the same independent of the quark content.

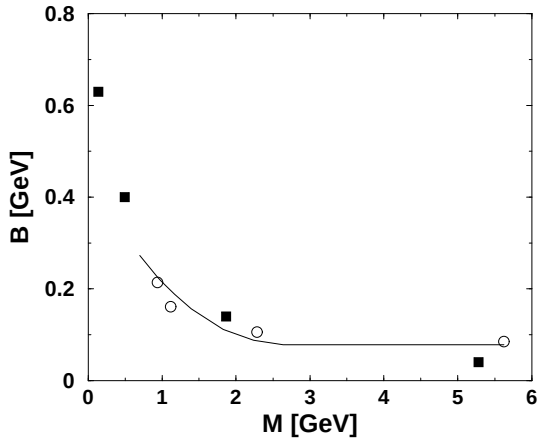

Figure 3. Low-lying hadron binding energy (B) of the low-lying hadrons as a function of the corresponding ground state mass. Experimental data for pseudoscalar mesons from Table I [13] (full squares). Experimental data for the low-lying spin 1/2 baryons comes from Table II [13] (empty circles). The results of the light-front model from the solution of Eqs.1 and 2 are shown by solid line.

The results from the numerical solution of the coupled equations (1) and (2) are obtained for a fixed $m_u = 0.386$ GeV (we kept the value found in Ref. [8]) which together with the nucleon mass of 0.938 GeV implies in $M_d = 0.695$ GeV. For the given value of the diquark mass M_d and different values of m_Q, we obtain the binding energy for the spin 1/2 baryons Λ^0, Λ_c^+ and Λ_b^0 as a function of m_Q ($Q = s, c, b$). For the baryon mass above 2.3 GeV,the bound Qqq system of the light-front model goes to the diquark threshold. This gives the saturation value of $0.077GeV$ seen in this figure. The theoretical results are in excelent agreement with the baryon data, consequently the dynamical assumption of the flavor independence of the effective interaction is indeed reasonable. In Fig. 4, we shown the baryon binding energy as a function of m_Q, again we observe the agreement between the model and the attributed experimental values for the binding energies and constituent quark masses.

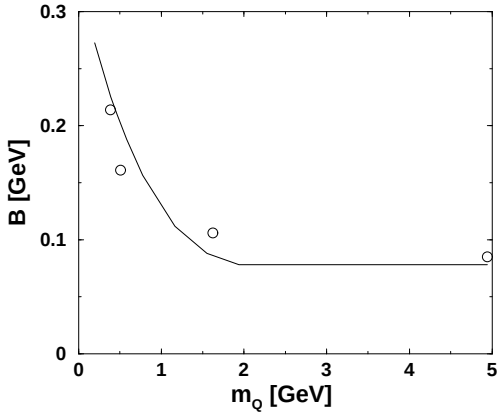

Figure 4. Low-lying baryon binding energy (B) as a function of the constituent quark mass (M_Q). Experimental data for the low-lying spin 1/2 baryons comes from Table II [13] (empty circles) and the constituent quark masses are given in Table I. The results of the light-front model from the solution of Eqs.1 and 2 are shown by solid line.

IV Conclusions

The binding energy of the constituent quarks forming the the low-lying spin 1/2 baryonic states of the nucleon, Λ^0, Λ_c^+ and Λ_b^0, obtained from the experimental values of baryon masses and constituent quark masses derived from the low-lying vector mesons masses was studied within a light-front model. The effective interaction between the constituent quarks was chosen of a contact form and spin was averaged out. The motivation of chosing this particular interaction was two-fold: it was necessary to bring to the effective QCD-inspired model of the light-mesons the pion mass scale, and to give the observed splitting between the pion and rho meson spectrum [2], on one side, and on the other side, it was succesfull in describing the proton mass and radius simultaneously [8]. In the present study the contact interaction was used because it allows to introduce in the model the minimal number of physical scales necessary to describe the low-lying spin 1/2 baryons. Therefore, the relativistic three-quark model of the baryon defined on the light-front, with a falvor independent interaction, has as inputs the constituent quark masses and the diquark mass in the light sector, which defines the strength of the contact interaction. This model allowed a surprising reproduction of the trend and magnitude of the binding energies as a function of the distint quark mass. As the present light-front model is still very schematic, we believe that our conclusion of the flavor dependence of baryonic masses may still hold in a more realistic model, and support the extension of the QCD-inspired model applied previously only to mesons [1, 2] also to baryons.

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