Interior Problem in a Nonsymmetric Theory of Gravitation

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The field equations of a previous metric nonsymmetric theory of gravitation are considered for the interior of a static spherically symmetric perfect fluid with a view to a study of stellar equilibrium. The equations are put into a form of four first-order differential equations which are ready for numerical integration.

Keywords: Nonsymmetric theory; Gravitation

I. INTRODUCTION

In previous papers [1-I,II] a metric nonsymmetric theory of gravitation has been developed by one of us and the solution for a point mass source was obtained. The theory was shown to be consistent with the four classical tests of general relativity (GR). In [2] it was shown that the theory is definitely free of ghost-negative radiative modes even when expanded around a Riemannian GR background space. In a more recent paper [3] the conservation laws of energy and momentum were established. Some application of the results were made for a static spherically symmetric perfect fluid with a view to future study of stellar equilibrium. This is the subject of which we start developing in this paper. The sources of the field are the energy-momentum-stress tensor $T_{\alpha\beta}$ and the fermionic current density S^{α}. For a macroscopic system this current is taken to be

$$S^{\alpha} = \Sigma_a f_a n_a u^{\alpha}, \tag{1.1}$$

where f_a is the coupling of the fermion specie a to the geometry, n_a is its rest number density found, for instance, in the description of the interior of stars (electrons, protons and neutrons) and u^{α} is the common velocity.

In this paper we study the interior problem for a static and spherically symmetric perfect fluid. Therefore, only the component S^0 will survive. This will be focused with a view to a study of stellar equilibrium. It would be interesting to analyze what the modifications of the predictions of GR are for stellar equilibrium and collapse particularly by the fact that, together with the stress tensor, the matter current is also a source of gravitation.

The approach is similar to the one studied by Savaria [4] in the context of a different nonsymmetric theory. The equations are put into a form of four first-order differential equations which are ready for numerical integration. They are ready for application to the study of white dwarf and neutron stars, to discuss their stability as compared with the results of GR.

The set of four differential equations involve four quantities to be determined: the fermionic charge $F(r)$ below the radial distance r, the two metric coefficients $g_{00}(r)$ and $g_{11}(r)$, the internal pressure $p(r)$ and or the mass density $\rho(r)$. The system must be supplemented by an equation of state relating the pressure and density as in GR, and information about $f_a n_a(r)$.

The paper is organized as follows. In Sec. II the field equations are presented In Sec. III we set their form for a static spherically symmetric sphere. In Sec. IV the first-order equations are written down and in Sec. V the limits at the centre and boundary are stated. In Sec. VI we present our conclusions and highlight plans for future work.

II. THE FIELD EQUATIONS

The field equations of the theory are [1-I], with $\kappa = 8\pi G$,

$$U_{\alpha\beta} + \Lambda g_{(\alpha\beta)} = \kappa \overline{T}_{(\alpha\beta)} , \tag{2.1}$$

$$\mathbf{g}^{(\alpha\beta)}{}_{,\gamma} + \mathbf{g}^{(\alpha\sigma)}\Gamma^{\beta}_{(\sigma\gamma)} + \mathbf{g}^{(\beta\sigma)}\Gamma^{\alpha}_{(\sigma\gamma)} - \mathbf{g}^{(\alpha\beta)}\Gamma^{\sigma}_{(\sigma\gamma)} = 0 , \tag{2.2}$$

$$\mathbf{g}^{[\alpha\beta]}{}_{,\beta} = 4\pi \mathbf{S}^{\alpha} \tag{2.3}$$

and

$$\Lambda g_{[\alpha\beta,\gamma]} = \kappa \overline{T}_{[\alpha\beta,\gamma]} . \tag{2.4}$$

We use the notation $a_{(\alpha\beta)} = (a_{\alpha\beta} + a_{\beta\alpha})/2$ and $a_{[\alpha\beta]} = (a_{\alpha\beta} - a_{\beta\alpha})/2$ for the symmetric and antisymmetric parts of $a_{\alpha\beta}$ and the notation $a_{[\alpha\beta,\gamma]} = a_{[\alpha\beta],\gamma} + a_{[\gamma\alpha],\beta} + a_{[\beta\gamma],\alpha}$ for the curl of $a_{[\alpha\beta]}$. In the first equation

$$U_{\alpha\beta} = \Gamma^{\sigma}_{(\alpha\beta),\sigma} - \Gamma^{\sigma}_{(\sigma\alpha),\beta} + \Gamma^{\rho}_{(\alpha\beta)}\Gamma^{\sigma}_{(\rho\sigma)} - \Gamma^{\rho}_{(\alpha\sigma)}\Gamma^{\sigma}_{(\rho\beta)} , \tag{2.5}$$

symmetric because the second term is (see (2.12) below) and containing only the symmetric part of the connection, is the analogue of the Ricci tensor. Λ is the cosmological constant and in the right-hand side we have the symmetric part of the tensor

$$\overline{T}_{\alpha\beta} = T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T , \tag{2.6}$$

with $T = g^{\alpha\beta} T_{\alpha\beta}$. The next two equations involve the symmetric and antisymmetric parts of $\mathbf{g}^{\alpha\beta} = \sqrt{-g} g^{\alpha\beta}$ where g =det.$(g_{\alpha\beta})$ and $g^{\alpha\beta}$ is the inverse of $g_{\alpha\beta}$ as defined by

$$g^{\alpha\beta} g_{\alpha\gamma} = g^{\beta\alpha} g_{\gamma\alpha} = \delta^{\beta}_{\gamma} . \tag{2.7}$$

Equation (2.4) involves the curl of the antisymmetric part of the metric and of the tensor in (2.6). The second field equation, (2.2), can be solved for the symmetric part of the connection [1-I] giving

$$\Gamma^{\sigma}_{(\alpha\beta)} = \frac{1}{2} g^{(\sigma\lambda)} \left(s_{\alpha\lambda,\beta} + s_{\lambda\beta,\alpha} - s_{\alpha\beta,\lambda} \right)$$

$$+ \left(g^{(\sigma\lambda)} s_{\alpha\beta} - \delta^{\sigma}_{\alpha}\delta^{\lambda}_{\beta} - \delta^{\lambda}_{\alpha}\delta^{\sigma}_{\beta} \right) C_{,\lambda} \,, \qquad (2.8)$$

with

$$C = \frac{1}{4} \ln \frac{s}{g} \,, \qquad (2.9)$$

where $s_{\alpha\beta}$ symmetric and with determinant s is the inverse of $g^{(\alpha\beta)}$ as defined by

$$s_{\alpha\beta} g^{(\alpha\gamma)} = \delta_{\beta}{}^{\gamma} \,. \qquad (2.10)$$

Equation (2.4) came as the result of the equation

$$\Lambda g_{[\alpha\beta]} + \Gamma_{[\alpha,\beta]} = \kappa \overline{T}_{[\alpha\beta]} \,, \qquad (2.11)$$

where $\Gamma_{\alpha} = (\Gamma^{\beta}_{\alpha\beta} - \Gamma^{\beta}_{\beta\alpha})/2 = \Gamma^{\beta}_{[\alpha\beta]}$ is a vector involving contractions of the antisymmetric part of the connection. In deriving (2.8) from (2.2) we come across the relation

$$\Gamma^{\sigma}_{(\sigma\alpha)} = (\ln \frac{-g}{\sqrt{-s}})_{,\alpha} \,, \qquad (2.12)$$

which can be re-obtained from that equation. One then sees that the second term on the right of (2.5) is in fact symmetric.

The Lagrangian density we end up with in [1-I] is

$$\mathbf{L} = \mathbf{g}^{\alpha\beta}(U_{\alpha\beta} + \Gamma_{[\alpha,\beta]}) + 2\Lambda\sqrt{-g} + \mathbf{L}_M \,, \qquad (2.13)$$

where $\mathbf{L_M}$ is the matter part of the Lagrangian. It is modeled after the one of GR, containing here the generalized matter energy-momentum-stress tensor $T_{\alpha\beta}$ and the fermionic current S^{α}, as given by

$$\delta\mathbf{L}_M = \delta\mathbf{L}^T_M + 4\pi\mathbf{S}^{\alpha}\delta\Gamma_{\alpha} \,, \qquad (2.14)$$

where

$$\delta\mathbf{L}^T_M = -\kappa\sqrt{-g}T_{\alpha\beta}\delta g^{\alpha\beta} \,. \qquad (2.15)$$

As $\delta\sqrt{-g} = g_{\mu\nu}\delta\mathbf{g}^{\mu\nu}/2$, we have $\sqrt{-g}\delta g^{\alpha\beta} = \delta\mathbf{g}^{\alpha\beta} - g^{\alpha\beta}g_{\mu\nu}\delta\mathbf{g}^{\mu\nu}/2$, and (2.14) can also be written as

$$\delta\mathbf{L}_M = -\kappa\overline{T}_{\alpha\beta}\delta\mathbf{g}^{\alpha\beta} + 4\pi\mathbf{S}^{\alpha}\delta\Gamma_{\alpha}. \qquad (2.16)$$

Equations (2.1)-(2.4) can be re-obtained from Eq. (2.13) by variations with respect to $\mathbf{g}^{\alpha\beta}$, $\Gamma^{\sigma}_{(\alpha\beta)}$ and Γ_{α}. The first one gives $U_{\alpha\beta} + \Gamma_{[\alpha,\beta]} + \Lambda g_{\alpha\beta} = \kappa\overline{T}_{\alpha\beta}$, the symmetric and antisymmetric parts give (2.1) and (2.11), this last one leading to (2.4). The second variation gives (2.2) while the one with respect to Γ_{α} gives (2.3).

Together with the down-indices stress $T_{\alpha\beta}$ we will be working (1-I) with the upper-indices $T^{\mu\nu}$ defined by the variation with respect to $g_{\mu\nu}$,

$$\delta\mathbf{L}^T_M = \kappa\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu} \,, \qquad (2.17)$$

as in GR. This second stress is related to the first one by

$$T_{\alpha\beta} = g_{\alpha\nu}g_{\mu\beta}T^{\mu\nu} \qquad (2.18)$$

which follows from the relation $\delta g_{\mu\nu} = -g_{\alpha\nu}g_{\mu\beta}\delta g^{\alpha\beta}$ resulting from the variation of Eq. (2.7). It should be kept in mind that Eq. (2.18) does not imply a rule for lowering indices because this operation is not defined for a nonsymmetric metric. A better name for the upper-indices stress tensor would probably be $S^{\mu\nu}$ but we shall use the same T for both tensors. Notice that the inverse relation is $T^{\mu\nu} = g^{\mu\beta}g^{\alpha\nu}T_{\alpha\beta}$ and that both have the same trace $g^{\alpha\beta}T_{\alpha\beta} = g_{\mu\nu}T^{\mu\nu}$.

We end this section by writing the matter-response equation. Working directly with $\delta\mathbf{L}_M$ [1-I] or going through the variational principle [3] the equation is of the form

$$G(g_{\alpha\lambda}\mathbf{T}^{\alpha\beta}{}_{,\beta} + g_{\lambda\alpha}\mathbf{T}^{\beta\alpha}{}_{,\beta} + 2[\alpha\beta,\lambda]\mathbf{T}^{\alpha\beta}) + \Gamma_{[\lambda,\alpha]}\mathbf{S}^{\alpha} = 0 \qquad (2.19)$$

where

$$[\alpha\beta,\lambda] = \frac{1}{2}(g_{\alpha\lambda,\beta} + g_{\lambda\beta,\alpha} - g_{\alpha\beta,\lambda}) \,. \qquad (2.20)$$

Now consider the situation in which we are are interested in, dealing with a perfect fluid. Then it can be shown [3] that

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} - pg^{\mu\nu} \,, \qquad (2.21)$$

ρ being the matter rest density, p the pressure and u^{μ} the velocity.

In the next section we will see the form acquired by the field equations and of (2.19) in the interior of a static spherically symmetric perfect fluid.

III. THE STATIC SPHERICALLY SYMMETRIC SPHERE

The static and spherically symmetric metric tensor in polar coordinates $x^0 = t$, $x^1 = r$, $x^2 = \Theta$ and $x^3 = \Phi$ is of the form

$$\begin{aligned} g_{00} &= \gamma(r), \qquad g_{11} = -\alpha(r), \\ g_{22} &= -r^2, \qquad g_{33} = -r^2\sin^2\Theta, \\ g_{01} &= -\omega(r) = -g_{10}, \end{aligned} \qquad (3.1)$$

and all other components equal to zero. The non-zero components of the inverse matrix are then

$$\begin{aligned} g^{00} &= \frac{\alpha}{\alpha\gamma - \omega^2} \,, \qquad g^{11} = -\frac{\gamma}{\alpha\gamma - \omega^2} \,, \\ g^{22} &= -\frac{1}{r^2} \,, \qquad g^{33} = -\frac{1}{r^2\sin^2\Theta} \,, \\ g^{01} &= \frac{\omega}{\alpha\gamma - \omega^2} = -g^{10} \,. \end{aligned} \qquad (3.2)$$

We need also the inverse $s_{\alpha\beta}$ to $g^{(\alpha\beta)}$, whose non-zero components are

$$\begin{aligned} s_{00} &= \frac{\alpha\gamma-\omega^2}{\alpha} , \quad s_{11} = -\frac{\alpha\gamma-\omega^2}{\gamma} , \\ s_{22} &= -r^2 , \qquad s_{33} = -r^2\sin^2\Theta . \end{aligned} \tag{3.3}$$

The determinants have values

$$g = -(\alpha\gamma-\omega^2)r^4\sin^2\Theta \tag{3.4}$$

and

$$s = -\frac{(\alpha\gamma-\omega^2)^2}{\alpha\gamma} r^4\sin^2\Theta \ . \tag{3.5}$$

From (2.3) we have $\mathbf{S}^\alpha{}_{,\alpha} = 0$ guaranteeing the conservation of the fermionic charge. For the static sphere we have $u^i = 0$ so according to (1.1) only the zero component survives in (2.3). From (3.2) and (3.4) that equation yields

$$\frac{\omega r^2}{(\alpha\gamma-\omega^2)^{1/2}} = F(r) , \tag{3.6}$$

where

$$F(r) = \int_0^r 4\pi\sqrt{\alpha\gamma-\omega^2} S^0 r'^2 dr' . \tag{3.7}$$

is the conserved fermionic charge contained in a sphere of radius r. From (3.6) we have

$$\frac{\omega^2}{\alpha\gamma} = \frac{F^2}{F^2+r^4} \ . \tag{3.8}$$

With this result and equations (3.4) and (3.5), we have $s/g = r^4/(F^2+r^4)$ and equation (2.9) yields

$$C_{,\lambda} = \delta_{\lambda,1}\frac{F^2-\xi r^4}{r(F^2+r^4)} , \tag{3.9}$$

where for short

$$\xi = \frac{F^{2\,\prime}}{4r^3} \ . \tag{3.10}$$

For future use we note that from (3.7) we have

$$F' = 4\pi\sqrt{\alpha\gamma-\omega^2} S^0 r^2 \ . \tag{3.11}$$

By multiplication by F and using (3.6) we obtain

$$F^{2\,\prime} = 8\pi\omega S^0 r^4 \tag{3.12}$$

and, from (3.10),

$$\xi = 2\pi\omega S^0 r \ . \tag{3.13}$$

Let us go now to the calculation of the symmetric part of the connections, from (2.8). Putting the abbreviation $B = F^2 + r^4$ a straightforward calculation yields ($a' = da/dr$)

$$\begin{aligned} \Gamma^0_{(01)} &= \frac{\gamma'}{2\gamma} + \frac{F^2-\xi r^4}{rB} , \\ \Gamma^1_{00} &= \frac{\gamma'}{2\alpha} + \frac{\gamma}{\alpha}\frac{F^2-\xi r^4}{rB} , \\ \Gamma^1_{11} &= \frac{\alpha'}{2\alpha} + \frac{F^2-\xi r^4}{rB} , \\ \Gamma^1_{22} &= -\frac{(1+\xi)r}{\alpha} , \\ \Gamma^1_{33} &= -\frac{r\sin^2\Theta}{\alpha} , \\ \Gamma^2_{(12)} &= \Gamma^3_{(13)} = \frac{(1+\xi)r^3}{B} , \\ \Gamma^2_{33} &= -\sin\Theta\cos\Theta , \\ \Gamma^3_{(23)} &= \cot\Theta , \end{aligned} \tag{3.14}$$

and all other components vanishes. Using all this information the nonzero components of $U_{\alpha\beta}$ are

$$U_{00} = \left(\frac{\gamma'}{2\alpha} + \frac{\gamma}{\alpha}\frac{F^2-\xi r^4}{rB}\right)'$$

$$+\left(\frac{\gamma'}{2\alpha} + \frac{\gamma}{\alpha}\frac{F^2-\xi r^4}{rB}\right)\left(\frac{\alpha'}{2\alpha} - \frac{\gamma'}{2\gamma} + \frac{2(1+\xi)r^3}{B}\right) , \tag{3.15}$$

$$U_{11} = -\left(\frac{\gamma'}{2\gamma} + \frac{F^2+2r^4+\xi r^4}{rB}\right)'$$

$$+\left(\frac{\gamma'}{2\gamma} + \frac{F^2-\xi r^4}{rB}\right)\left(\frac{\alpha'}{2\alpha} - \frac{\gamma'}{2\gamma}\right)$$

$$+\frac{2(1+\xi)r^3}{B}\left(\frac{\alpha'}{2\alpha} + \frac{F^2-r^4-2\xi r^3}{rB}\right) , \tag{3.16}$$

$$U_{22} = -\left(\frac{(1+\xi)r}{\alpha}\right)' + 1$$

$$-\frac{r(1+\xi)}{\alpha}\left(\frac{\gamma'}{2\gamma} + \frac{\alpha'}{2\alpha} + \frac{2F^2-2\xi r^4}{rB}\right) \tag{3.17}$$

and $U_{33} = U_{22}\sin^2\Theta$.

From (2.18) and (2.21) equation (2.6) gives

$$\overline{T}_{\alpha\beta} = (\rho+p)g_{\alpha\nu}g_{\mu\beta}u^\mu u^\nu + \frac{1}{2}g_{\alpha\beta}(p-\rho) \ . \tag{3.18}$$

As our fluid is in hydrostatic equilibrium we have

$$\overline{T}_{(\alpha\beta)} = (\rho+p)(g_{(\alpha0)}g_{(\beta0)} + g_{[\alpha0]}g_{[\beta0]})u^0u^0$$

$$+\frac{1}{2}g_{(\alpha\beta)}(p-\rho)\ , \tag{3.19a}$$

$$\overline{T}_{[\alpha\beta]}=(\rho+p)(g_{(\alpha 0)}g_{[0\beta]}-g_{(\beta 0)}g_{[0\alpha]})u^0u^0$$

$$+\frac{1}{2}g_{[\alpha\beta]}(p-\rho) \tag{3.19b}$$

and, from $u^\alpha u_\alpha = 1$,

$$u^0=\frac{1}{\sqrt{\gamma}}\ . \tag{3.19c}$$

From now on we will neglect the contribution of the cosmological constant as in GR. For the relevant components of (2.1) we then have from (3.19a),

$$U_{00}=\frac{1}{2}\kappa\gamma(\rho+3p)\ , \tag{3.20}$$

$$U_{11}=\frac{1}{2}\kappa\alpha(\rho-p-\frac{2(\rho+p)\omega^2}{\alpha\gamma}) \tag{3.21}$$

and

$$U_{22}=\frac{1}{2}\kappa r^2(\rho-p)\ . \tag{3.22}$$

On the other hand (2.11) becomes (with $\Lambda=0$)

$$\Gamma_{[\alpha,\beta]}=\kappa\{(\rho+p)(g_{(\alpha 0)}g_{[0\beta]}-g_{(\beta 0)}g_{[0\alpha]})u^0u^0$$

$$+\frac{1}{2}g_{[\alpha\beta]}(p-\rho)\}\ . \tag{3.23}$$

Only the $0,1$ antisymmetric component of this relation is nonvanishing, giving

$$\Gamma_0{}'=-\kappa\omega(\rho+3p)\ . \tag{3.24}$$

The matter-response equation (2.19) acquires [3] the simple form

$$p'=-\frac{1}{2}(\rho+p)\frac{\gamma'}{\gamma}-\frac{1}{4G}\Gamma_0S^0\ . \tag{3.25}$$

On account of the previous equation and of (3.13) this equation becomes

$$p'=-\frac{1}{2}(\rho+p)\frac{\gamma'}{\gamma}+\frac{\xi}{r}(\rho+3p)\ . \tag{3.26}$$

IV. THE FIRST-ORDER EQUATIONS

As in GR we can obtain from (3.20) and (3.21) a relation involving only first-order derivatives of the metric by multiplying the first one by α/γ and adding the result to second one. The calculation is delineated in Appendix A. After multiplication of the whole thing by $B/2\alpha r^2$ we obtain

$$-\frac{r(1+\xi)'}{\alpha}+\frac{r(1+\xi)}{\alpha}(\frac{\gamma'}{2\gamma}+\frac{\alpha'}{2\alpha}-\frac{F^2-\xi r^4}{r(F^2+r^4)})$$

$$=\frac{1}{2}\kappa(\rho+p)r^2\ . \tag{4.1}$$

This result would hold even if Λ were present.

Next we use Eq. (3.17) in (3.22) and add and subtract to (4.1). After some algebra (see Apendix A) we obtain

$$1-\frac{4(1+\xi)}{\alpha}+\frac{3(1+\xi)^2r^4}{\alpha(F^2+r^4)}$$

$$-\frac{2r(1+\xi)'}{\alpha}-r(1+\xi)(\frac{1}{\alpha})'=\kappa\rho r^2 \tag{4.2}$$

and

$$-1+\frac{(1+\xi)}{\alpha}\left(2+\frac{r\gamma'}{\gamma}-\frac{(1+\xi)r^4}{F^2+r^4}\right)=\kappa p r^2\ . \tag{4.3}$$

Using equations (1.1), (3.8) and (3.13) in (3.11) we obtain

$$F'(r)=4\pi r^2\sqrt{\alpha}(1+\frac{F^2}{r^4})^{-1/2}\Sigma_a f_a^2 n_a\ . \tag{4.4}$$

Equations (4.2), (4.3), (4.4) and (3.26) are the equations that define the behavior of the interior of our system. As promised they constitute a system of equations for the two metric coefficients $\gamma(r)$ and $\alpha(r)$, the fermionic charge $F(r)$ below r,the density ρ, the pressure p and the fermionic coupling-density number $f_an_a(r)$ for each component. This last quantity together with an equation of state $f(\rho,p)=0$, as in GR, are required for the solution of the system of equations.

These equations can be put in a more interesting form by noticing that the last two terms on the left-hand side of (4.2) can be condensed as $-r(1+\xi)^{-1}((1+\xi)^2/\alpha)'$. Motivated by the form of α in GR we then introduce the variable $m(r)$ defined by

$$\frac{(1+\xi)^2}{\alpha}=1-\frac{2Gm(r)}{r}\ . \tag{4.5}$$

When ξ is null, that is, when ω is null this becomes the GR relation with $m(r)$ equals the mass within a sphere of radius r. In terms of this quantity our four equations can be written as

$$m'=(1+\xi)\left(4\pi\rho r^2-\frac{3}{2G}(1-\frac{2Gm(r)}{r})(1+\frac{F^2}{r^4})^{-1}\right)$$

$$-\frac{\xi}{2G}+\frac{3}{2G}(1-\frac{2Gm(r)}{r})\ , \tag{4.6}$$

$$r\frac{\gamma'}{\gamma}=-2+(1+\xi)\{1+\frac{F^2}{r^4}$$

$$+(1-\frac{2Gm(r)}{r})^{-1}(1+8\pi Gr^2p)\} \quad (4.7)$$

$$rp' = -(\rho+p)(1+\xi)\{(1-\frac{2Gm(r)}{r})^{-1}(4\pi Gr^2p+\frac{Gm(r)}{r})$$

$$-\frac{F^2}{F^2+r^4}\}+2p\xi \quad (4.8)$$

and

$$F'(r) = 4\pi r^2(1+\xi)(1-\frac{2Gm(r)}{r})^{-1/2}$$

$$\times(1+\frac{F^2}{r^4})^{-1/2}\Sigma_a f_a^2 n_a \ . \quad (4.9)$$

The coefficient α is obtained from (4.5) and ω comes from (3.8)

$$\omega = \sqrt{\gamma}(1+\xi)(1-\frac{2Gm(r)}{r})^{-1/2}\frac{F}{r^2}(1+\frac{F^2}{r^4})^{-1/2} \ . \quad (4.10)$$

Having all this the total mass of the star can be calculated from equation (5-10) of [3], which is,

$$M = \int \sqrt{-g}\left(T_0{}^0 - T_i^i - \kappa^{-1}g^{[0i]}\Gamma_{0,i}\right) drd\Theta d\phi \quad (4.11)$$

where

$$T_\alpha{}^\sigma \equiv \frac{1}{2}(g_{\alpha\nu}T^{\sigma\nu}+g_{\nu\alpha}T^{\nu\sigma}) \ . \quad (4.12)$$

We give the details of the calculation in Appendix B, quoting here the final result. We have

$$M = 4\pi\int_0^R \frac{\sqrt{\gamma}(1+\xi)}{(1-\frac{2Gm(r)}{r})^{1/2}}(1+\frac{F^2}{r^4})^{1/2}(\rho+3p)r^2dr \ . \quad (4.13)$$

As it will be discussed in the next section, this quantity can also be calculated from the continuity of α across the surface boundary R of the star.

V. CONDITIONS AT THE CENTER AND BOUNDARY

Because of divergent terms at $r=0$ we expand the equations around that point. This will permit to perform the numerical integration from near the center on, all the way up to the radius R of the star.

Starting with (4.5) we see that $m(0)=0$ to have $\alpha(0)$ finite. Thus, from (4.9) we see that F near the origin goes as r^3 because $F(0)=0$ from (3.7).Then, from (3.10) it follows that ξ goes as r^2. Then (4.6) tell us that m goes as r^3, $\rho(0)$ being finite. Next, from (4.8) p goes as $p(0)$ plus a term of order r^2. Finally, (4.7) tell us that γ goes as $\gamma(0)$ plus a term of order r^2. We fix now the coefficients. Calling

$$n_0 = \frac{4\pi}{3}\Sigma_a f_a^2 n_a \quad (5.1)$$

we have

$$F(r) = n_0r^3 + O(r^4) \ , \quad (5.2)$$

$$\xi(r) = \frac{3}{2}n_0{}^2r^2 + O(r^4) \ , \quad (5.3)$$

$$m(r) = (\frac{4\pi}{3}\rho(0) - \frac{n_0{}^2}{2G})r^3 + O(r^4) \ , \quad (5.4)$$

$$\gamma = \gamma(0)(1+br^2) + O(r^3) \ , \quad (5.5)$$

where

$$b = 4\pi G(p(0)+\frac{1}{3}\rho(0)) + \frac{3}{2}n_0{}^2 \ , \quad (5.6)$$

and

$$p(r) = p(0) - er^2 + O(r^3) \quad (5.7)$$

where

$$e = 2\pi G[\rho(0)+p(0)][\frac{1}{3}\rho(0)+p(0)] \quad (5.8)$$

$$-\frac{1}{2}n_0{}^2(2p(0)-\rho(0)) \ .$$

Having (5.3) and (5.4), equation (4.5) gives for $\alpha(r)$

$$\alpha(r) = 1 + (\frac{8\pi G}{3}\rho(0) + 2n_0{}^2)r^2 + O(r^3). \quad (5.9)$$

At the surface the parameters ρ, p and S^0 or ξ all vanish. Having $\alpha(r)$ inside, the mass of the star can be calculated from the continuity at $r=R$. We have

$$\alpha(R) = \alpha(R)_{ext} \ . \quad (5.10)$$

The exterior value is determined by the solution obtained outside in [1-II],

$$\alpha(R)_{ext}^{-1} = (1+\frac{F^2(R)}{R^4})[1-\frac{2GM}{R}(1+\frac{F^2(R)}{R^4})^{-1/2}] \ . \quad (5.11)$$

VI. CONCLUSIONS

The field equations of the metric nonsymmetric theory of gravitation developed in [1-I,II] have been analyzed for the interior of a static spherically symmetric perfect fluid with a view to a study of structure of stars.

The field equations have been reduced to a set of four first-order differential equations which is ready for numerical integration.

The obtained set of equations is appropriate to the study of the behavior of white dwarf and neutron stars. It will be interesting to see what the modifications of the prediction of GR are for their state of equilibrium.

Another topic for future work would be to establish the form of the field equations for a non-rotating perfect fluid aiming as a model to the study of stellar collapse.

APPENDIX. A: PROOF OF (4.1)-(4.3)

After multiplication of (3.15) by α/γ, a differentiation by parts of its first term gives

$$\frac{\alpha}{\gamma}U_{00}=\left(\frac{\gamma'}{2\gamma}+\frac{F^2-\xi r^4}{rB}\right)'$$

$$+(\frac{\gamma'}{2\gamma}+\frac{F^2-\xi r^4}{rB})(\frac{2(1+\xi)r^3}{B}-\frac{\alpha'}{2\alpha}-\frac{\gamma'}{2\gamma})\,. \qquad (A1)$$

When added to (3.16) we obtain

$$\frac{\alpha}{\gamma}U_{00}+U_{11}=-\frac{2r^3(1+\xi)'}{B}$$

$$+\frac{2(1+\xi)r^3}{B}(\frac{\gamma'}{\gamma}+\frac{\alpha'}{\alpha}-\frac{F^2-\xi r^4}{rB})\,. \qquad (A2)$$

From (3.20) and (3.21) we find, using (3.8),

$$\frac{\alpha}{\gamma}U_{00}+U_{11}=\kappa\alpha\frac{(\rho+p)r^4}{B}\,. \qquad (A3)$$

Using (A2) and multiplying the whole thing by $B/2\alpha r^2$ equation (4.1) follows.

Next, from (A3) and (3.22) we find

$$\frac{B}{2\alpha r^2}(\frac{\alpha}{\gamma}U_{00}+U_{11})\pm U_{22}=\kappa r^2\{{}^{\rho}_{p}\}\,. \qquad (A4)$$

Using (3.7) and (A2), equations (4.2) and (4.3) follow.

APPENDIX. B: PROOF OF (4.13)

In our case only the $g^{[01]}$ component survives in the last term inside the parenthesis of (4.11). From (2.21) equation (4.12) gives

$$T_\alpha{}^\sigma=g_{(\alpha\nu)}(\rho+p)u^\sigma u^\nu-p\delta^\sigma_\alpha\,. \qquad (B1)$$

Thence, for our hydrostatic fluid, $T_0{}^0=\rho$ and $T_i{}^i=-3p$. Next, using (3.8) in the last relation of (3.2) we have

$$g^{[01]}=\frac{F^2}{\omega r^4}\,. \qquad (B2)$$

Taking this together with (3.24) in (4.11) gives

$$M=\int\sqrt{-g}\,(\rho+3p)\,(1+\frac{F^2}{r^4})drd\Theta d\phi\,. \qquad (B3)$$

From (3.4) and (3.8)

$$\sqrt{-g}=\sqrt{\alpha\gamma}(1+\frac{F^2}{r^4})^{-1/2}r^2\sin\Theta\,. \qquad (B4)$$

Using here (4.5) and taking the result into (B3) equation (4.13) follows.

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[3] S. Ragusa, Braz. J. Phys. **35**, 1020 (2005). A factor G^{-1} should be added to the last term of equation (6.4). Equation (3.26) should be written as in the present equation (2.19) and equation (3.28) should be dropped.
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