Confinement in the 3-Dimensional Gross-Neveu Model at Finite Temperature

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We study the N-component (2+1)-dimensional Gross-Neveu model bounded between two parallel planes separated by a distance L at finite temperature (T). We obtain a closed expression for the large-N effective coupling constant $g = g(L,T,\lambda)$. Different behavior depending on the magnitude of the fixed coupling constant λ is found to lead to a "critical" value λ_c. If $\lambda < \lambda_c$, only short-distance and/or high-temperature asymptotic freedom is found. For $\lambda \geq \lambda_c$ one also observes spatial confinement, which is destroyed by temperature effects. We find a confining length, $L_c \simeq 1.61\, fm$, that is close to the proton charge diameter ($\approx 1.74\, fm$) and a deconfining temperature, $\simeq 138\, MeV$, which is comparable to the estimated value of $\approx 200\, MeV$ for hadrons.

Keywords: Gross-Neveu model; Four-point function; Spatial confinement

Effective models in quantum field theories have been employed over the last decades in trials to obtain clues about the behavior of strongly interacting particles. Among them, the Gross-Neveu model [1], dealing with the direct four-fermions interaction, has been analyzed at finite temperature as an effective model for QCD and for superconducting systems (see for instance Ref. [2, 3]). Calculation of the effective potential of the ϕ^4 theory at finite temperature has also been performed [4].

Recently, we have studied the N-components tridimensional Gross-Neveu model at zero temperature, bounded between two parallel planes [5]. A closed expression was derived for the large-N effective coupling constant $g(L,\lambda)$ as a function of the distance L between the planes. From this result, the behavior of $g(L,\lambda)$ depending on the magnitude of the free space fixed coupling constant λ was found, such that for small λ, the model presents asymptotic freedom at short distances. On the other hand, for large enough values of λ both spatial confinement and short distance asymptotic freedom are simultaneously present. In this context, the analysis of the effect of temperature is crucial, since it affects the confinement properties. The main objective of the present Letter is to study the spatial confinement and thermal deconfinement properties of the Gross-Neveu model.

We recall that even though it is perturbatively non-renormalizable for dimensions $D > 2$, the massive Gross-Neveu model in Euclidean tridimensional (3-D) space has been shown to exist and has been explicitly constructed in the large-N [6]. A decisive physical point that brings consistency to this derivation is a theoretical result [7] supporting the idea that perturbatively non-renormalizable models do exist and have a physical meaning. For the $N = 1$ case, some operators can be made more relevant in the low energy region if the fermionic field is minimally coupled to the Chern-Simons field [8, 9].

We consider the N-component 3-D massive Gross-Neveu model, in the large-N limit, compactified along the imaginary-time axis and also along one of the spatial directions. From a physical point of view, in terms of a generalized Matsubara formalism [10], the model is intended to describe fermions bounded between two parallel planes, a distance L apart from one another and in thermal equilibrium with a reservoir at temperature $T = \beta^{-1}$. From the four-point function, we define an effective renormalized coupling constant $g(L,\beta,\lambda)$ in the large-N limit, which presents different behavior with L and β if the fixed coupling constant λ is below or above some "critical" value λ_c: high-T and short-distance asymptotic freedom, if $\lambda < \lambda_c$; with $\lambda \geq \lambda_c$, we obtain simultaneously asymptotic freedom and spatial confinement, for low enough temperatures. As the temperature is increased, a deconfining transition occurs. This is the first time, to our knowledge, that such an analytical calculation has been performed for an effective *ab initio* model.

Considering the Gross-Neveu model as an effective theory for the strong interaction between quarks and taking the constituent quark mass ($m \approx 350\, MeV$) as the fermion mass, we find a confining length of $1.61\, fm$ which is close to the proton charge diameter of $\approx 1.74\, fm$. Also, the temperature destroying the confinement, $138\, MeV$, is comparable to the estimated deconfinement temperature for hadrons ($\approx 200\, MeV$).

A central ingredient in our approach is the topological nature of the Matsubara imaginary-time formalism. To calculate the partition function in a quantum field theory, the Matsubara prescription is equivalent to a path-integral approach on $R^{D-1} \times S^1$, where S^1 is a circle of circumference $\beta = 1/T$. As a consequence the Matsubara formalism can be thought, in a generalized way, as a mechanism to deal also with spatial constraints in a field theory model. In this situation, for consistency, the fields fulfill periodic (anti-periodic) boundary conditions for bosons (fermions). We infer from this discussion that we are justified to consider in this paper the Matsubara mechanism as a path-integral formalism on $R^{D-2} \times S^1 \times S^1$ to deal simultaneously with temperature effects and spatial constraints. These ideas have been applied in different physical situations: for spontaneous symmetry breaking in the compatified ϕ^4 model [10, 11]; for second-order phase transitions in films, wires and grains [12]; for the Casimir effect for bosons [13] and for fermions in a box [14]; and, in particular, for the Gross-Neveu model at zero temperature [5]. It is worth emphasizing that for the fermionic field, the boundary (anti-periodic) conditions coincide with the physical bag-model conditions [5, 15, 16].

Our starting point is the Wick-ordered massive Gross-Neveu Lagrangian in a D-dimensional Euclidean space,

$$\mathcal{L} =: \bar{\psi}(x)(i\nabla\!\!\!\!/ + m)\psi(x) : + \frac{u}{2}(: \bar{\psi}(x)\psi(x) :)^2, \qquad (1)$$

where m is the mass, u is the coupling constant, x is a point of $\mathbf{R}^D$ and the γ's are either 2×2 or 4×4 matrices. The quantity $\psi(x)$ is a spin $\frac{1}{2}$ field having N (flavor) components, $\psi^a(x)$, $a = 1,2,...,N$. Summation over flavor and spin indexes is understood. Here we consider the large-N limit ($N\to\infty$), which permits considerable simplification. We use natural units, $\hbar = c = k_B = 1$.

We consider the system bounded between two parallel planes, normal to one of the spatial axis, a distance L apart from one another and in thermal equilibrium with a reservoir at temperature $T = \beta^{-1}$. We work with Cartesian coordinates $x = (x_0, x_1, \mathbf{z})$, where x_0 corresponds to the imaginary time-coordinate, x_1 is the constrained spatial coordinate of the system, and $\mathbf{z}$ is a $(D-2)$-dimensional vector. The corresponding momenta are specified by the notation $\mathbf{k} = (k_0, k_1, \mathbf{q})$, where $k_0 = w$ is the frequency and $\mathbf{q}$ is a $(D-2)$ -dimensional vector in momentum space. We assume that the field $\psi(x_0,x_1,\mathbf{z})$ satisfies the bag-model anti-periodic boundary condition [15, 16]. This constraint of the x_1-dependence of the field to be restricted to a segment of length L allows us to proceed, with respect to x_1, in a manner analogous to the imaginary-time in the Matsubara formalism in field theory. That is, the Feynman rules should be modified following the prescription [5, 10, 11],

$$\int \frac{dk}{2\pi} \to \frac{1}{\xi}\sum_{n=-\infty}^{+\infty}, \qquad k \to \frac{2(n+\frac{1}{2})\pi}{\xi}, \qquad (2)$$

for each one of the components k_0 and k_1, where $\xi = L$ in the case of the spatial constraint ($k = k_1$), and $\xi = \beta$, with $\beta = 1/T$, for the time constraint (for $k = k_0$).

Then the L and T-dependent four-point function, at leading order in $1/N$ and at zero external momenta, from which we will define an effective coupling constant between the fermions, has the following formal expression:

$$\Gamma_D^{(4)}(0;L,\beta,u) = \frac{u}{1+Nu\Sigma(D,L,\beta)}, \qquad (3)$$

where $\Sigma(D,L,\beta)$ is the expression for the L and T-dependent Feynman one-loop subdiagram,

$$\Sigma(D,L,\beta) = \frac{1}{\beta L}\sum_{l,n=-\infty}^{\infty}\int \frac{d^{D-2}q}{(2\pi)^{D-2}}\left[\frac{m^2 - \mathbf{q}^2 - \omega_n^2 - \nu_l^2}{(\mathbf{q}^2 + \omega_n^2 + \nu_l^2 + m^2)^2}\right], \qquad (4)$$

with $\omega_n = 2(n+\frac{1}{2})\pi/\beta$ and $\nu_l = 2(l+\frac{1}{2})\pi/L$. For the sake of regularization, let us introduce the following dimensionless quantities: $q'_i = q_i/2\pi m$ $(i = 1,2,...,D-2)$; $a = (mL)^{-2}$ and $b = (m\beta)^{-2}$.

To proceed we extend the method developed in [10] to generalize the results presented in [5] for the Gross-Neveu model at zero temperature. We use a modified minimal subtraction scheme, employing concurrently dimensional and analytical regularizations, where the counterterms are poles of the Epstein-Hurwitz *zeta*−functions [5]. The calculations go through the following steps: (i) using dimensional regularization techniques, Eq. (4) becomes

$$\begin{aligned}\Sigma(D,a,b) &= m^{D-2(s-1)}\sqrt{ab}\left[\frac{1}{4\pi^2}U_D(s;a,b)\right.\\ &\quad - \frac{D-2}{2(s-1)}U_D(s-1;a,b)\\ &\quad \left. + \frac{1}{s-1}\left(a\frac{\partial}{\partial a} + b\frac{\partial}{\partial b}\right)U_D(s-1;a,b)\right]_{s=2} \qquad (5)\end{aligned}$$

with

$$\begin{aligned}U_D(\mu;a,b) &= \frac{\pi^{D/2-2\mu+1}}{2^{2(\mu-1)}}\frac{\Gamma(\mu-\frac{D}{2}+1)}{\Gamma(\mu)}\\ &\quad \times \sum_{n,l=-\infty}^{\infty}\frac{1}{\left[a(n+\frac{1}{2})^2 + b(l+\frac{1}{2})^2 + (2\pi)^{-2}\right]^{\mu-D/2+1}} \qquad (6)\end{aligned}$$

(ii) transforming the summations over half-integers into sums over integers, Eq. (6) can be written as

$$\begin{aligned}U_D(\mu;a,b) &= \frac{\pi^{D/2-2\mu+1}}{2^{2(\mu-1)}}\frac{\Gamma(\mu-\frac{D}{2}+1)}{\Gamma(\mu)}\left[4^\nu Z_2^{4r^2}(\nu,a,b)\right.\\ &\quad -4^\nu Z_2^{4r^2}(\nu,4a,b) - 4^\nu Z_2^{4r^2}(\nu,a,4b)\\ &\quad \left. + Z_2^{r^2}(\nu,a,b)\right], \qquad (7)\end{aligned}$$

where $\nu = \mu - D/2 + 1$, $r = (2\pi)^{-1}$ and $Z_2^{h^2}(\nu,z,p) = \sum_{l,n=-\infty}^{\infty}[zn^2 + pl^2 + h^2]^{-\nu}$ is the double Epstein-Hurwitz *zeta*-function, which possesses the following analytical extension to the whole complex ν-plane [10],

$$\begin{aligned}Z_2^{h^2}(\nu,z,p) &= \frac{\pi}{\sqrt{zp}\,\Gamma(\nu)}\left\{\frac{\Gamma(\nu-1)}{h^{2(\nu-1)}}\right.\\ &\quad + 4\sum_{n=1}^{\infty}\left(\frac{\pi n}{h\sqrt{z}}\right)^{\nu-1}K_{\nu-1}\left(\frac{2\pi h n}{\sqrt{z}}\right)\\ &\quad + 4\sum_{l=1}^{\infty}\left(\frac{\pi l}{h\sqrt{p}}\right)^{\nu-1}K_{\nu-1}\left(\frac{2\pi h l}{\sqrt{p}}\right)\\ &\quad + 8\sum_{n,l=1}^{\infty}\left(\frac{\pi}{h}\sqrt{\frac{n^2}{z}+\frac{l^2}{p}}\right)^{\nu-1}\\ &\quad \left.\times K_{\nu-1}\left(2\pi h\sqrt{\frac{n^2}{z}+\frac{l^2}{p}}\right)\right\}, \qquad (8)\end{aligned}$$

where $K_\nu(x)$ is the Bessel function of the third kind; and (iii) the first term in Eq. (8) leads to a contribution for Σ which is divergent for even dimensions $D \geq 2$ due to the pole of the Γ-function. We renormalize Σ by subtracting this contribution, corresponding to a finite renormalization when D is odd.

For $D = 3$, using the formula for the Bessel functions $K_{\pm 1/2}(z) = \sqrt{\pi} e^{-z}/\sqrt{2z}$ and performing explicitly the calculations, we obtain the following expression for the L and T-dependent renormalized single-loop subdiagram:

$$\begin{aligned}\frac{\Sigma_R(L,\beta)}{m} &= 2\pi\left[\frac{1}{mL}\log(1+e^{-mL}) + \frac{1}{m\beta}\log(1+e^{-m\beta})\right. \\ &\quad -2H(L,\beta)+4H(L,2\beta)+4H(2L,\beta) \\ &\quad \left. -8H(2L,2\beta)\right] \\ &\quad -\frac{4\pi^2+1}{4\pi}\left[\frac{e^{-mL}}{1+e^{-mL}} + \frac{e^{-m\beta}}{1+e^{-m\beta}}\right. \\ &\quad -2G(L,\beta)+4G(L,2\beta)+4G(2L,\beta) \\ &\quad \left. -8G(2L,2\beta)\right], \end{aligned} \tag{9}$$

where the functions G and H are defined by

$$G(y,z) = \sum_{n,l=1}^{\infty} \exp\left(-\sqrt{(my)^2 n^2 + (mz)^2 l^2}\right), \tag{10}$$

$$H(y,z) = \sum_{n,l=1}^{\infty} \frac{\exp\left(-\sqrt{(my)^2 n^2 + (mz)^2 l^2}\right)}{\sqrt{(my)^2 n^2 + (mz)^2 l^2}}. \tag{11}$$

Now, taking as usual $Nu = \lambda$ fixed and using Eq. (3), we find the large-N effective (L and T-dependent) renormalized coupling constant, for $D = 3$, as

$$g(L,\beta,\lambda) = N\Gamma_{3R}^{(4)}(0,L,\beta,u) = \frac{\lambda}{1+\lambda\Sigma_R(L,\beta)}. \tag{12}$$

This is the basic result for subsequent analysis. We notice immediately that the behavior of $g(L,\beta,\lambda)$ is dictated by the dependence of Σ_R on L and β and the value of the fixed coupling constant.

The numerical computation of $\Sigma_R(L,\beta)$ is greatly facilitated by the fact that the double series defining the functions $G(L,\beta)$ and $H(L,\beta)$ are rapidly convergent. ¿From Eqs. (9-12), we see that $\lim_{L,\beta\to\infty}\Sigma_R(L,\beta) = 0$ and therefore g reduces to λ, the renormalized fixed coupling constant in free space at zero temperature. On the other hand, for either $L \to 0$ or $\beta \to 0$, $\Sigma_R(L,\beta)$ diverges implying that we have ultraviolet asymptotic freedom for short distances and/or for high temperatures, irrespective of the value of λ. The overall behavior of the renormalized subdiagram can be acquainted from Fig. 1, where we draw contour plots of $\Sigma_R(L,\beta)/m$, taking L and β in units of m^{-1}. The full line in Fig. 1 is the locus of the points such that $\Sigma_R(L,\beta) = 0$, which for large L (β) approaches the straight line $\beta = 2.07\,m^{-1}$ ($L = 2.07\,m^{-1}$); $\Sigma_R(L,\beta)$ is positive below this curve, negative above it, and reaches an absolute minimum, $\Sigma_R^{min} \simeq -0.0624\,m$, at the point $L = \beta \simeq 3.13\,m^{-1}$. This minimum (negative) value of $\Sigma_R(L,\beta)$ defines a "critical value" for λ, $\lambda_{min} = -(\Sigma_R^{min})^{-1} \simeq 16.03\,m^{-1}$, for which the denominator of Eq. (12) vanishes and the effective coupling constant diverges; for $\lambda < \lambda_{min}$ this never occurs. Also, if we take L and β different from $3.13\,m^{-1}$ (but still in the region of negative values of Σ_R), $0 > \Sigma_R(L,\beta) > \Sigma_R^{min}$, the denominator of Eq. (12) vanishes for larger values of λ ($\lambda > \lambda_{min}$).

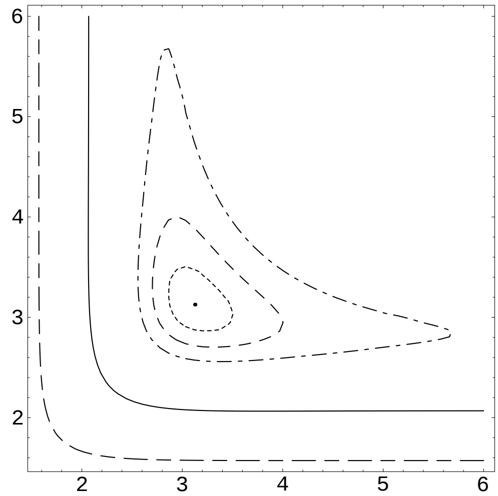

FIG. 1: Contour plots of $\Sigma_R(L,\beta)/m$, with L and β in units of m^{-1}. The open dashed line corresponds to $\Sigma_R(L,\beta)/m = 0.2$, the full line gives the points where $\Sigma_R = 0$, while the closed curves are for negative values of Σ_R/m, -0.053, -0.058 and -0.061 (dashed-dotted, dashed and dotted lines respectively). The dot is the location of the absolute minimum of Σ_R.

The existence of a region in the parameter space (L,β) where Σ_R is negative leads naturally to the onset of spatial confinement and thermal deconfinement, if the fixed coupling constant is high enough.

Consider initially the situation at $T = 0$. In this case, $\Sigma_R(L)$ has a zero at $L_c^{min} \simeq 2.07\,m^{-1}$, is negative for larger values of L, reaching a minimum ($\Sigma_R^{(0)min} \simeq -0.052\,m$) at $L_c^{max} \simeq 2.82\,m^{-1}$. We present in Fig. 2 $\Sigma_R(L)$ as a function of L. The existence of such a minimum implies that, for $\lambda \geq \lambda_c = -(\Sigma_R^{(0)min})^{-1} \simeq 19.16\,m^{-1}$, $\lambda/g(L,\lambda)$ has a non-positive minimum value and vanishes for a length $L_c^{(0)}(\lambda)$ belonging to the interval $\left(L_c^{min}, L_c^{max}\right]$. This means that the system will be confined in a length $L_c^{(0)}(\lambda)$, that is, starting with L small (in the region of asymptotic freedom) the length can not go above $L_c^{(0)}(\lambda)$ since $g(L,\lambda) \to \infty$ as $L \to L_c^{(0)}(\lambda)$ [5].

In Fig. 3, we show the effective coupling constant $g(L,\lambda)$ as a function of L, for some values of λ. We see clearly that, in the strong coupling regime, we have simultaneously asymptotic freedom and spatial confinement, in the sense described above.

Let us now consider the effect of temperature, taking $\lambda \geq \lambda_c$. For low (fixed) T, $\lambda/g(L,\beta,\lambda)$ vanishes at a value $L_c^{(\beta)}(\lambda) < L_c^{(0)}(\lambda)$, its minimum (negative) value being slightly lower than the zero temperature case. Further raising the temperature, $L_c^{(\beta)}(\lambda)$ and the minimum value of λ/g increase and, at the temperature $T_d(\lambda) = \beta_d^{-1}(\lambda)$, the minimum of $\lambda/g(L,\beta,\lambda)$ vanishes. The behavior of λ/g, as a function of

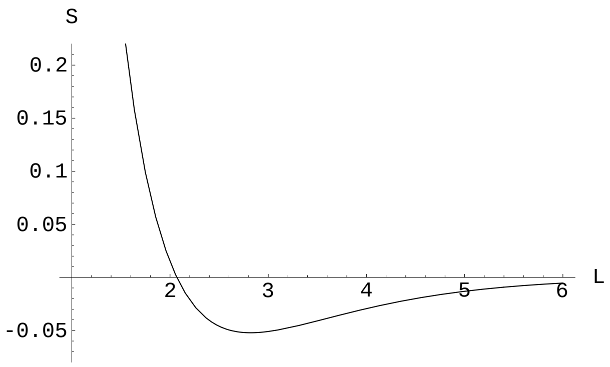

FIG. 2: Plot of $S = \Sigma_R(L)/m$, with L in units of m^{-1}.

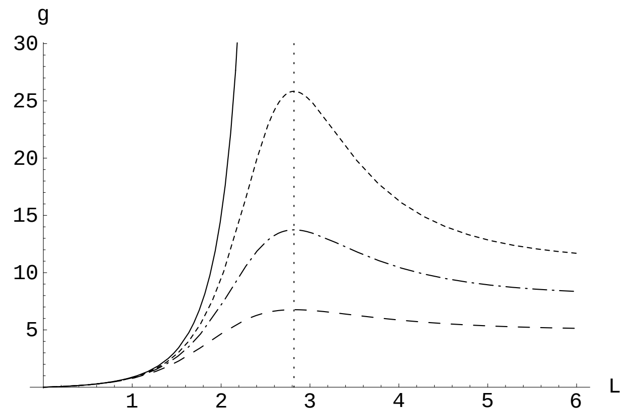

FIG. 3: Plots of the effective coupling constant $g(L,\lambda)$ as a function of L (in units of m^{-1}), for some values of λ (in units of m^{-1}): 5.0 (dashed line), 8.0 (dotted line), 11.0 (dotted-dashed line) and 19.16 (full line). The dotted vertical line, passing by $L_c^{max} \simeq 2.82$, is plotted as a visual guide.

L, is shown in Fig. 4 where we take $\lambda = 25.0\,m^{-1}$ and some values of β. Therefore, for $\beta < \beta_d(\lambda)$, $\lambda/g(L,\beta,\lambda)$ becomes positive for all values of L and then the system is unconfined. Thus, $T_d(\lambda)$ corresponds to the deconfining temperature for the given fixed coupling constant $\lambda \geq \lambda_c$.

Our finding that fermion spatial confinement exists in the strong coupling regime of the compactified Gross-Neveu model, being destroyed by raising the temperature, may acquire a physical meaning if we consider the Gross-Neveu model as an effective theory for the strong interaction between quarks. This corresponds to shrinking the gluon propagator similarly to the Fermi treatment of the weak force between leptons. In this sense, we will take the fermion mass as the constituent quark mass, $m \approx 350\,MeV \simeq 1.75\,fm^{-1}$ [17], in order to estimate the confining length and the deconfining temperature. We also take the fixed coupling constant with the minimum strength for confinement, $\lambda = \lambda_c \simeq 19.16\,m^{-1}$, corresponding to the maximum confining length at zero temperature, $L_c \simeq 2.82\,m^{-1}$. For this case, we find $\beta_d \simeq 2.54\,m^{-1}$. This choice leads to $L_c \simeq 1.61\,fm$ and $T_d \simeq 138\,MeV$. These

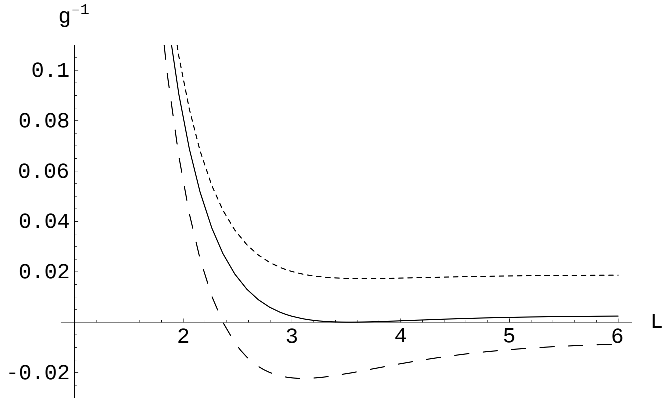

FIG. 4: Inverse of the effective coupling constant g^{-1} (in units of λ^{-1}), with $\lambda = 25.0\,m^{-1}$ fixed, as a function of L (in units of m^{-1}), for some values of β (in units of m^{-1}): 3.2, 2.356 and 2.2 (dashed, full and dotted lines respectively).

values are of the order of the experimentally measured proton charge diameter ($\approx 1.74\,fm$) [18] and the estimated deconfining temperature ($\approx 200\,MeV$) for hadronic matter [19], respectively. It should be noticed that, despite of the crudeness of this estimate, our result is in the range of the expected deconfining temperature of QCD.

In summary we have shown that, in the weak coupling situation ($\lambda < \lambda_c$), the 3-D Gross-Neveu model presents only short-distance and high-temperature asymptotic freedom. For the strong coupling regime ($\lambda > \lambda_c$), we analytically demonstrate the simultaneous existence of asymptotic freedom and (for low enough temperatures) a singularity in the effective renormalized coupling constant at a length $L_c(\lambda)$, signalizing spatial confinement. This means that, if we start with a system of a *quark-antiquark* (understood as quanta of the fermionic Gross-Neveu model) pair bounded between two planes a distance L ($< L_c(\lambda)$) from one another (at some, low enough, temperature), it would not be possible to separate them a distance larger than $L_c(\lambda)$. This *spatial* confinement of the *quark-antiquark* pair could be interpreted as the existence of bound states ("baryon-like" states), characteristic of the model in the strong coupling regime. By raising the temperature, we find that the spatial confinement disappears at the deconfining temperature $T_d(\lambda)$. Notice that we refer to this property of the Gross-Neveu model as *confinement*, understood in the sense described above, *not* of color confinement as it should happen for QCD. To account for color confinement we should consider a model that would accommodate gauge bosons, for instance Large-N QCD. This is the subject of a future investigation.

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