Zero-Temperature Superconducting Transition in Frustrated Josephson-Junction Arrays

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The critical behavior of zero-temperature superconducting transitions which can occur in disordered two-dimensional Josephson-junction arrays are investigated by Monte Carlo calculation of ground-state excitation energies and dynamical simulation of the current-voltage characteristics at nonzero temperatures. Two models of arrays in an applied magnetic field are considered: random dilution of junctions and random couplings with half-flux quantum per plaquette $f = 1/2$. Above a critical value of disorder, finite-size scaling of the excitation energies indicates a zero-temperature transition and allows an estimate of the critical disorder and the thermal correlation length exponent characterizing the transition. Current-voltage scaling is consistent with the zero-temperature transition. The linear resistance is nonzero at finite temperatures but nonlinear behavior sets in at a characteristic current density determined by the thermal critical exponent. The zero-temperature transition provides an explanation of the washing out of structure for increasing disorder at $f = 1/2$ while it remains for $f = 0$, observed experimentally in supercondoucting wire networks.

I. Introduction

Inhomogeneous superconductors in the form of an array of superconducting grains embedded in a non-superconducting host, may exhibit many properties that arise essentially from the phase-coherence among the grains, due to superconducting coupling [1-8]. The coupling between the grains can occur by Josephson tunnelling through an insulating host or by proximity effect in a normal-metal host. These systems can be physically realized as granular superconductors and are particularly important in high-T_c superconductors [2] where phase fluctuations effects from temperature and disorder play an important role. They can be artificially fabricated in two dimensions as arrays of Josephson junctions [4, 5], with well controlled parameters, and are also closely related to superconducting wire networks [9, 10], where the nodes of the network act like coupled effective "grains". In the simplest model only the phase coupling between the superconducting order parameter $\Psi_j = |\Psi_j|e^{i\theta_j}$ of point grains, defined on a lattice labelled by j, are taken into account. Each junction between nearest-neighbor grains contributes to the energy of the system as [11] $E_{ij} = -J_{ij}\cos(\theta_i - \theta_j - A_{ij})$. In the case of Josephson-junction arrays, the coupling J_{ij} can be related to the junction critical current I_{ij} as $J_{ij} = \hbar I_{ij}/2e$ and for wire networks it is proportional to the superconductor condensation energy of the wire material. The bond variable $A_{ij} = \frac{2\pi}{\phi_o}\int_i^j \vec{A}\cdot dl$ is the line integral of the vector potential $\vec{A}$ between grains in units of the flux quantum $\phi_o = hc/2e$ due to an applied magnetic field $\vec{B} = \nabla \times \vec{A}$. The resulting Hamiltonian of the coupled Josephson junction array can then be written as

$$H = -\sum_{<ij>} J_{ij}\cos(\theta_i - \theta_j - A_{ij}) \qquad (1)$$

where the sum is taken over nearest neighbors $< ij >$. This model is equivalent to an XY (two-component) spin system in which the gauge field A_{ij} leads to frustration effects in the spin alignment [12]. The frustration introduced by the magnetic field on a given plaquette p of the lattice can be defined as $f_p = \Phi_p/\Phi_o$, where Φ_p is the flux threading the plaquette, giving in general a fractional number of flux quanta. In this model, one also assumes that quantum-phase fluctuations due to charging effects [3] can be neglected. In a granular material, disorder arises naturally from the random location of grains leading to randomness in J_{ij} and A_{ij} (or equivalently on f_p). In general, different models can then be considered depending on the assumed random distribution of J_{ij} and A_{ij}.

Even when artificially fabricated as two-dimensional periodic lattices, in which case $f_p = f$ is a constant and the model in Eq. 1 is periodic in f with period $f = 1$, Josephson-junction arrays or wire networks will always contain some degree of disorder [13]. For example, inhomogeneities in the links of a regular superconducting wire network of YBa_2CuO_7 has been suggested as an importance source of disorder [14]. As discussed in the present work, if such disorder is modelled by Eq. 1 with random J_{ij}, it may explain the washing out of resistance minima observed at half-integer f in this system, in contrast to the case with integer f where sharp minima is observed.

On the other hand, disorder can also be intentionally introduced to study its effects [9, 15, 16] and compare with theoretical predictions [7, 13]. In this regard, random dilution of grains, in an otherwise two-dimensional periodic array, is probably the simplest way to introduce disorder in a controllable manner [9, 16]. Such system can be modelled by Eq. 1 assuming that a concentration x of junctions are removed randomly. In this case, recent theoretical work [17, 18] indicates that for $f = 1/2$, the superconducting transition should vanish above a critical value of disorder x_s but below the percolation dilution threshold x_p. In principle this can be verified experimentally through resistivity measurements in arrays. The disappearance of structure at $f = 1/2$ for increasing disorder already observed experimentally in diluted wire networks [9] supports this result.

In experiments as well in numerical simulations, the superconducting transition is usually identified from the behavior of the nonlinear current-voltage characteristics. However, the defining property of the superconducting phase is the vanishing linear resistivity. In order to extract information of the linear response and underlying transition, a scaling analysis [2, 19] is then required from which the critical temperature and related exponents can be determined. The interpretation of the experimental data also relies on the expected behavior of the model used to describe the system. In particular, knowledge of the so called lower critical dimension of the model [20] is essential for a meaningful application of the current-voltage scaling theory. This is because, below the lower critical dimension, there is a phase transition only at zero temperature but its effects can still show up at finite temperatures as a crossover behavior, rather than a thermodynamic transition. Thus, it is important to study in detail the lower critical dimension and transport properties of the basic models in order to provide support for these interpretations as well as to make predictions for the realistic physical system.

In this work, models of two-dimensional superconducting arrays with possible zero-temperature transitions relevant for Josephson-junction arrays and wire networks are considered. Monte-Carlo simulated annealing is used to obtain the ground-state excitation energies in order to study the lower-critical dimension while Langevin-type simulation is used to study the scaling of the current-voltage characteristics at nonzero temperatures. Two different models of disordered arrays in an applied magnetic field are considered: random dilution of junctions and random couplings with half-flux quantum per plaquette. Above a critical value of disorder, finite-size scaling of the excitation energies indicates that such models are below the lower-critical dimension and a zero-temperature transition is expected. At nonzero temperatures, the linear resistance is nonzero but nonlinear behavior sets in at a characteristic current density determined by the thermal critical exponent. These results are a possible explanation for the washing out of structure at $f = 1/2$ observed in some wire networks [9, 14].

II. Models and simulation

The diluted Josephson-junction array can be modelled by Eq. 1 assuming a junction dilution concentration x corresponding to J_{ij} being zero or J_o with probabilities x and $1 - x$, respectively, and is related to the fraction of junctions present p by $x = 1 - p$. Since any closed loops of nonzero bonds J_{ij} have an area which is an integer multiple of the elementary area S, the properties of this model remain periodic in $f = BS/\Phi_o$ with period 1. Similarly, the random coupling model is defined by Eq. 1 assuming $J_{ij} = J_o \pm D$ with equal probability. As defined, these two models are just particular cases of a more general model with random Josephson couplings where J_{ij} is given by an arbitrary probability distribution. In both cases, we consider in this work a square lattice and an external magnetic field corresponding to half-flux quantum per plaquette, $f = 1/2$.

To study the stability of the ground state, the energy change $\Delta E(L)$ of a defect in the ground state, corresponding to a low-energy excitation, is calculated for small system sizes L by Monte Carlo simulated annealing [17]. The defect-energy renormalization analysis [20] is then used to infer the behavior in the large system limit. Stability of the ground state against thermal fluctuations requires that $[\Delta E]_d$, where $[\ ]_d$ denotes a disorder average, is finite or increases with L. In a strongly disordered phase, as in the vortex-glass state, $[\Delta E]_d = 0$ but the width of defect energy distribution

$W_L = \{[(\Delta E)^2]_d - ([\Delta E]_d)^2\}^{1/2}$ is expected to scale as

$$W_L \propto L^\theta \tag{2}$$

for large enough L. If the stiffness exponent θ is positive there is long-range order in the system and a nonzero critical temperature. However, if $\theta < 0$, there exists a length scale where $L^\theta \sim W_L \sim kT$, beyond which thermal fluctuations destroy the order at any nonzero temperature and so the critical temperature vanishes. This length scale can be identified as a correlation length $\xi \propto 1/T^\nu$, with a critical exponent given by $\nu_T = 1/|\theta|$. As discussed below, this critical exponent determines the current-voltage scaling of the zero temperature transition.

The current-voltage characteristics can be obtained by numerical simulation of the resistivelly shunted Josephson-junction (RSJ) model for the current flow between grains [21, 22, 23]. The Langevin-type equations for this model can be written as

$$C_o \frac{d^2\theta_i}{dt^2} + \frac{1}{R_o}\sum_j \frac{d(\theta_i - \theta_j)}{dt} = - I_c \sum_j \sin(\theta_i - \theta_j - A_{ij}) + I_i^{ext} + \sum_j \eta_{ij} , \tag{3}$$

where I^{ext} is the external current, η_{ij} represents Gaussian thermal fluctuations satisfying

$$\begin{aligned} < \eta_{ij}(t) > &= 0 \\ < \eta_{ij}(t)\, \eta_{kl}(t') > &= \frac{2k_B T}{R_o}\delta_{ij,kl}\delta(t - t') \end{aligned} \tag{4}$$

and a capacitance to the ground C_o is allowed, in addition to the shunt resistance R_o, in order to facilitate the numerical integration [22]. The parameter $I_c R_o^2 C_o = 0.5$ used in the simulations corresponds to the overdamped regime. Hereafter, dimensionless quantities are used in units where $\hbar/2e = 1$, $R_o = 1$ and $I_c = 1$. The above equations can be integrated numerically and the results averaged over different realizations of the disorder.

To determine the nonlinear resistivity (or resistance in two dimensions), $\rho_{nl} = E/J$, a uniform external current I is imposed with density $J = I/L$ along one of the principal directions of the lattice using fluctuating boundary conditions [24]. The average voltage drop V across the system is computed as

$$V = \frac{1}{L}\sum_{j=1}^{L}\left(\frac{d\theta_{1,j}}{dt} - \frac{d\theta_{L,j}}{dt}\right) \tag{5}$$

and the average electric field by $E = V/L$. The linear resistance, $R_L = \lim_{J\to 0} E/J$, can also be computed without finite current effects, directly from the long-time equilibrium fluctuations of the phase difference across the system [23] $\Delta\theta(t) = \sum_{j=1}^{L}(\theta_{1,j} - \theta_{L,j})/L$ as

$$R_L = \frac{1}{2T}(\Delta\theta(t) - \Delta\theta(0))^2/t \tag{6}$$

which can be obtained from Kubo formula of equilibrium voltage-voltage fluctuations,

$$R_L = \frac{1}{2T}\int dt < V(t)V(0) >, \tag{7}$$

using the Josephson relation $V = d\theta/dt$.

III. Results and discussion

IV. Defect-energy scaling

To determine the stability of the ground state against low-energy excitations, a defect is created in a system of size $L \times L$ by imposing a change in the boundary conditions in one direction. The change $\Delta E(L)$ in the ground state energy for small systems is calculated for a large number of samples by directly searching for the minimum energy using Monte Carlo simulated annealing. The defect energy is obtained from the energy difference $\Delta E = E_a - E_p$ between periodic E_p and antiperiodic E_a boundary conditions in the phases θ_i. This energy difference provides a measure of phase coherence and can related to the renormalized stiffness constant by $J^*(L) = 2\Delta E/\pi^2$. In the thermodynamic limit, J^* is finite in the phase coherent state and vanishes in the incoherent state. In presence of disorder, ΔE fluctuates between samples, with a distribution that can be characterized by its moments. Stability of the ground state against thermal fluctuations requires that the average $[\Delta E]$, where [ ] denotes a disorder average, is finite or increases with L.

Fig. 1(a) shows the behavior of $[\Delta E]$ as function of L for the diluted array on a square lattice for increasing dilution x. The behavior is similar to that observed for the same model on triangular lattice [17]. For small x, it increases with L and eventually saturates, indicating the existence of long-range phase coherence. The increasing trend is a small size effect. For large L, it should scale as the phase stiffness which is proportional to L^{d-1} where d is the dimension of the system. On the other hand, for sufficiently large x it clearly decreases for increasing L, indicating a disordered phase. The change in the behavior yields an estimate of the phase-coherence threshold $x_s = 0.15(4)$. We note that this value is consistent with the one inferred from the behavior of the zero-temperature critical current in dynamical simulations [18]. Surprisingly, the same estimate is also obtained for the triangular lattice, $x_s = 0.14(1)$, even though the percolation dilution thresholds [25], $x_p = 0.5$ and 0.652, for the square and triangular lattices respectively, are quite different. Since it is known [25] that the transition temperature in absence of magnetic field, $f = 0$, vanishes at x_p, there is a range of disorder $x_s < x < x_p$ where the transition for $f = 1/2$ only occurs at $T = 0$ while it occurs at nonzero temperatures for $f = 0$. Measurements of the temperature-magnetic field phase boundary for increasing disorder in diluted wire networks [9] are consistent with a phase coherence threshold x_s for $f = 1/2$, which shows up as a washing out of the structure at half-integer f, while it remains essentially unaffected for f integer.

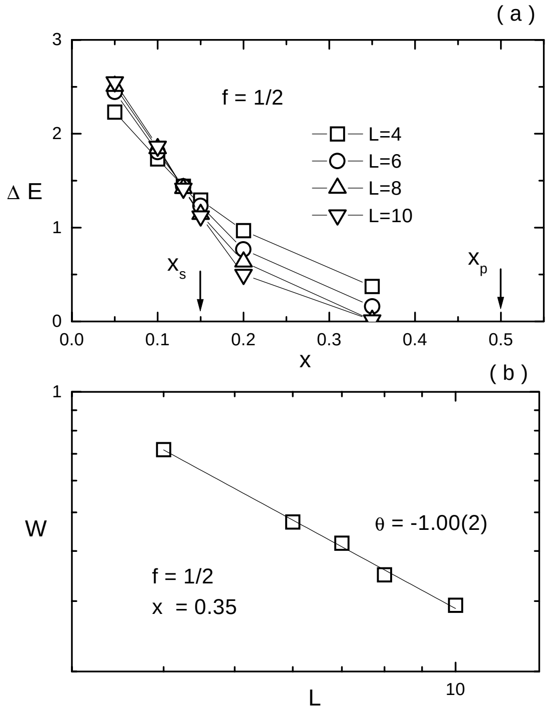

Figure 1. (a) Finite size behavior of defect energy [ΔE] for increasing dilution x and various system sizes L, on a square lattice. Arrows indicate the phase-coherence and percolation dilution thresholds. (b) Finite size behavior of the width of defect energy distribution W at a dilution concentration $x = 0.35$ in the range $x_s < x < x_p$.

The disordered phase for $x_s < x < x_p$ can be regarded as a vortex glass, since it also lacks long-range order in the vortex lattice [17]. The stability of this glass phase against thermal fluctuations is determined by the size dependence of the second moment of the defect-energy excitations W which is expected to have a power-law behavior given by Eq. 2. As shown in Fig. 1(b), the power-law exponent θ for a value of $x = 0.35$ in this region is negative. As a consequence, this vortex glass phase is below its lower-critical dimension and the phase transition only occurs at $T = 0$. From the power-law behavior of $W(L)$ the exponent $\nu_T = 1/ \mid \theta \mid$ of the thermal correlation length is estimated to be $\nu_T \sim 1$.

The results of defect-energy scaling for the array with random Josephson couplings are shown in Fig. 2. The behavior is similar to that observed for the diluted array discussed above. For small disorder, ΔE increases with L, indicating long range phase coherence, while for sufficiently large D it decreases for increasing L, indicating a disordered phase. The change in the behavior yields an estimate of the phase-coherence threshold $D_s = 0.6(4)$ for $f = 1/2$ and $D_o \sim 0.9$ for $f = 0$. Again, the power-law exponent θ for a value of disorder $D = 0.8$ larger than the critical value D_s is negative as shown in Fig. 2(b) and the vortex glass phase for $D > D_s$ is below its lower-critical dimension and correspondingly the phase transition for $f = 1/2$ should occur at $T = 0$. From the power-law behavior of $W(L)$ the exponent of the thermal correlation length is estimated to be $\nu_T \sim 1$. As for the diluted model, there is a range of disorder $D_s < x < D_o$, where the transition for $f = 1/2$ only occurs at $T = 0$ while it occurs at nonzero temperatures for $f = 0$. Thus, magnetoresistance measurements just above the $f = 0$ critical temperature, in arrays with disorder in this range, should display resistance minima only at integer values of f. This behavior has been observed in regular superconducting wire network [14] of YBa_2CuO_7 with high inhomogeneity in the links which in principle could lead to coupling disorder in this range.

V. Current-voltage scaling

The nonlinear resistance E/J as a function of current density J and temperature T for the array with random Josephson couplings is shown in Fig. 3(a) for a degree of disorder $D = 0.75$ above the critical value D_s estimated above. Similar behavior is found for the diluted array [18] above the phase coherence threshold x_s. The data shows the expected behavior for a $T = 0$ superconducting transition [2, 19]. In Fig. 3(a), the ratio E/J tends to a finite value for small J, corresponding to the linear resistance $R_L = \lim_{J\to 0} E/J$, which depends strongly on the temperature. For increasing J, there is a smooth crossover to nonlinear behavior that appears at smaller currents for decreasing temperatures. If the transition only occurs at $T = 0$, as indicated by the defect-energy scaling analysis discussed above, then the correlation length should diverge for decreasing temperature and a temperature-dependent crossover is expected. The linear resistance R_L is finite at any nonzero temperature but thermally activated,

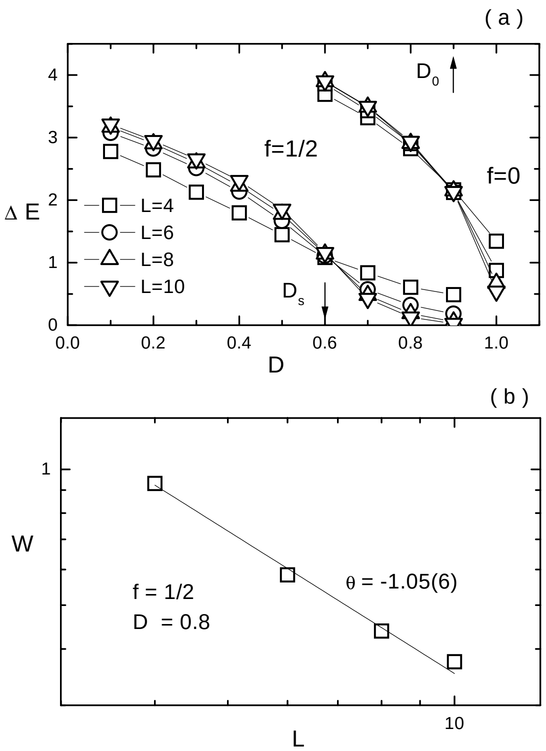

Figure 2. (a) Finite size behavior of defect energy $[\Delta E]$ for $f = 1/2$ and $f = 0$ for increasing disorder D and various system sizes L for the array with random couplings on a square lattice. Arrows indicate the corresponding phase-coherence thresholds. (b) Finite size behavior of the width of defect energy distribution W at a value of disorder $D = 0.75$ above D_s.

$R_L \propto \exp(-E_b/kT)$, where E_b is an energy barrier. If one assumes that the correlation length diverges as a power-law $\xi_T \propto T^{-\nu_T}$ then the behavior of the nonlinear resistivity normalized to R_L can be cast into the scaling form [2, 19]

$$\frac{E}{JR_L} = g(\frac{J}{T^{1+\nu_T}}) \tag{8}$$

in $d = 2$ dimensions, where g is a scaling function with $g(0) = 1$. A crossover from linear behavior, when $g(x) \sim 1$, to nonlinear behavior, when $g(x) >> 1$, is expected to occur when $x \sim 1$ which leads to a characteristic current density J_{nl} at which nonlinear behavior sets in that decreases with temperature as a power law $J_{nl} \propto T^{1+\nu_T}$.

One can then proceed to verify the scaling hypothesis and obtain a numerical estimate of the critical exponent ν_T. Fig. 3(b) shows a scaling plot according to Eq. (8) obtained by adjusting the parameter ν_T so that a best data collapse is obtained. The data collapse supports the scaling behavior of Eq. (8) and provides an estimate of $\nu_T = 2.2$. This estimate differs significantly from the value obtained by the above defect-energy scaling analysis for system sizes $L \leq 10$. At the present, the origin of this discrepancy is not quite clear. It is possible that the true asymptotic value of ν_T can only be obtained from the defect-energy scaling for much larger system sizes, which would require more accurate determination of the ground-state energy.

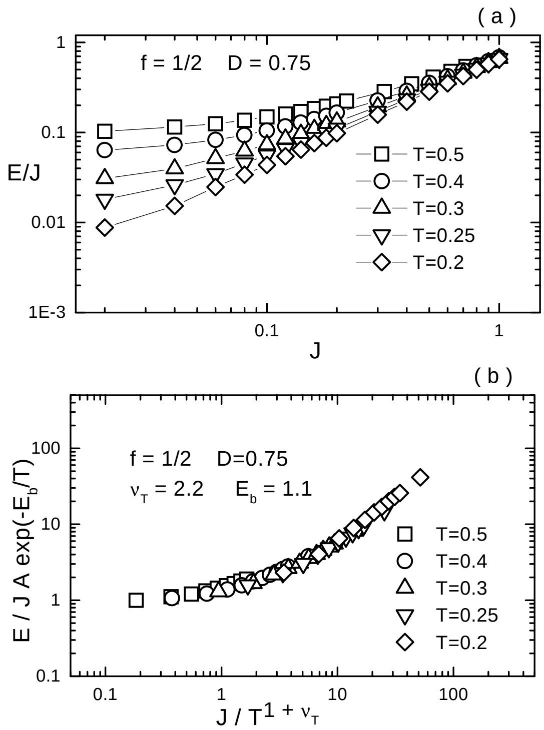

Figure 3. (a) Nonlinear resistance E/J as a function of temperature T for the array with random couplings, for $D = 0.75$ and system size $L = 64$, above the phase-coherence threshold D_s. (b) Scaling plot of the data in (a) for the lowest temperatures and current densities according to a $T = 0$ transition.

VI. Conclusions

The results of the ground state defect-energy calculations and dynamical simulations discussed here show that disorder in two-dimensional Josephson-junction arrays has important effects which show up in the current-voltage characteristics as a manifestation of a zero-temperature superconducting transition. It is interesting to note that even in the absence of magnetic field, arrays with random couplings can display similar current-voltage scaling near percolation threshold [26] and, without disorder, if the magnetic field corresponds to an irrational frustration [27]. For the diluted and random-coupling arrays studied in this work we find that, in presence of a magnetic field corresponding to $f = 1/2$ flux quantum per plaquette and above a critical value of disorder, finite-size scaling of the excitation energies indicates a zero-temperature transition and allows an estimate of the critical disorder and the thermal correlation length exponent characterizing the transition. Correspondingly, resistivity calculations at nonzero temperature shows that the linear resistance is nonzero but nonlinear behavior sets in at a characteristic current density determined by the thermal critical exponent. Since the critical disorder is smaller than the corresponding value for $f = 0$, there is a range of disorder where the superconducting transition occurs at nonzero temperatures for $f = 0$ while it only occurs at zero temperature for $f = 1/2$. Some experimental results on superconducting wire networks with inhomogeneous [14] and diluted links [9] support these conclusions in the sense that a washing out of the structure at $f = n/2$ (n integer) is found due to disorder while the structure at $f = n$ is essentially unaffected. Nevertheless, a detailed comparison between theory and experiment still awaits more accurate data and further predictions of measurable quantities.

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