Dynamic Modeling of Overload in Scale Free Networks

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We introduce a simple dynamic model to investigate the fragmentation of transport networks. The transport properties like as the size of largest connected cluster, the length of the minimum paths and the optimal paths between a pair of nodes of the network were evaluated upon continuously increasing the load on the system. We use two load insertion strategies: an uniform random distribution of loads and a Cohen-like immunization strategy (one node is selected with a uniform probability p and one of its first neighbours, randomly selected, receives the load). Both strategies may be classified as local strategies but the resulting effects are qualitatively different. Evaluating the physical quantities as a function of time we observe that for the random distribution strategy there is a crossover from a fully connected cluster to a non-connected state in the sense that all links become unavailable. On the other hand, following the Cohen-like strategy we found a sudden change in transport properties which is may be interpreted as a percolation-like transition induced by the cumulative process of load.

Keywords: Transport Networks; Load Management; Percolation

I. INTRODUCTION

Many real complex networks show heterogeneous structures with power-law degree distribution $P(k) \propto k^{-\gamma}$, where k is the number of links of a randomly chosen node in the network and γ is the scaling exponent [1, 2]. This algebraic distribution means that, in contrast to random networks, the probability for a node to possess a large number of links is not exponentially small [3, 4]. Due the ubiquity of scale-free networks in natural and manmade systems problems related to the security of these networks and their resilience have attracted a great interest. A myriad of important aspects concernig complex networks including epidemics, disturbances in power transmission systems, effects of network growth, cascading failures triggered by intentional attacks, avalanche size distributions and congestion instabilities have been discussed in the literature [4]-[9].

Recently the load concept was introduced to address the pattern of transport but considering both links and nodes as identical in terms of their functional roles in the network [1, 10]. Aiming to take into account the heterogeneity of elements such studies have been generalized by introducing weights to the links [11–13] and the congestion effects in transport networks may be taken into account by investigating the weight of a link as a function of its cumulative load (called cost function)[13–15]. Using simple models of load distribution the congestion effects could be described as function of time. In this scope we investigate the fragmentation phenomenon in a free-scale transport network computing how the nodes become unavailable due the overload of their links. Our work differs on the previous in the sense that the overload of the links is triggered by dynamic processes of continuous increase of load. The transport properties like as the size of largest connected cluster, the length of the minimum paths and the optimal paths between a pair of nodes which are usually used to define the efficience of the network were evaluated as a function of time.

This paper is organized as follow. In Section II we introduce our model motivated by studies of percolation models defined on complex networks and introduce the load insertion strategies. Section III describes the details of the investigated properties and discuss the results of the simulation taking into account the different strategies designed in the Section II. The Section IV summarizes the work taking a brief overview of the main results.

II. THE DYNAMIC PERCOLATION MODEL

Since the recognition that many technological, biological and social real systems could be mathematically represented by complex networks models the percolation tools have been applied to investigated their large scale connectivity properties like the resilience and robustness [1, 4, 5]. In these works the real system is modeled as a graph (or a mathematical network) in which the vertices or sites represent the elements and the edges represent their interactions. Such kind of investigation have contributed to improve our knowledge about the structure of complex systems and have provide useful clues to design efficient management strategies of real networks. In percolation works each site of the network can be considered inactive (or removed from the network) with a probability p or active (remain in the network) with a probability $(1-p)$. Upon increasing p in the interval $[0,1]$ exists a value of p for which the network is broken into a huge number of small connected clusters, that is, the network is disrupted like as a percolation transition. Such value of p may be referred as the percolation threshold p_c.

The removal probability p may be uniform or a function of some property of the network, defining different removal strategies. Many removal strategies of sites have been applied

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in scale-free networks producing a variety of interesting results [1, 4]. Among the strategies the random removal [1] (sites are removed with a uniform probability independent of its state or the state of its neighbouring) is the simplest one. It is a local removal strategy which aims to mimic sporadic failures on nodes of a network. On the other hand, the target removal strategy [1] may be classified as a global one because it requires the knowledge of the connectivity of all sites since the highest connected sites are sequentially removed (in decreasing order) until a fraction p of the whole system be removed. Such strategy intend to describe coordinate attacks to the network. The ideas of the percolation theory may be also used to design strategies aiming to protect the networks against external attacks and to propose efficient immunization strategies against epidemics [7, 8]. In the context of epidemics Cohen *et al* [11] proposed an efficient immunization strategy based on the immunization of acquaintances that, being local, exhibits properties of a global one.

Although much attention has been payed to the investigation of percolation properties of complex structures the major part of such studies focus static properties of the networks, that is, the behaviour of the system properties when a fraction p of their elements are *simultaneously* removed. However, systems subject to fluctuation of load could reach its limiting capacity though cumulative processes of individual failures (of nodes or links) which could not be characterized neither as sporadic failures nor as external attacks. Interesting cases are the communication networks. In a coarse grained approximation the transport capacity of a communication network is limited by two factors: link capacity (bandwidth) and node processing power (in the case of communication networks, the outer latency quantified by the packet insertion rate at which jamming may occurs). Such kind of system have been investigated using weighted networks [12–14] but also in these cases, the associated weigths are static quantities disregarding that dynamic features should be taken into account in order to describe changes in the transport properties occurring while the system is running [16, 17]. In this work we introduce a model in which the weigth associated to the links change in time and apply percolation tools to investigate it. Due their finite capacity (or bandwidth) the links may become overloaded and the transport through them is interrupted. The overload of links isolate finite clusters of sites and the system undergoes a percolation-like transition. In contrast to standard model used in previuos works, in our model the remotion of sites is not (necessarily) simultaneous and is time dependent.

Let us begin describing the details of our dynamic model defined on a scale-free network. For simplicity we consider a scale free network of mean connectivity $<k>=4$ constructed following the Barabási-Albert (BA) prescription [2]. To each link connecting a pair of sites we assign a maximum capacity of load management (or bandwidth) $\mathcal{L}_{max}=1$ and an initial load $\mathcal{L}_0$ in the interval $[0.5,1]$ while the sites have infinite capacity (the minimal initial load was fixed to be $\mathcal{L}_0=0.5$ since, in real networks, the links are designed to assure some minimal capacity of load transport). A site of the network selected and an additional load β is attributed to it. This load is distributed among its links proportionally to the avaliable capacity of the links, i.e, links with greater individual load receive a smaller part of additional load. For a site j of connectivity k_j the load $\Delta\mathcal{L}_i$ attributed to each one of its links i is calculated, for each time step, as

$$\Delta\mathcal{L}_i = z^{-1}\,\beta\,(1-\mathcal{L}_i) \tag{1}$$

where $\mathcal{L}_i$ is the load of the link i at time t and z is a normalization constant calculated in a such way that

$$\sum_{i=1}^{k_j}\Delta\mathcal{L}_i = \beta. \tag{2}$$

If the load attributed to a link exceeds its capacity ($\mathcal{L}_i > \mathcal{L}_{max}$) the link becomes overloaded. In this case the link becomes unavailable to new loads and its contribution to the networks transport properties is null. It could happen that a site is connected to the network only through overloaded links. In this case it becomes totally unattainable from any other site and may be considered as a removed site. We should stress that there is a second kind of unattainable sites. A cluster of sites could be locally connected through internal non-overloaded links but this cluster may be totally isolated from the whole network. So, depending on the total inserted load the network may be broken in many isolated clusters similar to those observed in percolation. Sites belonging to percolation clusters could yet be selected to receive load until becoming removed (we will refer as removed sites to those having all links overloaded).

One can define different load introduction strategies by choosing different functions to the probability p. Here we follow two strategies: (a) an uniform random distribution of loads (the sites selected to receive tthe load are randomly chosen with a uniform probability p) and (b) a Cohen-like immunization strategy [11](one node is selected with a uniform probability p and one of its first neighbours, randomly selected with uniform probability, receives the load). The next section describes the details of the simulation as well as the obtained results for both strategies.

III. SIMULATION AND RESULTS

We investigate the behaviour of the connectivity properties of scale free networks under continuous load insertion. Traditionally in percolation studies in scale free networks the quantities evaluated are the fraction of sites belonging to the giant connected cluster (S) and the length of minimum paths (l_{min}) connecting a pair of sites. In a finite system the giant cluster may be identified as the number of sites belonging to the largest connected cluster while the minimum paths are evaluated counting the minimum number of links connecting a pair of sites i and j and, in general the minimum paths could be used to estimate the efficience of the network in transport information [13]. In static percolation, upon increasing the network dilution the length of minimum paths increases and the network efficience diminishes. For $p > p_c$ the network

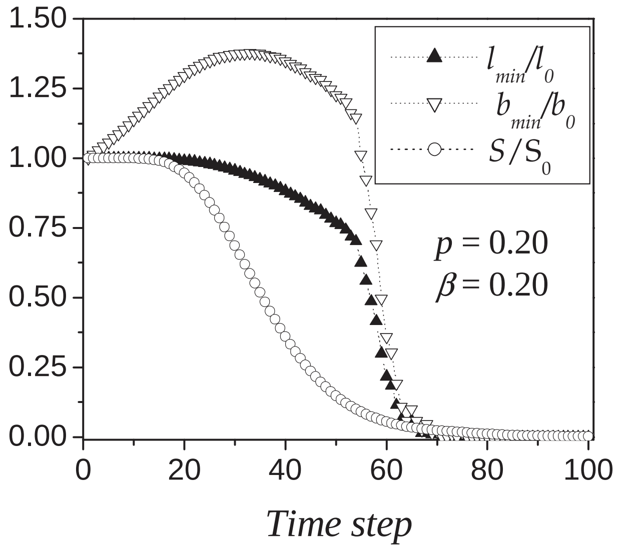

FIG. 1: Size of largest connected cluster, length of minimum paths and optimum paths as a funtion of time for the random insertion strategy.

is globally disconnected and the efficience vanishes. The increase of l_{min} occurs because the distance between a pair of nearest neighbours is unity and independent of the load of the link. However in communication systems it is possible to find a set of paths with the same minimum length but they could not be equivalent in exchange information. It could be more efficient exchange information along the path with the minimum total load [13], which is called optimum (or best) path. The optimum path could be defined as follows: at a time t each link has a weigth $b_{n,m}(t)$ representing the load between a pair of nearest neighbours n e m. The optimum path $b_{i,j}(t)$ will be that which minimizes the sum of weigths along all minimum paths connecting the pair of sites (i, j). This quantity could be useful in the investigation of systems under increasing and redistribution of loads since such alternative computation could represent better the efficience of a communication network since the links may manage a finite load without endanger (significantly) the network's efficience. So, besides the size of the largest clusters and the length of the minimum paths we also evaluated the optimum paths.

A. Random Insertion Estrategy

Maybe the random insertion estrategy is the simplest one. In this case the fraction p of sites of the network is randomly selected to receive an additional load β which is distributed among its links following the Equation 1. One time step is completed when all selected sites are consulted. Fig. 1 shows the behaviour of the size of the largest connected cluster, minimum and optimum paths (normalized by their corresponding values at $t = 0$) as a function of time steps.

At $t = 0$ the network is fully connected and the density of sites in the largest connected cluster is $S/S_0 = 1$. At low values of time the sites remain in the network since the load distribution does not imply in remotion of sites. However, while the total load increases with the time steps the sites are continuously removed from the network due the overload of their links and the size of largest cluster diminishes approaching a finite value, i.e., $S/S_0 \to 0$ for large t. It important to stress that the random insertion of load affect mainly the low connected sites since they have a few links to distribute the inserted load β. Moreover, as the low connected sites have a small contribution to the maintenance of the overall connectivity, the network remains connected in despite of being progressively diluted and a crossover to a disconnected state is observed. Let us now describe the time behaviour of the minimum (and optimum) paths are calculated on the largest connected cluster. In this case it is verified that while $S/S_0 = 1$ the l_{min} remains constant since the set of paths does not change. As the time goes on the network becomes more and more diluted and the size largest connected cluster diminishes, the corresponding mean distance between a pair of sites is reduced and also the length of minimum paths. At a first sight this is in contrast with all the previous results obtained in static percolation but we should stress that, in our model, the effect of dilution by overload of links is quite different. Indeed such process should hold similarities with a static bond percolation model in which the bonds of low connected sites are preferentially removed. In static site percolation the remotion of a site i implies on the simultaneous remotion of all (k_i) links connected to it and all the paths passing through the site i are interrupted. If the site selected to removal is a highly connected one, alternative routes of same length could not possible and the length of the minimum paths increases despite of the reduction of the size of the largest cluster. In our model the behaviour is qualitatively different. If the load is attributed to a highly connected site i, the distribution of the load among its links and the occasional overload of some links could not be sufficient to isolate i (which could remain connected to the network through some non-overloaded links) but some low-connected nearest neighbours of i could have their links overloaded and become isolated. So, the size of the largest connected cluster diminishes but the length of the minimum paths crossing i could not increase (indeed, the average of l_{min} could decrease since the size of the largest cluster diminishes). For the optimum paths the behaviour is quite different. At short times b_{min} increases reflecting the distribution of the additional load inserted among the sites of the network. Since the insertion is random each link has its weigth updated to a larger value and the total weigth of the paths increases. For intermediary time scales the size of the largest cluster and shortest distance between a pair of sites decrease but the weigth of the optimum path increases since the capacity of manage some load without become overloaded counterbalances the reduction of cluster size. However, due the limited capacity of the individual links ($\mathcal{L}_{max} = 1$) the increase of b_{min} is limited. Moreover if the weigths links of the highest connected sites reach the unity that sites are isolated and the lengths of paths increase and the optimum paths exhibits a maximum value. At long times the size of the largest cluster undergoes a quick reduction and the length of the paths is

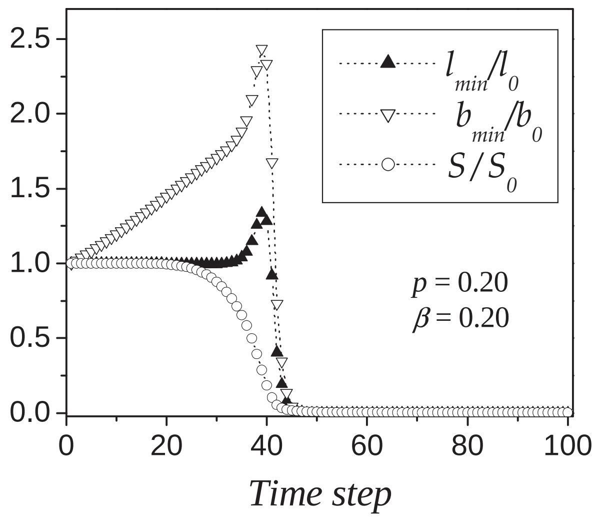

FIG. 2: Size of largest connected cluster, length of minimum paths and optimum paths as a funtion of time for the Cohen-like strategy.

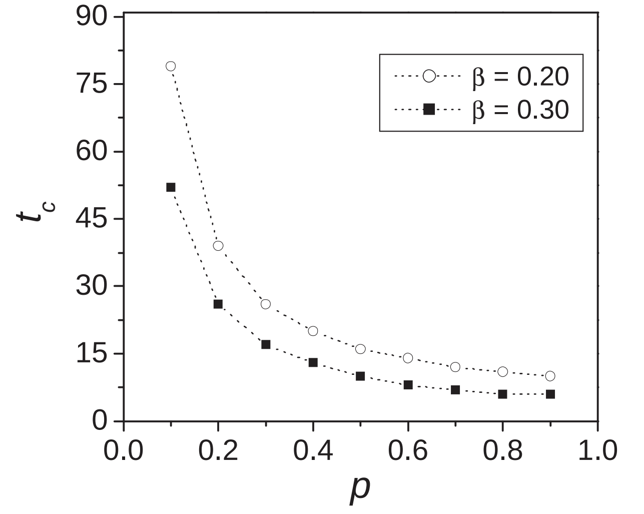

FIG. 3: Fragmentation time as a function of p for different values of β.

also reduced since the high connected sites remaining in the network begin to be isolated.

B. Cohen-like Insertion Strategy

In this section we investigate an insertion strategy based on the acquaintance immunization strategy introduced by Cohen *et al*. In each Monte Carlo step (or time) a fraction p of the nodes is randomly chosen. Once a site i is chosen the set of its k_i first neighbours (its acquaintances) is identified. The load is inserted in a site $j \in k_i$ (chosen at random with uniform probability) and distributed among the links of j, i.e., one of the sites in the acquaintances receives the additional load. Following this scheme each site could be chosen once in each Monte Carlo step (each one with probability p) but may be "indicated to receive the load" more than once since they could belong to the neighbouring of many sites.

Fig. 2 is a representative sample of the system behaviour for $p = 0.20$ and $\beta = 0.20$. The results are qualitatively different from that of observed to the random insertion case. Here, at short times the network remains totally connected and the largest connected cluster coincides with the network's size. A sudden change to an insignificant fraction of the network's size around a step t is observed indicating that the network is broken in a large number of small connected clusters. The abrupt change in the size of largest cluster takes place since the sites of higher connectivity are preferentially selected to receive the additional load. Due the heterogeneous properties of the network the high connected sites should be in the neighborhood of many low-connected ones and may receive load more than once at each Monte Carlo step since they can be "indicated to receive the load" by different low-connected sites. As a consequence the links of the most connected sites tend to become overloaded first. While the links have available bandwidth to support the load without becoming overloaded, the network is not diluted. Upon increasing the time the continuous load accumulation induces the overload of links and the consequent removal of high connected sites. It is surprising that the remotion of the high connected sites takes place almost simultaneously, or at least, in a very short time interval. We could argue that when t approaches a specific value t_c (t_c being a function of p and β) the most connected sites have their links overloaded and are removed. Such behaviour is similar to a percolation transition and we can say that the system undergoes a percolation-like transition. Our percolation-like argument is corroborated by the behaviour of the minimum and optimum paths. While the network remains connected the length of the minimum paths remains constant while the optimum paths increase reflecting the growth of total load on the system. Upon increasing the time the continuous load accumulation induces the overload of links and the consequent removal of high connected sites belonging to the minimum paths and longer alternative routes connecting the pairs of sites must be found. When t approaches t_c the network approaches its maximum dilution (remaining connected) and l_{min} has a peak. Besides, the total load along the optimum paths increases continuously and shows a fast change near t_c reflecting the sudden increase of the paths occurring when the network breaks up into small clusters.

It is interesting to note that the continuous insertion of load carries out to a transition similar to a simultaneous removal of a fraction f of sites following the standard strategy of Cohen. Aiming to analize this point we evalute the fraction f of totally isolated sites at the disruption time t_c for different sets of (p,β). Although t_c is a function of p and β (as showed in the Fig. 3), we got that f is a constant taking the value $f \cong 0.55$ for all sets (p,β). It means that independently of the site selection probability or the strength of inserted load the network will be broken when a fixed fraction of sites are totally removed. The total inserted load may correspond to a massive attack to the network continuously inserting a little quantity of load in a high quantity of sites as well as a localized attack targeting preferentially few high connected sites with a high load. Both attacks will affect the transport properties of the network the only difference being the time taken until the damage is maximum.

IV. SUMMARY

We investigate a simple dynamic model aiming to describe disruption phenomenon in transport network. In our model the transport properties like the size of largest connected cluster, the length of the minimum paths and the optimal paths between a pair of nodes were evaluated upon increasing the load on the system. We perform Monte Carlo simulations using two load insertion strategies: uniform random insertion of loads and (b) a Cohen-like immunization strategy. Both strategies may be classified as local strategies but the resulting effects are qualitatively different. Evaluating the physical quantities as a function of time we observe that for the random strategy there is a crossover from a fully connected cluster to a non-connected state in the sense that all links become unavailable. On the other hand, by following the Cohen-like strategy we found a sudden change in transport properties which is may be interpreted as a percolation-like transition induced by the cumulative process of load along the time.

Finally, we would like to stress that our results is similar to that of the recent work of Zhang *et. al.*[17]. They investigated a model in which each node generates R/N packets, where N is the size of the network and R is a parameter tuning the generation rate and such packets are exchanged between a pair of sites through the minimum paths. By evaluating the transport properties they found a phase transition between the free flow and the congestion state driven by the packets generation rate R. Moreover, the trasmission efficiency is enhanced deleting the links connected to sites of large betweeness since they are overloaded first. Such conclusion is quite similiar to ours, although in the Zhang's work the model is different.

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