T_c and $\triangle_o$ in a Phenomenological "Pseudogap" Model

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We study numerically superconductivity in a system characterized by the presence of a phenomenological "pseudogap", E_g, in the energy spectrum, for $0 \leq T \leq T^s$. T^* is a crossover temperature. As a simplification, the pseudogap and the superconducting gap have the same s–wave symmetry. We find that for $\forall E_g \neq 0$ we require a critical value of the superconducting interaction, V_d, to produce a finite superconducting critical temperature, T_c and another one for $\Delta_o \neq 0$.

1 Introduction

After their discovery by Bednorz and Muller[1] in 1986, the high–temperature superconductors ($HTSC$) are still attracting a lot of interest due to their unusual physical properties, both in the normal and in the superconducting phases. For example, the $HTSC$ exhibit a pseudogap in the energy spectrum in the temperature range $0 \leq T \leq T^*$. T^* is defined by Maier et al.[2] as the crossover temperature where the spin–susceptibility is a maximum.

There is experimental evidence by the group of Tallon and Loram[3] where the pseudogap persists below T_c, being independent of the superconducting gap. This interpretation is in agreement with the experiment of energy gap evolution in the tunneling spectra of $Bi2Sr2CaCu2_{8+\delta}$ performed by Dipasupil et al.[4]. They find that the pseudogap smoothly develops into the superconducting state gap with no tendency to close at T_c. Another proof that the pseudogap and the superconducting gap are independent of each other is given in the experiments of Krasnov et al.[5] where they apply a magnetic field to their superconducting samples and they destroy the superconducting gap, but the pseudogap remains. They conclude that the pseudogap and the superconducting gap coexist in Bi–2212 using intrinsic tunneling spectroscopy.

Rubio Temprano, Trounov and M$\ddot{u}$ller[6] have recently studied the isotope effects on the pseudogap in the high–temperature superconductor $La_{1.81}Ho_{0.04}Sr_{0.15}CuO_4$ by neutron crystal field spectroscopy. They have found evidence for the opening of an electronic pseudogap at $T* \approx 60K$, above the superconducting critical temperature, $T_c \approx 32K$.

We exploit the consequences of the psudogap on two macroscopic quantities in the superconducting state, namely, the superconducting critical temperature, T_c, and the superconducting order parameter at $T = 0$, Δ_o.

This paper is organized as follows. In Section 2, we present the pseudogap model, following the steps of Ţifrea, Grosu and Crisan[7]. In Section 3 we present our numerical results. In Section 4 we present our discussion and conclusions.

2 The "pseudogap" model

We assume[7] that the PG and the normal one–particle self–energy are given by

$$\Sigma(\vec{k}, i\omega_n) \equiv -E_g^2 G_o(\vec{k}, -i\omega_n) \quad , \tag{1}$$

where $G_o(\vec{k}, i\omega_n)$ is the free one–particle Green function. $\vec{k}$ is the wave vector and $\omega_n = 2\pi T(n + 1/2)$ is the odd Matsubara frequency, with n an integer. With this choice of self–energy (Eq. (1)) is easy to show that the "PG" Green function is given by

$$G(\vec{k}, i\omega_n) = \frac{u_{\vec{k}}^2}{i\omega_n - E_{\vec{k}}} + \frac{v_{\vec{k}}^2}{i\omega_n + E_{\vec{k}}} \quad , \tag{2}$$

where we have chosen the pseudogap of pure s–symmetry, since we want to look for details overlooked in Ref. [7]. For example, the authors of Ref. [7] did not find critical pairing interactions to have $T_c \neq 0$ and $\Delta_o \neq 0$. These considerations have been properly taken into account by Pistolesi and Nozières[8] in a similar model to ours. A word of caution is in order here. The model we are discussing here appears to be more of a semiconductor type, as recognized in Ref. [8]. To transform the present model into a real pseudogap model, we should include a damping factor, as it has been done by Andrenacci and Beck[9].

The superconducting state in the $HTSC$ is obtained from the two two BCS equations as follows

$$
\begin{aligned}
G^{-1}(\vec{k}, i\omega_n)\mathcal{G}(\vec{k}, i\omega_n) + \Delta\mathcal{F}^{\dagger}(\vec{k}, i\omega_n) &= 1 \\
\Delta^{*}\mathcal{G}(\vec{k}, i\omega_n) - G^{-1}(\vec{k}, -i\omega_n)\mathcal{F}^{\dagger}(\vec{k}, i\omega_n) &= 0
\end{aligned} \quad ,(3)
$$

where $\mathcal{G}(\vec{k}, i\omega_n)$ and $\mathcal{F}^{\dagger}(\vec{k}, i\omega_n)$ are the diagonal and off–diagonal BCS Green functions, respectively. This interpretation of the PG is equivalent to make the following choice in the T-matrix approximation[10, 11, 12] for the superconducting self–energy:

$$
\sum(\vec{k}, i\omega_n) = \begin{pmatrix} \Sigma(\vec{k}, i\omega_n) & \Delta \\ \Delta^{*} & \Sigma(\vec{k}, i\omega_n) \end{pmatrix} \quad (4)
$$

This assumption produces two gaps, one coming from the PG and the other one from Δ in Eq. (4). Our approach is completely different from the one in Refs. [13-16] where they have an effective gap, given by $\Delta_{eff} = \sqrt{\Delta^2 + E_g^2}$. Their approach is equivalent to taking $\Delta = 0$ in our Eq. (4) and substituting $\Sigma(\vec{k}, i\omega_n)$ by the diagonal self–energy (Eq. (1)), with $E_g \to \sqrt{E_g^2 + \Delta^2}$.

Solving Eq. (3), we obtain

$$
\mathcal{G}(\vec{k}, i\omega_n) = \sum_{i=1}^{4} \frac{\alpha_i(\vec{k})}{i\omega_n - \omega_i(\vec{k})} \quad ; \quad (5)
$$

$$
\mathcal{F}(\vec{k}, i\omega_n) = \Delta \sum_{i=1}^{4} \frac{\beta_i(\vec{k})}{i\omega_n - \omega_i(\vec{k})} \quad , \quad (6)
$$

where

$$
\begin{aligned}
\omega_{\pm}^2 &= E_{\vec{k}}^2 + \frac{|\Delta|^2}{2} \pm |\Delta| \sqrt{E_g^2 + \frac{|\Delta|^2}{4}} && (7) \\
\omega_1(\vec{k}) &= |\omega_+| \ ; \ \omega_2(\vec{k}) = -|\omega_+| \ ; \ \omega_3(\vec{k}) = |\omega_-| && (8) \\
\omega_4(\vec{k}) &= -|\omega_-| \ ; \ E_{\vec{k}}^2 \equiv \epsilon^2(\vec{k}) + E_g^2 \ , && (9)
\end{aligned}
$$

and $\epsilon(\vec{k}) = -2t(\cos(k_x) + \cos(k_y))$ is the free tight binding band in two dimensions. We are not considering here the presence of the chemical potential, which we leave for a future publication [17]. In Section 3 we choose $t = 1$, as our unit of energy. In Eqs. (5–6), the spectral weights, $\alpha_i(\vec{k})$ (normal ones) and $\beta_i(\vec{k})$ (superconducting ones), with $i = 1, 2, 3, 4$, are given by

$$
\begin{aligned}
\alpha_1(\vec{k}) &= \frac{\left(\frac{u_{\vec{k}}^2}{\omega_1(\vec{k})+E_{\vec{k}}} + \frac{v_{\vec{k}}^2}{\omega_1(\vec{k})-E_{\vec{k}}}\right)^{-1} \left((\omega_1)^2 - (\epsilon(\vec{k}))^2\right)}{4\Delta\ \omega_1(\vec{k})\sqrt{E_g^2 + \frac{|\Delta|^2}{4}}} \\
\alpha_2(\vec{k}) &= \frac{\left(\frac{u_{\vec{k}}^2}{\omega_1(\vec{k})-E_{\vec{k}}} + \frac{v_{\vec{k}}^2}{\omega_1(\vec{k})+E_{\vec{k}}}\right)^{-1} \left((\omega_1)^2 - (\epsilon(\vec{k}))^2\right)}{4\Delta\ \omega_1(\vec{k})\sqrt{E_g^2 + \frac{|\Delta|^2}{4}}} \\
\alpha_3(\vec{k}) &= \frac{-\left(\frac{u_{\vec{k}}^2}{\omega_3(\vec{k})+E_{\vec{k}}} + \frac{v_{\vec{k}}^2}{\omega_3(\vec{k})-E_{\vec{k}}}\right)^{-1} \left((\omega_3)^2 - (\epsilon(\vec{k}))^2\right)}{4\Delta\ \omega_3(\vec{k})\sqrt{E_g^2 + \frac{|\Delta|^2}{4}}} \\
\alpha_4(\vec{k}) &= \frac{-\left(\frac{u_{\vec{k}}^2}{\omega_3(\vec{k})-E_{\vec{k}}} + \frac{v_{\vec{k}}^2}{\omega_3(\vec{k})+E_{\vec{k}}}\right)^{-1} \left((\omega_3)^2 - (\epsilon(\vec{k}))^2\right)}{4\Delta\ \omega_3(\vec{k})\sqrt{E_g^2 + \frac{|\Delta|^2}{4}}} \\
\beta_1(\vec{k}) &= -\beta_2(\vec{k}) = \frac{(\omega_1)^2 - (\epsilon(\vec{k}))^2}{4\Delta\ \omega_1(\vec{k})\ \sqrt{E_g^2 + \frac{|\Delta|^2}{4}}} \\
\beta_3(\vec{k}) &= -\beta_4(\vec{k}) = \frac{(\omega_3)^2 - (\epsilon(\vec{k}))^2}{4\Delta\ \omega_3(\vec{k})\ \sqrt{E_g^2 + \frac{|\Delta|^2}{4}}} . \qquad (10)
\end{aligned}
$$

We have to solve the T_c equation and the gap equation, Δ_o, at $T \equiv 0$. They are given by

$$
\begin{aligned}
\frac{1}{V_d} &= \frac{1}{N_x \times N_y} \sum_{nx,ny} \frac{1}{2\sqrt{(\epsilon(\vec{k}))^2 + E_g^2}} \\
&\quad \times tanh\left(\frac{\sqrt{(\epsilon(\vec{k}))^2 + E_g^2}}{2\ k_B\ T_c}\right) \equiv F(T_c) \quad (11)
\end{aligned}
$$

$$\frac{1}{V_d} = \frac{1}{N_x \times N_y} \sum_{nx,ny} \frac{1}{2\Delta_o \sqrt{\Delta_o^2 + 4E_g^2}} \times \left(\frac{A_o^2}{\sqrt{(\epsilon(\vec{k}))^2 + A_o^2}} - \frac{B_o^2}{\sqrt{(\epsilon(\vec{k}))^2 + B_o^2}} \right) \quad (12)$$

$$A_o^2(B_o^2) \equiv E_g^2 + \frac{1}{2}\left(\Delta_o^2 \pm \Delta_o \sqrt{\Delta_o^2 + 4E_g^2}\right) = E_g^2 + \frac{1}{2}\left(\Delta_o^2 \pm \Delta_{eff}^2\right) \quad (13)$$

where $k_x = 2n_x\pi/N_x$ and $k_y = 2n_y\pi/N_y$, with $n_x = 0, 1, ..., N_x - 1$, and $n_y = 0, 1, ..., N_y - 1$, since we are solving our discrete system in two dimensions. We have chosen $N_x = N_y = 1000$ in our numerical calculations. V_d is the absolute value of the pairing interaction. We have used a precision of 10^{-5} to solve Eqs. (11-12). From these equations we conclude that $A_o^2 = \Delta_o^2$ and $B_o^2 = 0$ when $E_g = 0$.

3 Numerical results

In Fig. 1 we present T_c vs V_d for several values of the pseudogap parameter, E_g, for the case of pure s–wave symmetry. We observe that there is a critical value of the interaction potential, $V_{d,c}^{T_c}$, in order to have T_c. As we will see in the results for Δ_o vs V_d, there is also a critical value of the pairing potential below which $\Delta_o = 0$. In the case of $V_{d,c}$ coming from the $\Delta_o \to 0$, these two critical pairing interactions are different. These critical pairing interactions were not discussed in Ref. [7]. However, they were considered in a similar model by Pistolesi and Nozières[8].

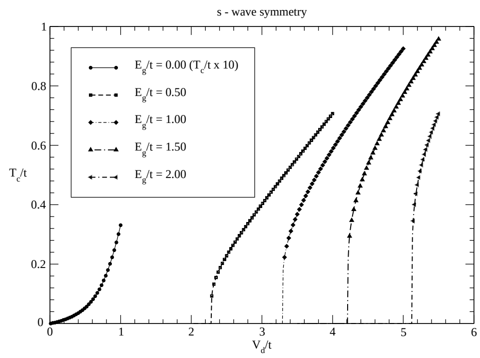

Figure 1. T_c/t vs V_d/t for several values of the pseudogap parameter, E_g/t, for the case of pure s–wave symmetry. For $E_g/t \neq 0$ there is a critical interaction potential, $V_d/t = V_{d,c}^{T_c}/t$, below which $T_c/ = 0$. For example, for $E_g/t = 0.50$ we find $V_{d,c}^{T_c}/t \approx 2.25$ in units of t.

In Fig. 2 we present Δ_o vs V_d for several values of E_g, when the pseudogap and the superconducting order parameter, at $T = 0$, have the same symmetry, namely, pure s–wave. We need a critical interaction potential, $V_{d,c}^{\Delta_o} \neq 0$, when $E_g \neq 0$, in order to have $\Delta_o \neq 0$. From Fig. 2, for $E_g = 0.50$, $V_{d,c}^{\Delta_o} \approx 3.00$. Comparing Figs. 1 and 2, we see that for a fixed value of E_g, $V_{d,c}^{T_c} \leq V_{d,c}^{\Delta_o}$. This result implies that the ratio $2\Delta_o(V, E_g)/k_B T_c(V, E_g)$ is well defined only for $V \geq V_{d,c}^{\Delta_o}$, for a fixed value of E_g.

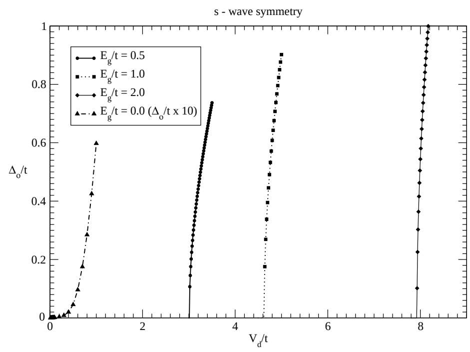

Figure 2. Δ_o/t vs V_d/t for several values of the pseudogap parameter, E_g/t. The superconducting order parameter, when $E_g = 0/t$ has been re–escaled by a factor of 10. For $E_g/t \neq 0$ there is a critical interaction potential, $V_d/t = V_{d,c}^{\Delta_o}/t$, below which $\Delta_o/t = 0$. For example, for $E_g/t = 0.50$ we find $V_{d,c}^{\Delta_o}/t \approx 3.00$ in units of t.

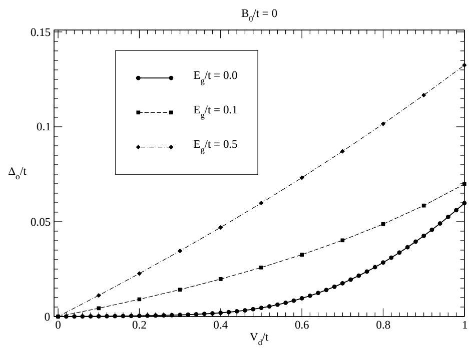

Figure 3. Δ_o vs V_d for several values of the pseudogap parameter, E_g. Following the approximation of Ref. [7], we have taken $B_o \equiv 0$

From Fig. 3 we plot Δ_o/t vs V_d/t for several values of the pseudogap parameter, E_g/t, when we adopt the aproximation of Ref. [7], namely, $B_o/t \equiv 0$. This approximation does not produce a critical value of the interaction potential. In consequence, $V_d^{\Delta_o}/t = 0, \forall\ E_g/t$. As our Eq. (11) does not have the presence of the factor B_o/t, we cannot perform this approximation. Because of this, T_c/t always needs a critical value of the interaction, for $\forall\ E_g/t \neq 0$.

4 Discussion and conclusions

In this paper we have considered that the pseudogap and the superconducting order parameter both have the same symmetry, namely, pure s–wave symmetry. This is not a crazy idea, because it has been shown that the observed symmetry of the order parameter cannot be fitted with only the lowest harmonics of the d–wave order parameter[18, 19, 20, 21]. Furthermore, a recent experiment with twisted Josephson junctions in the Bi–cuprates[22], is in favor of an extended s–wave order parameter, and has shown the absence of a d–wave part in the order parameter. However, the inclusion of order parameters with another symmetry is not a difficult task in our working scheme. With this point of view in mind and following the model of Ţifrea, Grosu and Crisan[7] we have studied their model to treat delicate points like the critical interaction potential.

In summary, we have numerically implemented a model which has a pseudogap (really, it is a semiconductor gap, since damping effects have been neglected in our calculations) in the one–particle energy spectrum of quasi–particles in the temperature range $0 \leq T \leq T^*$. We have investigated the effect of E_g on the two basic parameters of the BCS theory, T_c and Δ_o. We have found that for $E_g \neq 0$ two critical pairing potentials emerge from our calculations. In consequence, in order to define the ratio $R \equiv 2\Delta_o(V_d, E_g)/k_B T_c(V_d, E_g)$ we need to be above the bigger of the two critical pairing potentials. When $E_g = 0$, $R \approx 3.5$ in the BCS–approximation. We have briefly discussed the case when both order parameters, E_g and Δ_o, have the same symmetry, namely, pure s–wave symmetry. However, the d–wave symmetry is not difficult to study and we leave for a future publication. Another aspect we could study is the crossover phase diagram from the BCS limit to the Bose–Einstei regime[23].

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