Langevin Dynamics of the Deconfinement Transition for Pure Gauge Theory

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We investigate the effects of dissipation in the deconfinement transition for pure $SU(2)$ and $SU(3)$ gauge theories. Using an effective theory for the order parameter, we study its Langevin evolution numerically. Noise effects are included for the case of $SU(2)$. We find that both dissipation and noise have dramatic effects on the spinodal decomposition of the order parameter and delay considerably its thermalization. For $SU(3)$ the effects of dissipation are even larger than for $SU(2)$.

Keywords: Quark-gluon plasma; Deconfinement; Dynamics of phase transitions; Spinodal decomposition

I. INTRODUCTION

Recent results from lattice QCD [1], corroborated by experimental data from BNL-RHIC [2], indicate that strongly interacting matter under extreme conditions of temperature and pressure undergoes a phase transition to a deconfined plasma. Such extreme conditions are believed to have happened in the early universe, and might also be found in the core of neutron stars [3].

The process of phase conversion during the deconfinement transition can occur in different ways. For a pure gauge $SU(N)$ theory, the trace of the Polyakov loop provides a well-defined order parameter [4–6], and one can construct an effective Landau-Ginzburg field theory based on this quantity [7, 8]. The effective potential for $T << T_c$ has only one minimum, at zero, where the whole system is localized. With the increase of the temperature new minima appear (N minima for $Z(N)$, the center of $SU(N)$). At the critical temperature, T_c, all the minima are degenerate, and above T_c the new minima become the true vacuum states of the theory, so that the system starts to decay. In the case of $SU(3)$, whithin a range of temperatures close to T_c there is a small barrier, and the process of phase conversion will be guided by bubble nucleation. For larger T, the barrier disappears and the system explodes in the process of spinodal decomposition. For $SU(2)$, the transition is second-order, and there is never a barrier to overcome.

In this paper, we consider pure $SU(2)$ and $SU(3)$ gauge theories, without dynamical quarks, that are rapidly driven to very high temperatures, well above T_c, and decay to the deconfined phase via spinodal decomposition. We are particularly interested in the effect of noise and dissipation on the time scales involved in this “decay process”. In what follows, we adopt an effective model proposed in Ref. [8] for the order parameter and the effective potential. Numerical calculations for the evolution of the order parameter are performed on a lattice, using a local Langevin equation.

The paper is organized as follows. Section II briefly describes the effective model for the order parameter. In Section III, we consider the Langevin evolution, discussing how to fix the dissipation coefficient from lattice simulations, and present our results for $SU(2)$ and $SU(3)$. Section IV contains our final remarks.

II. THE EFFECTIVE MODEL

The model proposed in [8] intends to provide a better representation of lattice results for the gluon plasma equation of state as compared to the usual bag model. It is obtained combining a few phenomenological inputs with $Z(N)$ symmetry and some known features of the perturbative equation of state. In particular, in a temperature range going from the deconfinement temperature T_d to $5T_d$ the model gives reasonable results and exhibits a thermodynamic behaviour that is coherent with data obtained from lattice simulations.

In this approach, thermodynamic properties are determined by functions of the Polyakov loop, defined in Euclidean finite temperature gauge theories as [4]:

$$P(\vec{x}) = \mathcal{T} exp\left[ig\int_0^{1/T} d\tau\, A_0(\vec{x},\tau)\right] , \qquad (1)$$

where $\mathcal{T}$ denotes Euclidean time ordering, g is the gauge coupling constant and A_0 is the time component of the vector potential. We work with $SU(2)$ and $SU(3)$, representing the color degrees of freedom. Consequently, we have a $Z(N)$ symmetry for the case of pure gauge theories that is spontaneously broken. It would be explicitly broken in the presence of quarks.

Working in the imaginary time framework, we have bosonic fields being periodic and fermionic fields being antiperiodic in the imaginary time τ:

$$A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0) \ \ , \ q(\vec{x},\beta) = -q(\vec{x},0) \, . \qquad (2)$$

Any gauge transformation periodic in τ respects these boundary conditions. However, as demonstrated by ’t Hooft [5], one can consider more general gauge transformations which are only periodic up to the center of the group: $\Omega(\vec{x},\beta) = \Omega_c \ \ , \ \ \Omega(\vec{x},0) = 1$.

Color adjoint fields are invariant under these transformations, while those in the fundamental representation are not:

$$A^{\Omega}(\vec{x},\beta) = \Omega_c^{\dagger} A_\mu(\vec{x},\beta)\Omega_c = A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0) \, , \qquad (3)$$

$$q^{\Omega}(\vec{x},\beta) = \Omega_c^{\dagger} q(\vec{x},\beta) \neq -q(\vec{x},0) \,. \tag{4}$$

Consequently, pure gauge theories have a global $Z(N)$ symmetry, which is spoiled by the addition of dynamical quarks.

Thus the action is invariant under $Z(N)$ transformations, but $\langle \mathrm{Tr}_F P(\vec{x})\rangle$ is not. Symmetry requires $z\langle \mathrm{Tr}_F P(\vec{x})\rangle = \langle \mathrm{Tr}_F P(\vec{x})\rangle$, which implies $\langle \mathrm{Tr}_F P(\vec{x})\rangle = 0$. When the phase transition occurs, the $Z(N)$ symmetry is spontaneously broken and $\langle \mathrm{Tr}_F P(\vec{x})\rangle$ assumes a non-vanishing value. So, one can use it as an order parameter for the transition and it is possible to write an effective theory for its dynamics.

The efective theory of Ref. [8] is based on a mean field treatment in which the Polyakov loops are constant throughout the space and the free energy is a function of its eigenvalues. A perturbative calculation of the free energy of gluons as a function of the Polyakov loop eigenvalues yields

$$f = -\frac{1}{\beta}\sum_{j,k=1}^{N} 2\left(1-\frac{1}{N}\delta_{jk}\right)\int \frac{d^3k}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n}e^{-\beta m w_k + in\Delta\theta_{jk}} \,, \tag{5}$$

where θ is defined through the eigenvalues of the Polyakov loop, $P_{jk} = \exp(i\theta_j)\,\delta_{jk}$, and $\Delta\theta_{jk} \equiv \theta_j - \theta_k$, which reduces to the usual blackbody formula in the case $A_0 = 0$.

This expression for the free energy of gluons propagating in the background of Polyakov loops predicts a gas of gluons that is always in the deconfined phase, with no indication that higher-order corrections will modify this result. This can be modified by the introduction of a mass scale into f. This mass scale will determine the deconfinement temperature T_d and is introduced in a phenomenological way. One ends up with the same expression of the perturbative calculation, but now $w_k = \sqrt{k^2+M^2}$. Parametrizing the Polyakov loop and representing the diagonal matrix as $diag[\exp(i\phi_{N/2},\ldots,i\phi_1,-i\phi_1,\ldots,-i\phi_{N/2}]$, it is possible to extract an effective potential as a function of ϕ. For $SU(2)$ one has only $\phi_1 = \phi$ and $\phi_{-1} = -\phi$, and the effective potential can be written as

$$\begin{aligned} V &= -\frac{\pi^2T^3}{15} + \frac{4T^3}{3\pi^2}\phi^2(\phi-\pi)^2 \\ &+ \frac{M^2T}{4} + \frac{M^2T}{\pi^2}\phi(\phi-\pi) \,. \end{aligned} \tag{6}$$

Notice that there is a symmetry $\phi \leftrightarrow \pi-\phi$ associated with the $Z(2)$ invariance. It is convenient to write this equation in terms of a new variable $\psi = 1-\phi\pi/2$ to make this symmetry more evident. One obtains:

$$\begin{aligned} V &= -\frac{\pi^2T^3}{15} + \frac{T^3\pi^2}{12}(1-\psi^2)^2 \\ &+ \frac{M^2T}{4} - \frac{M^2T}{4}(1-\psi^2) \,, \end{aligned} \tag{7}$$

where $\psi = 0$ represents confinement. It is interesting to connect the ψ used here and the trace of the Polyakov loop, used in Ref. [7] as the order parameter. For the diagonalyzed matrix we have the trace, according to our parametrization, as $e^{i\phi}+e^{-i\phi}$, namely $\mathrm{Tr}L = 2\cos(\phi)$, or, as defined above, $\mathrm{Tr}L = 2\cos(\pi(1-\psi)/2)$. So, when $\psi = 0$ we have $\mathrm{Tr}L = 0$, which represents confinement, and when $\psi \to 1$, then $\mathrm{Tr}L \to 1$, representing the deconfined state.

The phase transition in this case is second order, as expected [9]. The value of M can be determined from the deconfinement temperature through the relation $T_d = (3/2)^{1/2}M/\pi \approx 0.38985M$, so that it is possible to extract the deconfining temperature from the lattice and then fix M. The minimum of the potential occurs for

$$\psi_0 = \sqrt{1-\frac{3M^2}{2T^2\pi^2}} \,. \tag{8}$$

For $SU(3)$ there are three eigenvalues: $\phi_1 = \phi$, 0 and $\phi_{-1} = -\phi$. The potential assumes the form:

$$\begin{aligned} V &= -T^3\frac{8\pi^2}{45} + \frac{T^3}{6\pi^2}[8\phi^2(\phi-\pi)^2 + \phi^2(\phi-2\pi)^2] \\ &+ \frac{2TM^2}{3} + \frac{TM^2}{2\pi^2}[2\phi(\phi-\pi)+\phi(\phi-2\pi)] \,. \end{aligned} \tag{9}$$

Again, it is useful to rewrite the potential in terms of a new variable $\psi = 2\pi/3-\phi$, so that one obtains

$$\begin{aligned} V &= \frac{8\pi^2}{405}T^3 + \left(\frac{3}{2\pi^2}TM^2 - \frac{2}{3}T^3\right)\psi^2 \\ &- \frac{2}{3\pi}T^3\psi^3 + \frac{3}{2\pi^2}T^3\psi^4 \,. \end{aligned} \tag{10}$$

Now, M and T_d are related as follows:

$$T_d = \frac{9}{20\pi}\sqrt{10}M \approx 0.45296M \,, \tag{11}$$

and the minimum is at

$$\psi_0 = \frac{\pi T + 3\sqrt{T^2\pi^2 - 2M^2}}{6T} \,. \tag{12}$$

In this case, $\mathrm{Tr}L = e^{i\phi}+1+e^{-i\phi}$ and the connection with ψ becomes $\mathrm{Tr}L = \frac{2}{3}\cos\left(\frac{2\pi}{3}-\psi\right)+\frac{1}{3}$.

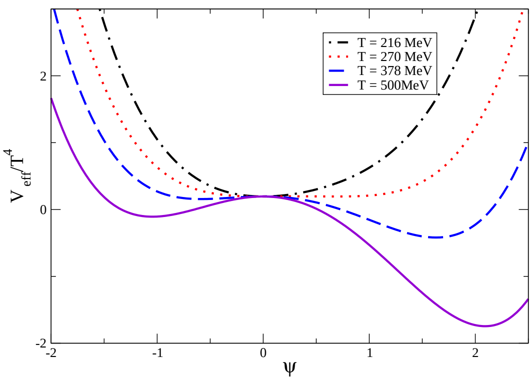

FIG. 1: Effective potential for $SU(3)$ for different values of the temperature.

Immediately above the critical temperature, the $SU(3)$ potential presents a barrier between the old and the new vacua. This barrier, however, is very small and quickly disappears with the increasing of the temperature. One should notice that above $2T_c$ the changes in the potential are negligible.

III. LANGEVIN EVOLUTION

Let us now consider the real-time evolution of the order parameter for the breakdown of $Z(N)$. We assume the system to be characterized by a coarse-grained free energy

$$F(\phi,T) = \int d^3x \left[\frac{B}{2}(\nabla\phi)^2 + V_{eff}(\phi,T)\right], \qquad (13)$$

where $V_{eff}(\phi,T)$ is the effective potential obtained in the last section, and $B = \pi^2 T/g^2$ for $SU(2)$ and $B = 4T/g^2$ for $SU(3)$. The time evolution of the order parameter and its approach to equilibrium will be dictated by the following Langevin equation

$$B\left(\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi\right) + \Gamma\frac{\partial\psi}{\partial t} + V'_{eff}(\psi) = \xi\,, \qquad (14)$$

where g is the QCD coupling constant, and Γ is the dissipation coefficient, which is usually taken to be a function of temperature only, $\Gamma = \Gamma(T)$. The function ξ is a stochastic noise assumed to be gaussian and white so that $\langle\xi(\vec{x},t)\rangle = 0$ and $\langle\xi(\vec{x},t)\xi(\vec{x}',t')\rangle = 2\Gamma\delta(\vec{x}-\vec{x}')\delta(t-t')$. The noise and dissipation terms are originated from thermal and quantum fluctuations resulting either from self-interactions of the Polyakov loop field or from the coupling to diferent fields (such as chiral fields). The case with only first-order time derivative was considered in Ref. [10].

This description is admittedly very simplified. A more complete analysis should consider different contributions of noise and dissipation terms and memory kernels instead of simple Markovian terms proportional to the first time derivate of the field [11, 12]. In general, one obtains a complicated dissipation kernel that simplifies to a multiplicative dissipation term which depends quadratically on the amplitude of the field as $\Gamma_1(T)\,\psi^2(\vec{x},t)\,\dot{L}(\vec{x},t)$ where Γ_1 is determined by imaginary terms of the effective action for ψ and depends weakly (logaritmically) on the couplings. The fluctuation-dissipation theorem implies, then, that the noise term will also contain a multiplicative contribution of the form $\psi(\vec{x},t)\xi(\vec{x},t)$, and be in general non-Markovian. The white noise limit is reobtained only for very high temperatures.

For the $SU(2)$ case we have fixed Γ in the following way. We have used pure-gauge Euclidean lattice Monte Carlo simulations in the line discussed in Ref. [8]. In this approach, spinodal decomposition is obtained on the lattice performing local heat-bath updates of gauge field configurations at $\beta = 4/g^2 = 3$, after thermalizing the lattice at $\beta = 4/g^2 = 2$. The critical value of β for deconfinement is found to be $\beta_d \sim 2.3$. Γ is then extracted by comparing the short-time exponential growth of the correlation function $\langle L(k,t)L(-k,t)\rangle$ predicted by the lattice simulations [13] and the Langevin description, assuming of course that both dynamics are the same. Making this comparison for the lowest lattice momentum mode, it is found that $\Gamma = 7.6\times10^3\,T^3/\mu$, where μ is a time scale relating Monte Carlo time and real time. Assuming that typical thermalization times are of the order of a few fm/c, we obtain $\Gamma \sim 10^3$ fm^{-2}.

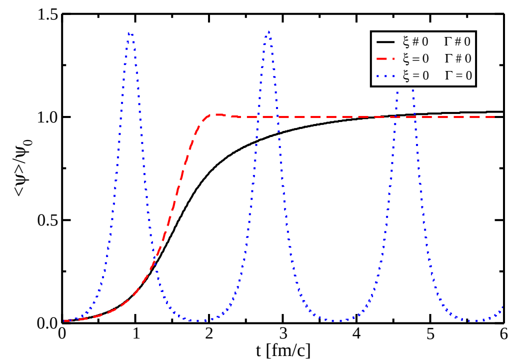

FIG. 2: Volume average of the $SU(2)$ order parameter normalized to the $\psi_0 > 0$ minimum of the bare effective potential.

In our numerical calculations we solve Eq. (14) on a cubic spacelike lattice with 64^3 sites under periodic boundary conditions. We use a semi-implicit finite-diference scheme for the time evolution and a finite-difference Fast Fourier Transform for the spatial dependence [14]. For $SU(2)$, the critical temperature is $T_d = 302$ MeV [15] and we obtain $M = 775$ MeV. We took the average of several realizations with random initial configurations around $\psi \sim 0$. We consider the time dependence of the volume average of ψ

$$\langle\psi\rangle = \frac{1}{N^3}\sum_{ijk}\psi_{ijk}(t)\,, \qquad (15)$$

where N is the number of lattice sites in each spatial direction, and $i,j,k = 1,\cdots,N$ are the lattice sites. In Fig. 2 we plot $\langle\psi\rangle/\psi_0$, where $\psi_0 > 0$ is the positive minimum of the bare effective potential, for three situations: no dissipation and no noise (dotted curve), no noise (dashed curve) and full solution (solid curve). When considering noise, we have added the appropriate counterterms to make the equilibrium solution independent of the lattice spacing [16]. All curves are for $T = 6.6\,T_d$.

Clearly seen in Fig. 2 is the large effect of dissipation, which delays the rapid exponential growth of the order parameter due to spinodal decomposition. The retardation seen here for the deconfinement transition is substantially larger than the corresponding delay seen for the chiral condensate evolution in Ref. [17]. The effect of noise is also in the direction of delaying equilibration, as expected. Also expected, and clearly shown in Fig. 2, is the effect of noise in the equilibrium value of ψ which is larger than ψ_0.

For $SU(3)$ one has $T_d = 263$ MeV [18], so that $M = 580$ MeV. The results of our simulations at $T = 6.6\,T_d$ are shown in Fig. 3. Here we are using the same lattice and same dissipation Γ as for $SU(2)$. As seen in this figure, the effect of dissipation is even more dramatic than for $SU(2)$, with the proviso of course that we are using the value of Γ extracted from $SU(2)$ lattice simulations. As mentioned earlier, immediately above the critical temperature the $SU(3)$ potential presents a barrier between a local minimum and an absolute minimum. However, this barrier has no effect on the delay

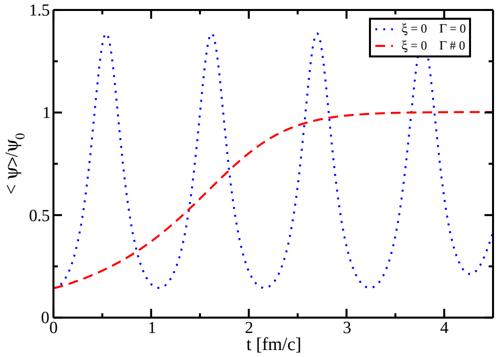

FIG. 3: Volume average of the $SU(3)$ order parameter normalized to the $\psi_0 > 0$ minimum of the bare effective potential.

seen in Fig. 3, since our simulations are done for high temperatures, $T \gg 2T_d$. We have not investigated the effect of noise in this case because the appropriate renormalization counterterms for an effective potential with a first-order transition are not yet available [16].

IV. SUMMARY AND OUTLOOK

We have investigated the effects of dissipation and noise in the deconfinement transition of $SU(2)$ and $SU(3)$ pure gauge theories. We have used the effective model proposed in Ref. [8], which combines phenomenological inputs with $Z(N)$ symmetry and some known features of the perturbative equation of state. The model provides a reasonable representation of lattice results for the pure-gluon plasma equation of state in the temperature range between T_c and $5T_c$. We have performed numerical simulations for the evolution of the order parameter on a spatial cubic lattice using a local Langevin equation. We find that both dissipation and noise have dramatic effects on the spinodal decomposition of the $SU(2)$ order parameter, delaying considerably its thermalization. Dissipation effects are even larger for $SU(3)$.

The present work must be improved in several aspects. Perhaps the most important one is in the method used to extract the dissipation coefficient Γ [13]. This was done using Euclidean lattice Monte Carlo simulations, in which spinodal decomposition of the order parameter is obtained performing local heat-bath updates of gauge field configurations above the deconfinement temperature. One of the major uncertainties in this approach is the relation between Monte Carlo updates and real time. Another source of uncertainties comes from a richer structure of noise and dissipation terms, including an evaluation of memory kernels. It is widely known that, in general, quantum corrections lead to complicated dissipation kernels that only in very special situations simplify to an additive noise term as used here. These issues will be considered in a future publication [19].

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