Superconducting Properties of Mesoscopic Squares

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We apply the complete nonlinear Time Dependent Ginzburg Landau (TDGL) equations to study the vortex dynamics in a mesoscopic type II superconductor using the numerical method based on the technique of gauge invariant variables. The solution of these equations shows how the vorticity penetrates into and goes out of the superconductor through the surface boundary. We calculate the spatial distribution of the superconducting electron density and the phase of the superconducting order parameter in a mesoscopic superconducting square sample containing two holes in the presence of a uniform perpendicular magnetic field. The dynamics of different vortex states are studied as a function of the external magnetic field.

Keywords: Vortex, Mesoscopics; Gizburg - Landau

I. INTRODUCTION

For a bulk type II superconductor, the magnetic field can penetrate in the form of vortices, each carrying one single flux quantum arranged in a triangular lattice. This so called mixed state takes place between the first and the second critical magnetic fields. For mesoscopic samples, i.e., for samples whose size is of the order of the penetration or the coherence lengths, the superconducting properties, such as the critical magnetic fields and the critical current, as well as the vortex configurations, can present new and very interesting properties. For example, it is observed an increase in the critical fields, the formation of chain like structure in superconducting strips and giant vortices carrying more than one quantum flux in mesoscopic disks. The possibility of improving the superconducting parameters by nanostructuring existing superconducting materials had called attention of both theoretical and experimental researchers. For example, L. F. Chibotaru *et al* [1] studied the formation of vortices and antivortices in mesoscopic superconductor square. C. Bolech and G. Buscaglia [2], reported numerical simulations of vortex arrays in thin superconducting films, L.R.E. Cabral *et al* [3], investigated the dynamics of stable vortex configurations in thin superconducting disk using both the nonlinear Ginzburg Landau theory and the London approximation. C.C. de Souza Silva and co-authors [4], studied the vortex dynamics in homogeneous superconducting films of arbitrary thickness under parallel magnetic field. G.R Berdiyorov *et al.* [5] used nonlinear Ginzburg-Landau theory (GL) to investigate theoretically the influence of various holes into the superconducting sample on the penetration and on the arrangement of vortices. Also the numerical method has been used to study, for example, the magnetic flux penetration and exit in a superconducting mesoscopic sample through the surface boundary [6,7].

II. THEORETICAL FORMALISM

In the present work, we considered a mesoscopic superconducting cylinder with rectangular cross section which is immersed in an insulating medium in the presence of an applied perpendicular uniform magnetic field ***Ha***. We used the time dependent Ginzburg Landau equations (TDGL) coupled with Maxwell equations, leading to the following mathematical problem for the order parameter ψ and the vector potential $\mathbf{A}$.

$$\frac{\partial\psi}{\partial t} = -\frac{1}{\eta}\left[(-i\nabla-\mathbf{A})^2\psi + (1-T)\left(|\psi|^2-1\right)\psi\right] + \tilde{f} \quad (1)$$

$$\frac{\partial\mathbf{A}}{\partial t} = (1-T)\,Re\left[\psi^*(-i\nabla-\mathbf{A})\psi\right] - \kappa^2\nabla\times\nabla\times\mathbf{A} \quad (2)$$

Where lengths have been scaled in units of $\xi(0)$, time in units of $t_0 = \pi\hbar/(96K_BT_C)$, $\mathbf{A}$ units of $H_{C2}(0)\xi(0)$, temperatures in units of T_C, η is a positive constant, $\tilde{f}$ a random force simulating thermal fluctuations.

In the following we considered a thin superconductor cylinder of square cross section, we will refer to this geometry as a square superconductor. Thus, we are allowed to take the order parameter and the local magnetic field invariant along to the z direction. The superconducting wave function satisfies the boundary conditions $\hat{n}.(-i\nabla-\mathbf{A})\psi = 0$ where $\hat{n}$ denotes the normal to the superconductor - vacuum interface, and the boundary conditions for $\mathbf{A}$, namely that $B_Z = \hat{e}_z.\nabla\times\mathbf{A}$ at the external surface must equal to ***Ha***, the applied field.

The Link Variable Method, two auxiliary fields U^x and U^y are related to $\mathbf{A}$ by:

$$U^x(x,y,t) = \exp\left(-i\int_{x_0}^{x} A_x(\xi,y,t)\,d\xi\right) \quad (3)$$

$$U^y(x,y,t) = \exp\left(-i\int_{y_0}^{y} A_y(x,\eta,t)\,d\eta\right) \quad (4)$$

The outline of this simulation procedure is as follows: The sample is divided in a rectangular mesh consisting of $N_x\times N_y$ cells, with mesh spacing $a_x\times a_y$ (in our simulation we use the simple Euler method with time step $\Delta t = 0.0005$, space step $a_x = a_y = 0.2$, the grid size 160×160, $\kappa = 2$, for 8×10^6 steps), each hole has dimensions $10\xi(0)\times10\xi(0)$, ***Ha*** is

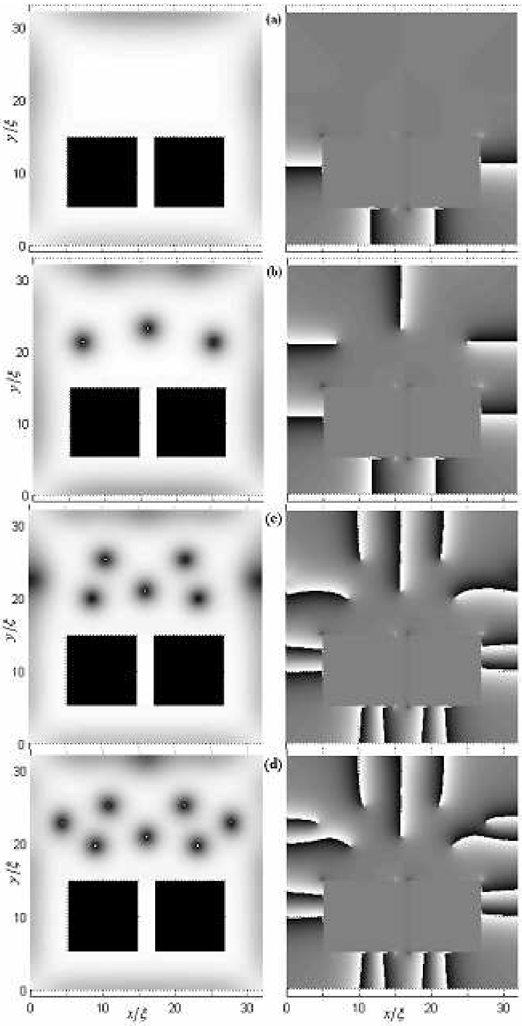

FIG. 1: The magnitude of the order parameter (left) and its phase (right) for the mesoscopic square with two square holes with (a) $L=4$, (b) $L=7$, (c) $L=13$, and (d) $L=15$ respectively, at $\boldsymbol{Ha}=0.200$, 0.225, 0.245 and 0.250, dark and bright regions represents values of the modulus of the order parameter (as well as the $\Delta\phi/2\pi$, from 0 to 1).

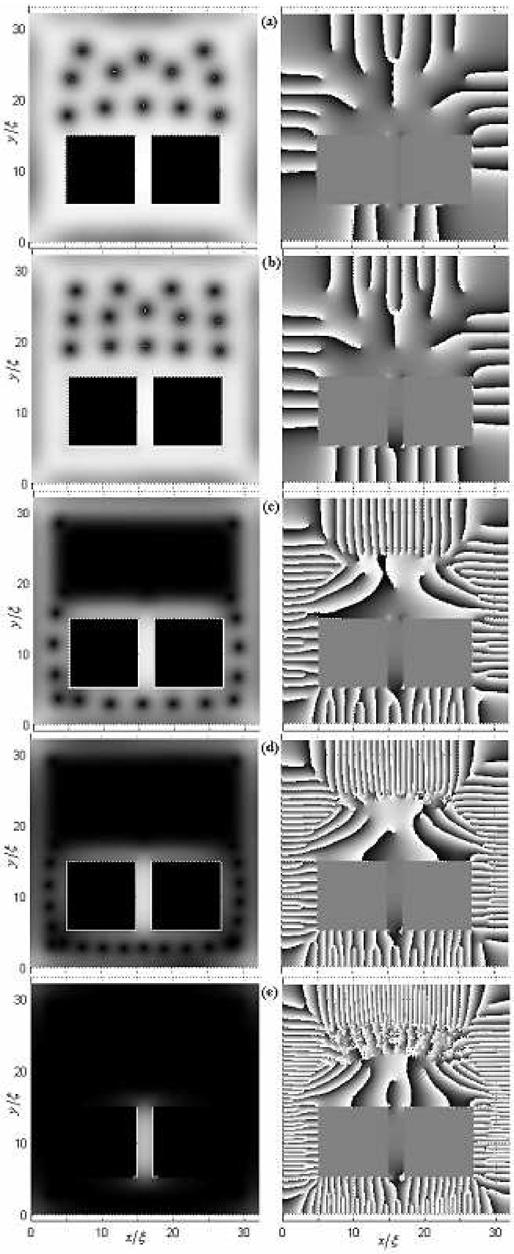

FIG. 2: The same as Fig. 1, but for (a) $\boldsymbol{Ha}=0.300$, (b) 0.330, (c) 0.585, (d) 0.750, (e) 0.965 respectively.

increased lineary with time from 0 to 1, with small intervals of $\Delta H = 2.5\times10^{-7}$. variables are homegeneously initialized to a perfect Meissner state $\psi(t=0)) = 1$, $\mathbf{A}(t=0)=0$ (for every point in the domain).

Let us first examine the time development of the superconducting state by integrating the TDGL equations starting with the perfect Meissner state.

The magnitude of the order parameter $|\psi|$ and its phase are plotted in Fig. 1 for the superconductor with two holes. When the holes are placed in the sample, the square symmetry is broken and the holes act like a pinning center. In Figs. 1(a - d), values of the phase close to zero are given by dark gray regions and close to 2π by light gray regions. The phase allows to determine the number of vortices in a given region, by counting the phase variation in a closed path around this region. If the vorticity in this region is L, then the phase changes by $\Delta\phi = 2\pi L$ [5].

In the Fig. 1(a) the first four vortices will sit in the holes symmetrically, two in each hole. Although they are not visible in the contour plot of the magnitude of order parameter, there is a change in the phase around each hole equal to $\Delta\phi = 4\pi$. The vortex entry occurs through the closest points in the outer sample surface to the holes. By increased the magnetic field three more vortices appear in the sample, as shown in Fig 1(b), they are localized in the superconductor region opposed to the holes position, for $L = 13$ Fig. 1(c), two vortex sit in each hole and five vortex are in the superconducting region, while four more vortices are entering in the sample (two in each hole, situated close to the holes). In Fig 1(d), with $L = 15$, four vortices are in each hole and seven in the superconducting region.

In Fig. 2, we can see the vortices in the superconducting region forming a vortex configuration symmetric to the geometry made by both holes. By increasing the magnetic field one vortex enter in each hole for $\boldsymbol{Ha} = 0.300$, one more vortex enter in each hole, and we have ten vortices in the holes together with twelve in the superconducting region [see Fig 2 (a)]. At $\boldsymbol{Ha} = 0.330$ [Fig. 2(b)], 29 vortices are in the sample, 15 of them inside the holes. For the states with higher vorticity Fig. 2 (c - e), vortices are too close to each other in the region opposed to the holes position (vortices can still be clearly distinguished from one another in Fig. 2 (c) in the space between the holes and the outer sample surface). A giant rectangle of depreciated superconductivity is observed, with an extended region of large vorticity inside (see in the right of Figs. 2(c) and 2(d)), because the vortices are overlapping. For magnetic fields near to $H_{C2}(0)$ the sample reach the normal state except in the region between the holes.

III. CONCLUSIONS

We investigated theoretically the spatial distribution of the vortices in a square mesoscopic superconductor with two square holes. The presence of the holes affects the vortex distribution, as well as the vortex entry. Vortices not in the holes are mainly located in the superconducting region opposed to the holes. At high magnetic field, there is a rectangular region of high vorticity symmetrical to the holes. Close to $H_{C2}(T)$ superconductivity survives only in the region of the superconductor between the holes.

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