QCD Sum Rule Approach to the New Mesons and the $g_{D_{sJ}DK}$ Coupling Constant

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We use diquark-antidiquark currents to investigate the masses and partial decay widths of the recently observed mesons $D_{sJ}^+(2317)$, $D_0^{*0}(2308)$ and $X(3872)$, considered as four-quark states, in a QCD sum rule approach. In particular we investigate the coupling constant $g_{D_{sJ}DK}$. We found that the $g_{D_{sJ}DK}$ obtained in this four-quark scenario is smaller than the coupling constant obtained when $D_{sJ}^+(2317)$ is considered as a conventional $c\bar{s}$ state.

Keywords: QCD Sum Rule

I. INTRODUCTION

The constituent quark model provides a rather successful descrition of the spectrum of the mesons in terms of quark-antiquark bound states, which fit into the suitable multiplets reasonably well. Therefore, it is understandable that the recent observations of the very narrow resonances $D_{sJ}^+(2317)$ by BaBar [1], $D_{sJ}^+(2460)$ by CLEO [2], $X(3872)$ by BELLE [3], and the very broad scalar meson $D_0^{*0}(2308)$ by BELLE [4], all of them with masses below quark model predictions, have stimulated a renewed interest in the spectroscopy of open charm and charmonium states. The difficulties to identify the mesons $D_{sJ}^+(2317)$ and $D_{sJ}^+(2460)$ as $c\bar{s}$ states are rather similar to those appearing in the light scalar mesons below 1 GeV (the isoscalars $\sigma(500)$, $f_0(980)$, the isodublet $\kappa(800)$ and the isovector $a_0(980)$), whose masses are also smaller than naive quark model predictions, and that can be interpreted as four-quark states [5, 6]. In the case of $X(3872)$, besides its small mass, the observation, reported by the BELLE collaboration [7], that the X decays to $J/\psi\pi^+\pi^-\pi^0$, with a strength that is compatible to that of the $J/\psi\pi^+\pi^-$ mode:

$$\frac{Br(X \to J/\psi\pi^+\pi^-\pi^0)}{Br(X \to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4 \pm 0.3 \,, \tag{1}$$

establishes strong isospin violating effects, which can not be explained if the $X(3872)$ is interpreted as a $c\bar{c}$ state.

Due to these facts, these new mesons were considered as good candidates for four-quark states by many authors [8]. In refs. [9, 10] the method of QCD sum rules (QCDSR) [11–13] was used to study the two-point functions for the mesons $D_{sJ}^+(2317)$, $D_0^{*0}(2308)$and $X(3872)$ considered as four-quark states in a diquark-antidiquark configuration. The results obtained for their masses are compatible with the experimental values and, therefore, in refs. [9, 10] the authors concluded that it is possible to reproduce the experimental value of the masses using a four-quark representation for these states.

Concerning their decay widths, the study of the three-point functions related to the decay widths $D_{sJ}^+(2317) \to D_s^+\pi^0$, $D_0^{*0} \to D^+\pi^-$ and $X(3872) \to J\psi\pi^+\pi^-$, using the diquark-antidiquark configuration for D_{sJ}, D_0^{*0} and X, was done in refs. [14–16]. The results obtained for their partial decay widths are given in Table I, from where we see that the partial decay widths obtained in refs. [14, 15], supposing that the mesons $D_{sJ}^+(2317)$ and D_0^{*0} are four-quark states, are consistent with the experimental upper limit for the total decay width.

TABLE I: Numerical results for the resonance decay widths

decay	$D_{sJ}^+ \to D_s^+\pi^0$	$D_0^{*0} \to D^+\pi^-$	$X \to J/\psi\pi^+\pi^-$
Γ (MeV)	$(6\pm2)\times10^{-3}$	120 ± 20	50 ± 15
Γ_{tot}^{exp} (MeV)	< 5	270	< 2.3

However, in the case of the meson $X(3872)$, the partial decay width obtained in ref. [16] is much bigger than the experimental upper limit to the total width.

In ref. [16] it was shown that it is possible to reduce the value of the estimated $X(3872)$ decay width, by imposing that the initial four-quark state has a non-trivial color structure. In this case, some diagrams are eliminated and the partial decay width can be reduced to $\Gamma(X \to J/\psi\pi^+\pi^-)) = (0.7\pm0.2)$ MeV. However, that procedure may appear somewhat unjustified and, therefore, more study is needed until one can arrive at a definitive conclusion about the structure of the meson $X(3872)$.

Concerning the meson $D_{sJ}^+(2317)$, although its mass and decay width can be explained in a four-quark scenario, they can also be reproduced in other approaches [8], and it is not yet possible to discriminate between the different structures proposed for this state. Therefore, it is important to find experimental observations that could be used to descriminate between the different quark structure of these mesons. As pointed out in ref. [17], a signal could be obtained by the analysis of certain heavy-ion collision observables. Another possibility is to study the $D_{sJ}^+(2317)$ production in photonucleon reactions. Whith the 12 GeV Upgrade of the CEBAF accelerator at Jefferson Lab., the $D_{sJ}^+(2317)$ can be produced in reactions of the type: $\gamma p \to \Lambda D_{sJ}^+\bar{D}^0$. Therefore, if the coupling constant, $g_{D_{sJ}DK}$, is found to be very different depending on the structure for $D_{sJ}^+(2317)$, then the photo-production of $D_{sJ}^+(2317)$ can be used as a signal to descriminate its structure.

II. THE $g_{D_{sJ}DK}$ COUPLING CONSTANT

The coupling, $g_{D_{sJ}DK}$, defined through the effective lagrangian

$$\mathcal{L}_{D_{sJ}DK} = g_{D_{sJ}DK}\,(\bar{D}_{sJ}DK + D_{sJ}\bar{D}\bar{K})\ , \tag{2}$$

was evaluated in ref. [18], supposing that the meson $D_{sJ}^{+}(2317)$ is a conventional $c\bar{s}$ state. They got:

$$g_{D_{sJ}DK} = (9.2\pm 0.5)\ \text{GeV} \tag{3}$$

Here, we extend the calculation done in refs. [14, 15] to study the hadronic vertex $D_{sJ}DK$. The QCDSR calculation for the vertex, $D_{sJ}DK$, centers around the three-point function given by

$$T_\mu(p,p',q) = \int d^4x d^4y\, e^{ip'.x}\, e^{iq.y}\langle 0|T[j_D(x) j_{5\mu}(y) j_{D_{sJ}}^{\dagger}(0)]|0\rangle, \tag{4}$$

where $j_{D_{sJ}}$ is the interpolating field for the scalar D_{sJ} meson [9]:

$$j_{D_{sJ}} = \frac{\varepsilon_{abc}\varepsilon_{dec}}{\sqrt{2}}\left[(u_a^T C\gamma_5 c_b)(\bar{u}_d\gamma_5 C\bar{s}_e^T) + u\leftrightarrow d\right], \tag{5}$$

where $a,\ b,\ c,\ \ldots$ are colour indices and C is the charge conjugation matrix. In Eq. (4), $p = p' + q$ and the interpolating fields for the kaon and for the D mesons are given by:

$$j_{5\mu} = \bar{s}_a\gamma_\mu\gamma_5 q_a, \quad j_D = i\bar{q}_a\gamma_5 c_a, \tag{6}$$

where q stands for the light quark u or d.

The calculation of the phenomenological side proceeds by inserting intermediate states for D, K and D_{sJ}, and by using the definitions: $\langle 0|j_{5\mu}|K(q)\rangle = iq_\mu F_K$, $\langle 0|j_D|D(p')\rangle = \frac{m_D^2 f_D}{m_c}$, $\langle 0|j_{D_{sJ}}(p)\rangle = \lambda$. We obtain the following relation:

$$T_\mu^{phen}(p,p',q) = \frac{\lambda m_D^2 f_D F_K\, g_{D_{sJ}DK}/m_c}{(p^2-m_{D_{sJ}}^2)(p'^2-m_D^2)(q^2-m_K^2)}\ q_\mu + \text{continuum contribution}\ , \tag{7}$$

where the coupling constant, $g_{D_{sJ}DK}$, is defined by the on-mass-shell matrix element: $\langle DK|D_{sJ}\rangle = g_{D_{sJ}DK}$. The continuum contribution in Eq.(7) contains the contributions of all possible excited states.

In the case of the light scalar mesons, considered as diquark-antidiquark states, the study of their vertex functions using the QCD sum rule approach at the pion pole [12, 13, 19], was done in ref.[20]. It was shown that the decay widths determined from the QCD sum rule calculation are consistent with existing experimental data. Here, we follow ref. [21] and work at the kaon pole. The main reason for working at the kaon pole is that one does not have to deal with the complications associated with the extrapolation of the form factor [22]. The kaon pole method consists in neglecting the kaon mass in the denominator of Eq. (7) and working at $q^2=0$. In the OPE side one singles out the leading terms in the operator product expansion of Eq.(4) that match the $1/q^2$ term. Since we are working at $q^2=0$, we take the limit $p^2 = p'^2$ and we apply a single Borel transformation to $p^2, p'^2 \to M^2$. On the phenomenological side, in the structure q_μ we get:

$$T^{phen}(M^2) = \frac{\lambda m_D^2 f_D F_K\, g_{D_{sJ}DK}}{m_c(m_{D_{sJ}}^2 - m_D^2)}\left(e^{-m_D^2/M^2} - e^{-m_{D_{sJ}}^2/M^2}\right) + A\, e^{-s_0/M^2} + \int_{u_0}^{\infty}\rho_{cc}(u)\, e^{-u/M^2}du, \tag{8}$$

where A and $\rho_{cc}(u)$ stands for the pole-continuum transitions and pure continuum contributions, with s_0 and u_0 being the continuum thresholds for D_{sJ} and D respectively [14, 15]. For simplicity, one assumes that the pure continuum contribution to the spectral density, $\rho_{cc}(u)$, is given by the result obtained in the OPE side. Therefore, one uses the ansatz: $\rho_{cc}(u) = \rho_{OPE}(u)$. In Eq.(8), A is a parameter which, together with $g_{D_{sJ}DK}$, has to be determined by the sum rule.

On the OPE side we single out the leading terms proportional to q_μ/q^2. Transferring the pure continuum contribution to the OPE side, the sum rule for the coupling constant, up to dimension 7, is given by:

$$\begin{aligned} C\ &\left(e^{-m_D^2/M^2} - e^{-m_{D_{sJ}}^2/M^2}\right) + A\ e^{-s_0/M^2} = \\ =\ &\frac{1}{\sqrt{2}}\left[\frac{\langle\bar{q}q\rangle + \langle\bar{s}s\rangle}{2^4\pi^2}\int_{m_c^2}^{u_0} du\, e^{-u/M^2} u\left(1-\frac{m_c^2}{u}\right)^2\right. \\ +\ &\frac{m_c m_s\langle\bar{s}s\rangle}{2^5\pi^2}\int_{m_c^2}^{u_0} du\, e^{-u/M^2}\left(1-\frac{m_c^2}{u}\right)^2 + \frac{m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{12}e^{-m_c^2/M^2} \\ -\ &\left.\frac{m_c\langle\bar{q}q\rangle(\langle\bar{q}q\rangle+\langle\bar{s}s\rangle)}{3}e^{-m_c^2/M^2}\right], \end{aligned} \tag{9}$$

with

$$C = \frac{\lambda m_D^2 f_D F_K}{m_c(m_{D_{sJ}}^2 - m_D^2)}\ g_{D_{sJ}DK}. \tag{10}$$

III. RESULTS AND CONCLUSIONS

In the numerical analysis of the sum rules, the values used for the meson masses, quark masses and condensates are: $m_{D_{sJ}} = 2.317$ GeV, $m_D = 1.87$ GeV, $m_c = 1.2$ GeV, $m_s = .13$ GeV $\langle\bar{q}q\rangle = -(0.23)^3$ GeV^3, $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$. For the meson decay constants we use $F_K = 160$ MeV and $f_D = 0.22$ GeV [23]. We use $u_0 = 6$ GeV^2 and for the current meson coupling, λ, we are going to use the result obtained from the two-point function in ref. [9]. Considering $2.6 \le \sqrt{s_0} \le 2.8$ GeV we get $\lambda = (2.9\pm 0.3)\times 10^{-3}$ GeV^5.

In Fig. 1 we show, through the dots, the right-hand side (RHS) of Eq.(9) as a function of the Borel mass. To determine $g_{D_{sJ}DK}$ we fit the QCDSR results with the analytical expression in the left-hand side (LHS) of Eq.(9):

$$C\left(e^{-m_{D_s}^2/M^2} - e^{-m_{D_{sJ}}^2/M^2}\right) + A\ e^{-s_0/M^2}, \tag{11}$$

Using $\sqrt{s_0} = 2.7$ GeV we get: $C = 4.53\times 10^{-4}$ GeV^7 and $A = -4.68\times 10^{-4}$ GeV^7. Using the definition of C in Eq.(10) and

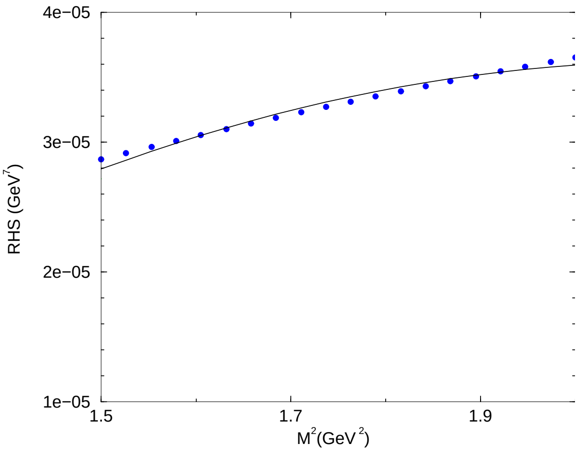

FIG. 1: Dots: the RHS of Eq.(9), as a function of the Borel mass. The solid line gives the fit of the QCDSR results through the LHS of Eq.(9).

$\lambda = 2.9\times10^{-3}$ GeV^5 (the value obtained for $\sqrt{s_0} = 2.7$GeV) we get $g_{D_{sJ}DK} = 2.8$ GeV. Allowing s_0 to vary in the interval $2.6 \leq \sqrt{s_0} \leq 2.8$ GeV, the corresponding variation obtained for the coupling constant is

$$2.5 \text{ GeV} \leq g_{D_{sJ}DK} \leq 3.8 \text{ GeV}. \quad (12)$$

Fixing $\sqrt{s_0} = 2.7$GeV and varying the quark condensate, the charm quark and the strange quark masses in the intervals: $-(0.24)^3 \leq \langle\bar{q}q\rangle \leq -(0.22)^3$ GeV^3, $1.1 \leq m_c \leq 1.3$GeV and $0.11 \leq m_s \leq 0.15$GeV, we get results for the coupling constant still between the lower and upper limits given above. it is important to mention that the agreement between the RHS and LHS of the sum rule in Fig.1 is not so good, in this case, as it was in the case of the couplings $g_{D_{sJ}D_s\pi}$ and $g_{D_0^{*0}D\pi}$ evaluated in refs. [14, 15]. One possible reason for that is the fact that the kaon mass is much bigger than the pion mass. Therefore, neglecting the kaon mass in Eq. (7) is not an approximation as good as it is in the case of the sum rule in the pion pole.

We have presented a QCD sum rule study of the vertex function associated with the hadronic vertex $D_{sJ}DK$, where the $D_{sJ}(2317)$ meson was considered as diquark-antidiquark state. Comparing the results in Eqs. (12) and (3) we see that when the meson $D_{sJ}(2317)$ is considered as a conventional $c\bar{s}$ state one gets a $g_{D_{sJ}DK}$ coupling constant much bigger than when $D_{sJ}(2317)$ is considered a four-quark state. This result can be usefull to experimentally investigate the quark structure of the meson $D_{sJ}(2317)$ through, for example, its photoproduction on nucleon targets.

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