Gribov Ambiguities in the Maximal Abelian Gauge

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The effects of the Gribov copies on the gluon and ghost propagators are investigated in $SU(2)$ Euclidean Yang-Mills theory quantized in the maximal Abelian gauge. By following Gribov's original approach, extended to the maximal Abelian gauge, we are able to show that the diagonal component of the gluon propagator displays the characteristic Gribov type behavior. The off-diagonal component is found to be of the Yukawa type, with a dynamical mass originating from the dimension two gluon condensate, which is also taken into account. Furthermore, the off-diagonal ghost propagator exhibits infrared enhancement. Finally, we make a comparison with available lattice data.

Keywords: Yang-Mills theories; Gribov ambiguities; Confinement; Maximal Abelian Gauge

I. INTRODUCTION

Nowadays, the main problem of Yang-Mills theories (YM) and consequently of quantum chromodynamics (QCD), is to explain theoretically the confinement phenomenon. Despite the fact that there are several approaches to treat the issue, there is no final answer to the confinement problem. Let us briefly point out the main ideas which will be the motivation of the present work.

A. Dual superconductivity and confinement

An appealing mechanism to explain the color confinement is the so called dual superconductivity mechanism [1]. According to this proposal, the low energy regime of YM would contain monopoles as vacuum configuration. The ensuing magnetic condensation would induce a dual Meissner effect in the chromoelectric sector. As for ordinary superconducting media, the potential between chromoelectric charges increases linearly with their length, characterizing a confinement picture.

The splitting between the diagonal and off-diagonal degrees of freedom in this approach [1, 2] indicates that the natural way to treat the problem would be to consider different gauge fixings for the diagonal and off-diagonal sectors of the theory. In fact, the class of Abelian gauges [2] shows itself to be the suitable framework to work with. In this class of gauges not only are the diagonal and off-diagonal sectors independently gauged, but also monopoles show up as defects in the gauge fixing. An interesting example of an Abelian gauge is the maximal Abelian gauge (MAG) [2], which will be used in this work and discussed in the next section.

B. Abelian dominance and dynamical gluon mass

Another important ingredient of the infrared regime of QCD is the hypothesis of Abelian dominance [3]. This principle states that in the infrared limit, QCD would be described by an effective theory constructed from only Abelian degrees of freedom. This effect has been confirmed using lattice numerical simulations [4, 5].

Recently, it has been argued that the off-diagonal gluon might acquire a large dynamical mass in the MAG [6], with the explicit $SU(2)$ value of $m \approx 2.25\Lambda_{QCD}$, due to the condensation of the off-diagonal gluon composite operator $A^a_\mu A^a_\mu$. This can be regarded as evidence in favour of the Abelian dominance, since for an energy scale below this mass, the off-diagonal gluons should decouple and the diagonal degrees of freedom would dominate the theory. At the same level, evidence of Abelian dominance has been advocated in the Landau gauge, due to the condensation of dimension two operators [7]. The Abelian degrees of freedom are identified here with the diagonal gluons.

C. Gribov ambiguities and the quantization of QCD

It is a fact that YM theories are plagued by Gribov ambiguities [8, 9], *i.e.*, after the gauge fixing of the model, there still remains a residual gauge symmetry spoiling a consistent complete quantization of QCD.

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The improvement of the Faddeev-Popov quantization formula [8] in the Landau gauge, yields modifications of the propagators of the theory. The gluon propagator turns out to be suppressed in the infrared limit, acquiring pure imaginary poles, indicating a destabilization of the gluon excitations. On the other hand, the ghost propagator shows itself to be more singular than the perturbative behavior, and can be related to the existence of long range forces. Thus, one can tacitly infer that the Gribov problem is related to the confinement phenomenon. This is a strong motivation to study Gribov ambiguities in the MAG. However, concerning the Gribov problem in gauges other than the Landau and Coulomb ones, the available information is very humble. To our knowledge, the only available results are those obtained in the linear covariant gauges [10] and in the MAG [11–13]. The latter one will be discussed here.

D. Lattice data

The final motivation for the present work is related to lattice numerical simulations, which provide a useful nonperturbative method to treat QCD. In the last decades the lattice has been used to extract nonperturbative effects of QCD. Mass parameters are commonly used to fit the lattice data with relative success. In particular, in the MAG, two mass scales have been employed to fit the data obtained for the diagonal and off-diagonal gluon propagators, yielding evidence in favour of the Abelian dominance [14, 15]. In [15], the off-diagonal mass has been found to be almost twice as big as compared to the diagonal one. Moreover, the most suitable fit for the transverse off-diagonal gluon propagator is found to be of the Yukawa type

$$D_{off}(q) = \frac{1}{q^2 + m_{off}^2} \,, \tag{1}$$

while for the diagonal sector, the best fit of the transverse propagator is given by a Gribov type formula

$$D_{diag}(q) = \frac{q^2}{q^4 + m_{diag}^4} \,, \tag{2}$$

where

$$m_{off} \approx 2 m_{diag} \,. \tag{3}$$

The numerical value of the off-diagonal mass is, in both works [14, 15], $m_{off} \approx 1.2 GeV$. Both propagators are suppressed in the infrared limit. In addition, a longitudinal component arises in the off-diagonal sector from the lattice data [15]. This component is very well described by a Yukawa fit, (1), using the same off-diagonal mass value. In the case of the diagonal sector, there is no longitudinal component of the propagator.

E. Analytic results

The above considerations are sufficient to justify the study of the MAG. Here, we shall summarize the analytic results obtained in [13], concerning the effects of the Gribov ambiguities and of the dynamical mass generation on the propagators of the theory, in the case of $SU(2)$ MAG.

The original Gribov approach, [8], can be essentially repeated in the case of the MAG, where the effect of the dynamical off-diagonal gluon mass [6] can also be taken into account. The resulting behavior of the propagators is as follows. The transverse off-diagonal gluon propagator is of the Yukawa type

$$D_{off}(q) = \frac{1}{q^2 + m^2} \,, \tag{4}$$

where m is the dynamical mass generated due to the condensation of the off-diagonal gluon operator $A_\mu^a A_\mu^a$. The diagonal propagator is purely transverse, being given by

$$D_{diag}(q) = \frac{q^2}{q^4 + \gamma^4} \,, \tag{5}$$

where γ is the so called Gribov parameter, with the dimension of a mass. Further, the off-diagonal ghost propagator turns out to be enhanced in the low energy region, according to

$$\lim_{q \to 0} G(q) \propto \frac{1}{q^4} \,. \tag{6}$$

To our knowledge this is the first result on the ghost propagator in the MAG when the Gribov ambiguities are taken into account.

These results might provide some physical insights on the nature of the mass parameters appearing in the lattice fits. Here, one can easily see from (4) and (5) that the Gribov ambiguities are responsible for the infrared suppression of the diagonal sector, while the dynamical mass enters only the off-diagonal sector, making it also suppressed in the infrared.

Despite the fact that the propagators (4) and (5) are in qualitative agreement with the lattice data (1) and (2), see [15], we were not able to provide specific values for γ and m due to the lack of a more formal framework to work with. Such a framework is currently only at hand in the Landau gauge where a local, renormalizable Lagrangian, which takes into account the Gribov ambiguities, is known [16]. In [17–19], this Lagrangian was used to make explicit computations. In the case of the MAG, such a Lagrangian was recently derived [20], but no explicit computations have been performed yet. Moreover, it will allow us to investigate the behavior of the longitudinal component of the off-diagonal gluon propagator, a feature which we were unable to address within the approximation employed in [13].

II. GRIBOV AMBIGUITIES IN THE MAG

First, we shall present the MAG in the case of $SU(2)$ Yang-Mills. Then, the Gribov ambiguities will be introduced and their main features will be briefly discussed.

A. The maximal Abelian gauge

In order to fix the gauge differently in the diagonal and off-diagonal sectors, we decompose the $SU(2)$ gluon field according to [21]

$$\mathcal{A}_\mu = A^a_\mu T^a + A_\mu T^3 \ , \tag{7}$$

where T^a, $a = 1,2$, stands for the off-diagonal generators of $SU(2)$, while T^3 denotes the diagonal one. In the same way the field strength decomposes as

$$\mathcal{F}_{\mu\nu} = F^a_{\mu\nu} T^a + F_{\mu\nu} T^3 \ , \tag{8}$$

so that, for the YM action we get

$$S_{YM} = \frac{1}{4}\int d^4x \left(F^a_{\mu\nu}F^a_{\mu\nu} + F_{\mu\nu}F_{\mu\nu} \right) \ . \tag{9}$$

Explicitly, for the components of the field strength we have

$$\begin{aligned} F^a_{\mu\nu} &= D^{ab}_\mu A^b_\nu - D^{ab}_\nu A^b_\mu \ , \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \ , \end{aligned} \tag{10}$$

where the covariant derivative D^{ab}_μ is defined with respect the diagonal gluon

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - g\varepsilon^{ab} A_\mu \ . \tag{11}$$

The action (9) is invariant under the gauge transformations

$$\begin{aligned} \delta A^a_\mu &= -D^{ab}_\mu \omega^b - g\varepsilon^{ab} A^b_\mu \omega \ , \\ \delta A_\mu &= -\partial_\mu \omega - g\varepsilon^{ab} A^a_\mu \omega^b \ , \end{aligned} \tag{12}$$

where $\{\omega^a, \omega\}$ are the infinitesimal gauge parameters. The presence of the gauge freedom requires a constraint for a consistent perturbative quantization. The MAG gauge fixing condition is attained by requiring that

$$D^{ab}_\mu A^b_\mu = 0 \ . \tag{13}$$

The residual local $U(1)$ gauge symmetry present in the diagonal sector is fixed by means of the Landau gauge

$$\partial_\mu A_\mu = 0 \ . \tag{14}$$

According to [13], the gauge fixing conditions (13), (14) yield the following partition function

$$\mathcal{Z} = \int DA^a DA \left(\det \mathcal{M}^{ab} \right) \delta(D^{ab}_\mu A^b_\mu) \delta(\partial_\mu A_\mu) e^{-S_{YM}} \ , \tag{15}$$

where $\mathcal{M}^{ab}$ is the off-diagonal Hermitian Faddeev-Popov ghost operator

$$\mathcal{M}^{ab} = -D^{ac}_\mu D^{cb}_\mu - g^2 \varepsilon^{ac}\varepsilon^{bd} A^c_\mu A^d_\mu \ . \tag{16}$$

B. The Gribov problem in the MAG

The existence of a residual gauge symmetry in the path integral (15) is recognized as the well known Gribov problem [8]. It consists in the existence of equivalent gauge field configurations, $\{\widetilde{A}^a_\mu, \widetilde{A}_\mu\}$, obeying the gauge conditions (13) and (14), namely

$$\begin{aligned} D^{ab}_\mu(\widetilde{A})\widetilde{A}^b_\mu &= 0 \ , \\ \partial_\mu \widetilde{A}_\mu &= 0 \ . \end{aligned} \tag{17}$$

At the infinitesimal level, conditions (17) yield

$$\mathcal{M}^{ab}\omega^b = 0 \ , \tag{18}$$

$$-\partial^2\omega - g\varepsilon^{ab}\partial_\mu(A^a_\mu\omega^b) = 0 \ , \tag{19}$$

implying the existence of zero modes for the Faddeev-Popov operator $\mathcal{M}^{ab}$. Thus, the Yang-Mills measure in the partition function (15) is ill-defined. Also, from eq.(19), one observes that the diagonal parameter ω is completely determined once eq.(18) has been solved for ω^a, according to

$$\omega = -g\varepsilon^{ab}\frac{1}{\partial^2}\partial_\mu(A^a_\mu\omega^b) \ . \tag{20}$$

As expected, this observation allows us to prove that the diagonal ghosts decouple, and can be in fact integrated out in the partition function (15). Thus, one can focus only on the zero modes of the off-diagonal Faddeev-Popov operator, eq.(18).

C. Facing the Gribov copies

According to [13], the existence of the Gribov copies in the MAG can be faced along the lines outlined by Gribov in the case of the Landau and Coulomb gauges [8], where the domain of integration in the Feynman path integral is restricted to a smaller region Ω, known as the Gribov region.

In the case of the MAG, the region Ω is identified with the set of field configurations obeying the gauge conditions (13) and (14), and for which the Faddeev-Popov operator, eq.(16), is strictly positive, namely

$$\Omega \equiv \left\{ \mathcal{A}_\mu \ \Big| \ D^{ab}_\mu A^b_\mu = 0 \ , \ \partial_\mu A_\mu = 0 \ , \ \mathcal{M}^{ab} > 0 \right\} \ . \tag{21}$$

The boundary $\partial\Omega$ of the region Ω is known as the Gribov horizon. In the MAG, the restriction of the domain of integration in the path integral to the region Ω is supported by the fact that for a field configuration belonging to Ω and lying near the Gribov horizon $\partial\Omega$, there is an equivalent configuration located on the other side of the horizon $\partial\Omega$, outside of the Gribov region Ω. This result is a generalization to the MAG of Gribov's original statement in the Landau gauge. The complete proof can be found in [13]. The restriction to the region Ω is achieved by modifying the partition function (15) in such a way that

$$\mathcal{Z} = \int DA^a DA \left(\det \mathcal{M}^{ab}\right) \delta(D_\mu^{ab} A_\mu^b) \delta(\partial_\mu A_\mu) e^{-S_{YM}} \mathcal{V}(\Omega) , \tag{22}$$

where the functional $\mathcal{V}(\Omega)$ implements the restriction to Ω in field space.

The functional $\mathcal{V}(\Omega)$ can be constructed recursively by means of a no-pole condition on the off-diagonal ghost propagator, which is nothing else but the inverse of the operator $\mathcal{M}^{ab}$. In fact, from the definition of the region Ω, it follows that the inverse of the Faddeev-Popov operator, $(\mathcal{M}^{ab})^{-1}$, see [8, 13], has to be positive and without singularities, except for those configurations which are located on the boundary $\partial\Omega$, where $\mathcal{M}^{ab}$ vanishes. Moving to momentum space, it can be shown that the Green function $\mathcal{G}(k) = \left\langle k \middle| \mathcal{M}^{-1} \middle| k \right\rangle$ has no poles at nonvanishing k^2, except for a singularity at $k^2 = 0$, corresponding in fact to the boundary $\partial\Omega$. According to the no-pole prescription, the first nontrivial term for the factor $\mathcal{V}(\Omega)$ if found to be [13]

$$\mathcal{V}(\Omega) = \exp\left\{ -\frac{\gamma^4}{2} \int \frac{d^4q}{(2\pi)^4} \frac{A_\mu(q) A_\mu(-q)}{q^4} \right\} , \tag{23}$$

where γ is the Gribov parameter. It is not a free parameter, being determined by the gap equation

$$\frac{3}{4} g^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4 + \gamma^4} = 1 . \tag{24}$$

The gap equation (24), together with the path integral (22) ensures the correct truncation of the integration domain up to the Gribov horizon.

D. Propagators I

The restriction of the domain of integration, eq.(22), has far reaching consequences on the behavior of the propagators. Looking at the form of the functional (23), one can easily see that it affects only the diagonal tree level propagator. In fact, the diagonal propagator is given by (5), which is infrared suppressed. We notice the appearance of imaginary poles in this propagator, indicating that the diagonal gluon does not belong to the physical spectrum of the model. This behavior is consistent with the confining character of the theory.

Concerning the off-diagonal gluon propagator, it is left unmodified at the tree level, coinciding with the transverse perturbative propagator

$$D_{off}^{pert} = \frac{1}{q^2} . \tag{25}$$

There is no longitudinal off-diagonal component at the tree level.

The off-diagonal ghost propagator shows itself to be more singular in the infrared region, as can be inferred from (6). We remark that the gap equation (24) is an essential ingredient for this behavior. Notice also that this enhancement is a manifestation of the Gribov horizon, where the ghost propagator is highly singular.

III. DYNAMICAL MASS IN THE MAG

In [13], the Gribov issue was also studied in the presence of the off-diagonal dynamical gluon mass. In [6], the condensation of the operator $A_\mu^a A_\mu^a$ was analysed in detail in the MAG, by evaluating the effective potential for this operator at one loop order. It was shown that this potential develops a nontrivial minimum, which lowers the vacuum energy, favoring thus the formation of the condensate $\left\langle A_\mu^a A_\mu^a \right\rangle$, which results in an effective dynamical off-diagonal gluon mass. Following [6], the dynamical mass generation can be described by adding to the Yang-Mills action the following term

$$S_m = \frac{1}{2\zeta g^2} \int d^4x \left[\sigma^2 + g\sigma A_\mu^a A_\mu^a + \frac{g^2}{4} \left(A_\mu^a A_\mu^a\right)^2 \right] , \tag{26}$$

where σ is a Hubbard-Stratanovich auxiliary field coupled to the composite operator $A_\mu^a A_\mu^a$. As shown in [6], the field σ develops a nonvanishing vacuum expectation value, $\langle\sigma\rangle \neq 0$, which is related to the gluon condensate $\left\langle A_\mu^a A_\mu^a \right\rangle$ through the relation [6]

$$\langle\sigma\rangle = -\frac{g}{2} \left\langle A_\mu^a A_\mu^a \right\rangle . \tag{27}$$

The parameter ζ is needed to account for the vacuum divergences present in the Green function $\left\langle A^2(x) A^2(y) \right\rangle$. At one loop order, for the dynamical gluon mass m one finds [6]

$$m^2 = \frac{\left\langle A_\mu^a A_\mu^a \right\rangle}{g\zeta} \approx (2.25\Lambda_{QCD})^2 , \tag{28}$$

Remarkably, the effective action

$$S = S_{YM} + S_m , \tag{29}$$

turns out to be gauge invariant, provided the auxiliary field σ transforms as

$$\delta\sigma = g A_\mu^a D_\mu^{ab} \omega^b . \tag{30}$$

As a consequence, the action (29) is still plagued by the Gribov ambiguities. In order to cure this pathology, one can perform the same procedure as was performed for the massless case. According to [13], both the functional (23) and the gap equation (24) remain the same.

A. Propagators II

Since the functional (23) depends only on the diagonal gluon field, the corresponding propagator is not affected, at the tree level, by the dynamical mass. Thus, the diagonal gluon propagator is given by (5).

The off-diagonal gluon propagator is, however, affected by the dynamical mass. A simple computation leads to the expression (4), exhibiting infrared suppression due to the dynamical mass. .

The ghost propagator, also not affected by the gluon mass, shows the typical infrared enhancement (6). Again, the gap equation (24) is fundamental for this result.

IV. CONCLUSIONS AND MORE

In this work we have reviewed the influence of the Gribov ambiguities and of the dynamical gluon mass on the propagators of $SU(2)$ Yang-Mills theories in four-dimensional Euclidean space time, quantized in the maximal Abelian gauge. The diagonal as well as the off-diagonal gluon propagators are infrared suppressed. The diagonal sector displays a Gribov type behavior, due to the presence of the Gribov parameter, see (5). The off-diagonal gluon propagator has a Yukawa type behavior, see (4).

The off-diagonal ghost propagator was computed in the infrared limit. The result (6) shows that it is enhanced in the low energy region. To some extent, this enhancement signals the influence of field configurations located near the boundary $\partial\Omega$ of the Gribov region, where the ghost propagator is highly singular [8, 24].

We recall here that, for the gluon propagators, our results are in qualitative agreement with the available lattice data [15], see (1) and (2). Unfortunately, till now, no data for the ghost propagator are available.

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