Dynamical Behavior of the Four-Body Transverse Ising Model with Random Bonds and Fields

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We study the effect of random bonds and fields on the dynamical behavior of the one-dimensional transverse Ising model with four-spin interactions. We consider finite chains of increasing size to determine the time-dependent correlation function and the longitudinal relaxation function of the infinite chain. In this fully disordered system we observe a crossover from a collective mode type of dynamics to that of a central regime.

The dynamical behavior of pure quantum spin systems has been the subject of much research activity in the past decades[1]. Only recently, more attention has been devoted to the investigation of the time evolution of disordered systems. A number of studies about the phase diagram and thermodynamic functions has demonstrated that the behavior of magnetic materials may be drastically affected by the presence of randomness. Among the systems studied, the transverse Ising model (TIM) has attracted considerable interest in recent years[2, 3, 4, 5] and is regarded as one of the simplest models with non-trivial dynamics.

In this work we are interested in the time evolution of the one-dimensional TIM with four-spin interactions. The model Hamiltonian is

$$H = -8\sum_{i=1}^{L} J_i\, S_i^z S_{i+1}^z S_{i+2}^z S_{i+3}^z - \sum_{i=1}^{L} B_i S_i^x, \qquad (1)$$

where S_i^α ($\alpha = x, y, z$) is the spin-1/2 operator at site i and L is the number of sites of the lattice. The four-spin interactions J_i and the magnetic field B_i are uncorrelated variables chosen at random from the probability distributions $P(J_i)$ and $P(B_i)$, respectively. There have been several studies on the model with uniform exchange interaction and zero magnetic field[6] by using mean-field theories[7, 8], renormalization group[9], series expansions[10] and Monte Carlo simulations[8, 11]. The resulting phase diagram has a tricritical point which separates regions of first and second-order transitions. The pure TIM was also employed to describe the phase transition in poly(vinylidene fluoride-trifluoroethylene) [P(VDF-TrFE)] copolymers[12]. More recently, some studies on the dynamics of this model in the presence of disorder showed that the system undergoes a crossover from a collective mode excitation regime to a central mode type of dynamics as a function of the bond and field energies[4, 13]. Differently from the usual two-body model [14, 15], the four-spin interaction model does not allow a Gaussian decay for the correlation function.

The problem of disorder in magnetic systems is a fascinating phenomenon that has become the object of intense study. In most materials, the presence of magnetic and nonmagnetic ions leads to a distribution of exchange couplings between pairs of spins. In the ideal case, the random interactions J_i for different pairs are totally uncorrelated. The simplest case occurs when all the couplings are of the same sign but vary in strength. Other interesting behavior occurs when the model includes random magnetic fields. In this case, there is a competition between the different energies with the spins tending to align under the exchange interactions

J_i or to follow the local field B_i. The interaction energies can be quite smaller than the thermal energy even, say, at room temperature. In those cases where $J_i/kT << 1$ and $B_i/kT << 1$ the dynamics can be studied in the infinite-temperature limit. In that limit, it is a well known fact that the dynamics is insensitive to the sign of the interaction, since only even powers of the interaction coupling and field come into the moments of the autocorrelation function [4,15,16].

The dynamics of the four-spin TIM can be obtained through the time-dependent autocorrelation function of S_i^z defined by

$$C(\{J_i\},\{B_i\};t) = \frac{< S_i^z S_i^z(t) >}{< S_i^z S_i^z >}. \qquad (2)$$

In the high temperature limit it takes the following form,

$$< S_i^z S_i^z(t) >= \frac{1}{2^L} \mathrm{Tr}\, S_i^z e^{iHt} S_i^z e^{-iHt}. \qquad (3)$$

We shall peform our calculations in finite rings, *i.e.*, linear chains with periodic boundary conditions. The exact behavior of the thermodynamic systems, at short times, is inferred from the dynamics obtained for systems of several sizes (L=7, 9 and 11). Numerical results will be presented only for the largest ring (L=11), which shows signs of having attained the asymptotic behavior of the thermodynamic system at large times.

In the general problem of quenched disorder, the physical properties of a system are obtained by performing a configurational average in the statistical ensemble of realizations of the random variables. Formally, the averaged autocorrelation function is defined as,

$$\overline{C(t)} = \int\int C(\{J_i\},\{B_i\};t) P(J_i) P(B_i) dJ_i dB_i. \qquad (4)$$

In our approach the mean autocorrelation function $\overline{C(t)}$ is obtained as follows. We first diagonalize the full Hamiltonian to determine the energies ϵ_n and eigenvectors $|n>$ for a large number of realizations in the statistical ensemble of energy coupling randomness. We employed 1000 to 10000 configurations to obtain averages over the random variables; so that the error bars fall within the thickness of the curves presented. The results are then used to determine the mean autocorrelation function, which is cast in the following form [17],

$$\overline{C(t)} = \frac{4}{2^L} \sum_{n,m} \overline{\cos(\epsilon_n - \epsilon_m)t| < n|S_i^z|m > |^2}, \qquad (5)$$

where the configurational average is also performed.

Another quantity of interest is the averaged longitudinal relaxation shape function, defined as

$$\Psi(\omega) = \int_0^\infty \overline{C(t)} e^{-i\omega t} dt, \qquad (6)$$

where $\overline{C(t)}$ is given by Eq. (5). The real part of $\Psi(\omega)$ gives directly a physically accessible quantity, the longitudinal relaxation function, $F(\omega) = \mathrm{Re}\,\Psi(\omega)$, which can be measured directly in nuclear magnetic resonance (NMR) experiments —the so-called NMR line shape [18]. The longitudinal relaxation function is also very useful in the understanding of the dynamics of the system, since different dynamic behaviors have distinct signatures in that quantity [19].

We investigated the four-body TI model chain considering bimodal probability distributions for the exchange coupling and magnetic field,

$$P(J_i) = (1-p)\,\delta(J_i - J_1) + p\,\delta(J_i - J_2), \qquad (7)$$

and

$$P(B_i) = (1-p')\,\delta(B_i - B_1) + p'\,\delta(B_i - B_2), \qquad (8)$$

where p is the concentration of couplings of type J_2, and p' is the concentration of magnetic field of type B_2. We determined the time-dependent autocorrelation function and the longitudinal relaxation function for the set (J_i, B_i) with several values of p and p'.

In our calculations we considered the values $J_1 = 1$ and $J_2 = 0.5$ in the bimodal probability distribution for the exchange interaction, and several values of the concentration p of J_2-couplings. The coupling $J_1 = 1$ is kept fixed and sets the energy and time scales. As to the field probability distribution we considered the cases where $B_1 = 1.5$ and $B_2 = 0.5$, also with several concentrations p'. For simplicity, we will present here only the particular case where $p = p'$. The convergence of our results as the chain size increases is reached with the $L = 11$ chain in the time domain shown in the figures, $0 \le t \le 8$. In all cases studied, our calculations also recover the results for the pure TI model with four-spin interactions ($p = 0$) [4].

The results for the time-dependent correlation function are shown in Figs. 1 and 2. The energy couplings are $J_1 = 1$ and $J_2 = 0.5$ with probability p, and the fields are $B_1 = 1.5$ and $B_2 = 0.5$, the latter with probability $p' = p = 0$, 0.2, 0.5 and 0.8. The curves for $p = 0$ corresponds to the pure case dominated by the stronger field energy. The correlation function is oscillatory and damped, falling off quickly towards zero. The longitudinal shape function shows a peak at a nonzero frequency. Hence, at $p = 0$ the system is at the collective mode regime, where the dynamics is mostly due to the precession of the spins about the external

field axis. As we increase p from 0 to 0.2, the correlation function still oscillates. However, the peak in the longitudinal shape function has now moved towards lower frequencies. When $p = 0.5$, the correlation function decays monotonically to zero. The shape function is now peaked at zero frequency, and all the system has undergone a crossover to a central mode behavior. As p is increased further, the correlation function decays more slowly, and the central peak is enhanced, as can be seen from the case $p = 0.8$.

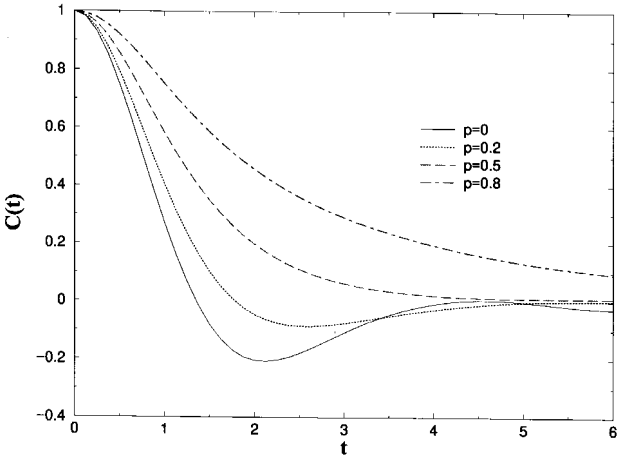

Figure 1. Time-dependent correlation function for the four-body transverse Ising model with random bonds and fields. The couplings and the magnetic fields are distributed according to bimodal probability distributions, and can assume the values J_1 and B_1 with probability $(1-p)$ and J_2 and B_2 with probability p. The energy and time scales are set by the coupling $J_1 = 1.0$. The curves represent the cases $J_2 = 0.5$, $B_1 = 1.5$ and $B_2 = 0.5$ for various values of the probability p.

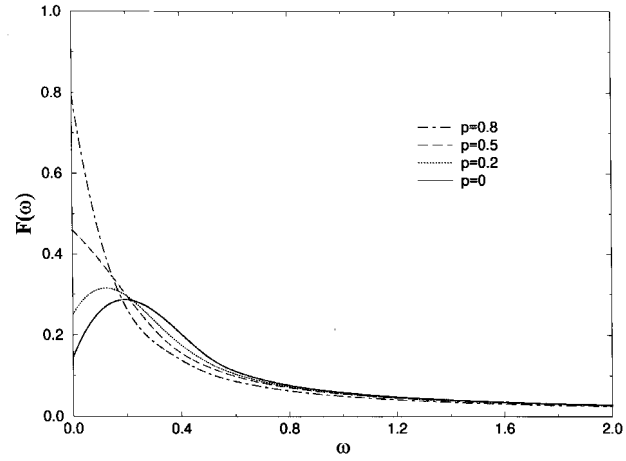

Figure 2. Longitudinal relaxation function (in arbitrary units) vs frequency. The system undergoes a crossover from collective mode behavior to a central mode as p is increased.

In conclusion, we have studied the dynamical behavior of the transverse Ising model with four-spin interactions in the presence of random bonds and fields via exact diagonalization of finite chains. We have calculated the time-dependent correlation functions and the longitudinal relaxation functions for system of sizes $L = 7$, 9 and 11. We present the results only for the size $L = 11$, for which the dynamic correlation functions have already converged to those at the thermodynamic limit. Our results show that disorder induces a crossover from a collective mode type of dynamics to a central mode behavior as a function of disorder.

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