The Role of the Boundary Conditions on the Critical Properties of Superconducting Rings Under the Action of a Tranversal Magnetic Field

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In the framework of the Ginzburg-Landau (GL) theory in the limit of a GL parameter much larger than unity, we study theoretically the critical behavior of a mesoscopic superconducting ring with negligible width in the presence of a magnetic field applied perpendicularly to the ring plane. It is assumed that the inner ring edge is in contact with a material whose properties are accounted in the de Gennes boundary condition with a parameter b (de Gennes extrapolation length), while the outer edge is in contact with vacuum. Special attention is devoted to the influence of the different materials contacting the inner ring edge of the superconductor on the features of the phase diagram, as well as the effect of the confinement on such thermodynamic property.

Keywords: Superconducting rings; Ginzburg-Landau

I. INTRODUCTION

In either of the semiconductor and superconductor systems, the concept of mesoscopics is directly related to that of low-dimensionality. In superconductors there are two fundamental length scales characterizing the confinement of the charge carriers. One is the penetration length λ over which an externally applied magnetic field is screened into the superconducting condensate. The other one is the coherence length ξ which limits the distance over which the superconducting order parameter can vary appreciably [1]. The ratio between the former length and the latter, known as the Ginzburg-Landau (GL) parameter κ, strongly affects the behavior of these systems under external magnetic fields.

Fluxoid quantization in superconducting systems has been known since many years ago. However, a resurgence of interest for the study of mesoscopic superconducting samples (for which their typical size is comparable to the coherence length and the magnetic field penetration length) such as rings [2,3] and disks [4,5] has been noted in the last decade. This interest can be due to the recent impressive progress in nanostructuring techniques such as e-beam lithography, single atom manipulation with a scanning tunneling microscope tip, etc. In particular, special attention has been paid to quantum coherence effects in superconducting rings and their arrays, which can be important for potential applications in quantum computing. The basic physics of such rings is governed by flux quantization, as in the Little-Parks experiment [6].

One of the remarkable effects observed in small superconducting samples is the paramagnetic Meissner effect (PME), for which the sample energy decreases upon increasing magnetic field [7]; as a consequence of this fact the sample presents paramagnetic behavior in certain domain regions of the phase diagram which alternates with the conventional diamagnetic behavior.

The superconducting condensate confined within these non-trivial topologies is subjected to severe constraints and the properties of such samples are strongly affected by the boundary conditions. In particular, for mesoscopic cylindrical superconductors, the influence of boundary conditions has been discussed in Ref. [8].

In this paper we use the Ginzburg-Landau theory in order to study the critical behavior of a mesoscopic superconducting ring with negligible thickness, under the action of a magnetic field applied perpendicularly to the ring plane. It is assumed that the inner ring edge is in contact with a material whose properties are accounted in the de Gennes boundary condition with a parameter b (de Gennes extrapolation length), while the outer edge is in contact with vacuum. Special attention will be paid to the influence of such conditions on the PME.

II. MODEL

We consider a circular ring in the plane z= 0, centered on the z axis, with external radius R and hole radius R_h, under an uniform magnetic field H normal to the plane of the film sample. The ring is considered mesoscopic in the sense that its characteristic size R satisfies the condition d¡¡ξ¡¡R¡¡λ, in which d is the thickness of the superconducting film. In this range we can neglect the screening of the magnetic field [2] and the vector potential of the applied field is given by $\mathbf{A} = \frac{1}{2}\mathbf{H}\times\mathbf{r}$.

In cylindrical coordinates ρ y φ, the normalized GL parameter $\psi = \Psi\Big/\sqrt{|\alpha|/\beta}$, where α and β are the usual GL coefficients, satisfies the equation:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{1}{x}\frac{\partial\psi}{\partial x} - \left(\frac{1}{x}\frac{\partial}{\partial\varphi} + \frac{\Phi}{\Phi_0}x\right)^2\psi + \frac{R^2}{\xi^2}\left(\psi - \psi^3\right) = 0 \tag{1}$$

under the boundary conditions:

$$\left.\frac{\partial\psi}{\partial x}\right|_{x=1} = 0, \qquad \left.\frac{\partial\psi}{\partial x}\right|_{x=R_h/R} = \frac{1}{b}\psi(R_h/R). \tag{2}$$

Here x=ρ/R, Φ_0 is the flux quantum, Φ=$\pi R^2 H$is the magnetic flux through the disk of radius Rin the absence of any flux expulsion and b is the de Gennes extrapolation length.

In order to obtain the upper critical field H_{c3} for the mesoscopic ring, using the linearized GL equation [9], we search for solutions of definite angular momentum n of the form $\psi_n(x,\varphi) = f_n(x)\exp(in\varphi)$. The solution of the linear version of Eq. (1) satisfying the first boundary condition (at the external ring edge), can be expressed in the form:

$$\psi_n(x,\varphi) = x^n \exp\left(-\frac{x^2}{2}\frac{\Phi}{\Phi_0} + in\varphi\right) \times \left[M\left(Y,n+1,x^2\frac{\Phi}{\Phi_0}\right) - \Lambda U\left(Y,n+1,x^2\frac{\Phi}{\Phi_0}\right)\right], \quad (3)$$

in which M and U are the independent solutions of the Kummer equation [10],

$$\Lambda = \left.\frac{(n-z)\,M(Y,n+1,z) + 2zY\,M(Y+1,n+2,z)}{(n-z)\,U(Y,n+1,z) + 2zY\,U(Y+1,n+2,z)}\right|_{z=z_0}, \quad (4)$$

and

$$z_0 = \left(R_h^2/R^2\right)\left(\Phi/\Phi_0\right).$$

The quantity

$$Y = \tfrac{1}{2} - \tfrac{1}{4}\left(R^2/\xi^2\right)\left(\Phi_0/\Phi\right)$$

is determined by the non-linear equation

$$\left(n\frac{R}{R_h} - \frac{\Phi}{\Phi_0}\frac{R_h}{R} - \frac{1}{b}\right)\left[M(Y,n+1,z_0) - \Lambda U(Y,n+1,z_0)\right] + \left(2\frac{\Phi}{\Phi_0}\frac{R_h}{R}Y\right)\left[M(Y+1,n+2,z_0) - \Lambda U(Y+1,n+2,z_0)\right] = 0 \quad (5)$$

which results from the boundary condition at the inner edge of the ring.

For a given flux Φ, the critical line $T_c(\Phi)$ is obtained by choosing the angular momentum value which minimizes the dimensionless quantity R^2/ξ^2.

III. NUMERICAL RESULTS

The trascendental equation (5) was solved in order to obtain R^2/ξ^2 as a function of the dimensionless flux given by Φ/Φ_0 for various values of the de Gennes extrapolation length b and different ratios R_h/R. The H_{c3}phase transition line is made up of the envelope line of the curves R^2/ξ^2 vs Φ/Φ_0 for different number $n$of flux quanta entering the ring.

The H_{c3}lines for a ring with R_h=0.5R and for different values of the de Gennes extrapolation length are shown in Fig. 1.

When the inner hole is filled with a perfect insulator ($b = \infty$) the H_{c3}phase transition line is made up of the envelpoe of a set of curves with n=0, 1, 2,... starting at Φ/Φ_0=0 and showing three paramagnetic domains located above the transitions n=0→ n=1, n=1→ n=2, n=2→ n=3 (Fig. 1(a)). The numerical analysis shows that this picture remains qualitatively unchanged for finite values of b satisfying b¿R. But for b¡R

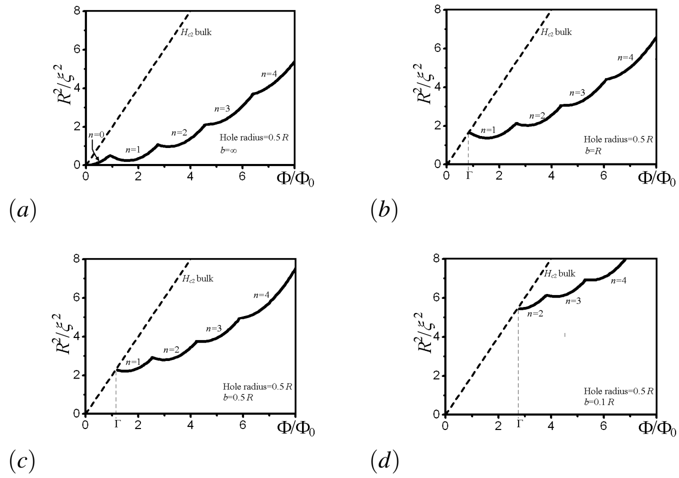

FIG. 1. The upper critical field H_{c3} (thick line) for a ring with R_h=0.5R and for different values of the de Gennes extrapolation length b: (a) $b = \infty$, (b)$b = R$, (c)b=0.5R,(c)b=0.1R. For finite bH_{c3}intersects the upper critical field H_{c2} (dashed line) at $\Phi/\Phi_0 = \Gamma$.

considerable qualitative changes in the phase diagram are observed. In order to illustrate them, in Fig. 1(b) we took an extrapolation length comparable with the external radius of the ring ($b = R$). The following remarcable features are observed: (i) the envelope line defining H_{c3} does not include the n=0 curve; *(ii)* the paramagnetic domains are located above the transitions n=1→ n=2, n=2→ n=3; *(iii)* the H_{c3} line intersects the H_{c2} line at a nonzero value Γ of Φ/Φ_0. This means that for magnetic fields satisfying Φ/Φ_0¡Γ the transition to the normal phase is not mediated by the superconducting states associated to the presence of the ring edges. When the de Gennes extrapolation length $b$further decreases, Γ shifts towards greater values of the applied magnetic field, as can be seen from Fig. 1($c-d$). This means that at the inner edge the supercondincting current is supressed for more values of the flux quanta.

In order to understand the role of the radial confinement on the behavior of the phase diagram, we have considered in Fig. 2 a ring with a hole radius R_h=0.75R for the same set of values of b as in Fig. 1. It is seen that the increasing of the radial confinement causes an increment of the number of domains exhibiting PME. Additionally, it is noted that the effect of the medium contacting with the internal edge of the ring is more notorius, than in the ring of Fig. 1. This can be understood if we consider the relation between two areas: (i) The area of the region where the order parameter extrapolates outside the inner edge and *(ii)* the superconducting ring area. With the increasing of the radial confinement, this relation also increases, which leads to the fact that the value of Γ at which the H_{c3} line intersects the H_{c2} line exhibits a faster desplacement along the H_{c2} line as b diminishes.

In Fig. 3 we have represented the H_{c3} field (in units of $\Phi_0/(\pi R^2)$) as a function of the reduced temperature T/T_{c0}, (T_{c0}being the transition temperature at zero magnetic field of a disk of a radius R) for a fixed value b=0.5R and for different values of the hole radius. For reference we have plotted in Fig. 3(a) the

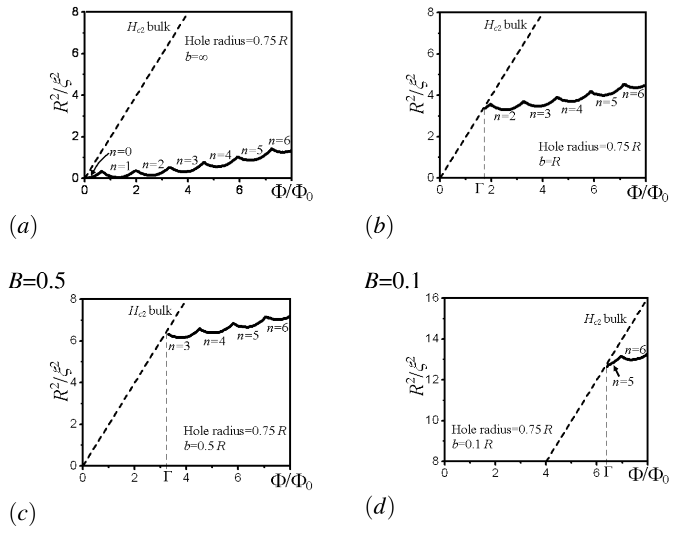

FIG. 2. The same as in Fig.1 but for a ring with R_h=0.75R

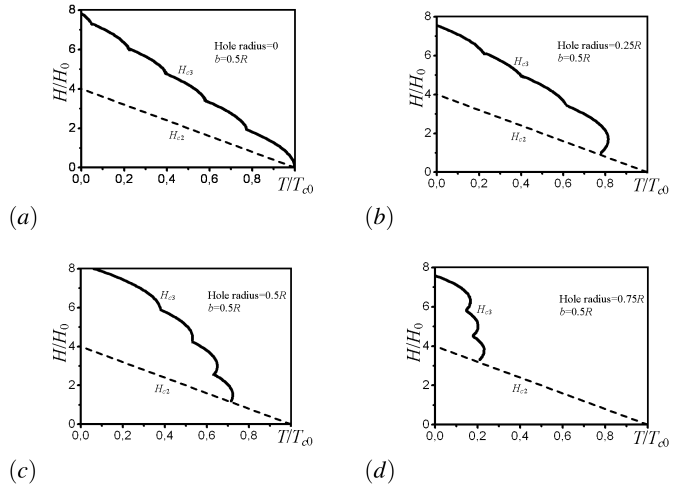

FIG.3. Temperature behavior of the upper critical field H_{c3} (in units of ($H_0 = \Phi_0/(\pi R^2)$)) for rings with a fixed de Gennes extrapolation length b=0.5R and different hole radii: (a)R_h=0, (b)R_h=0.25R, (c)R_h=0.5R,(d)R_h=0.75R.

case of a disk. We see that in the ring the superconductivity can not be preserved up to T/T_{c0}=1, caused by the proximity effect at the inner edge, which affects the superconducty in the ring region. The corresponding critical temperature at which the superconductivity associated to the presence of edges disappears diminishes as the radial confinement increases. A similar conclusion with relation of the role of the proximity effect on the superconducting properties of long cylinder samples in an external magnetic field has been pointed out in Ref. [7].

IV. CONCLUSIONS

We have studied (in the frame of the GL theory) the upper critical field of a type II mesoscopic superconducting ring with negligible width in the presence of a transversal magnetic field under the assumption that the order parameter at the inner ring edge extrapolates outside the boundary at a distance b. For a given value of the de Gennes extrapolating length, the oscillating behavior of the H_{c3} is controled by the size of the hole and has been observed to be more pronounced as the radial confinement in the ring increases, which causes an increment of the number of domains exhibiting the Paramagnetic Meissner effect.

For a given hole radius, we have shown that the characteristics of the material filling the hole area strongly influence the behavior of the upper critical field, which manifests in the appearance of a nonzero point at which the H_{c3} line intersects the H_{c2} line.

The obtained relations can be used to consider the influence of the radial confinement on the so called enhanced superconductivity [11]. Additionally, these results can be extended in order to discuss the influence of boundary conditions at the outer edge. A further extension can take into account also the effect of eccentricity; for semiconductor rings such problem has been discussed recently in Ref. [12]

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