

Connecting an Effective Model of Confinement and Chiral Symmetry to Lattice QCD

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We construct an effective model for the chiral field and the Polyakov loop in which we can investigate the interplay between the approximate chiral symmetry restoration and the deconfinement of color in a thermal SU(3) gauge theory with three flavors of massive quarks. The phenomenological couplings between these two sectors can then be related to the recent lattice data on the renormalized Polyakov loop and the chiral condensate close to the critical region.

Keywords: Confinement; Chiral phase transition; Quark mass effects

I. INTRODUCTION

Various theoretical developments accompanied by several numerical results from lattice simulations have advanced our understanding of QCD thermodynamics [1, 2]. Nevertheless, there are still some important qualitative questions that remain to be answered. For instance, as one increases the temperature above the critical one, which mechanism drives the QCD phase transition for a given quark mass? Is the QCD transition more deconfining or chiral symmetry restoring? Although lattice QCD might provide definitive responses soon, some current findings are still controversial. In particular, the numerical value of the critical temperature for the QCD phase transition [3], the uncertainty on the location and even the possible absence of a critical point in the temperature-baryon density plane [4], and the newly suggested possible difference between the critical temperatures for the chiral and the deconfinement transitions [5]. Therefore, it is useful to address these issues from a phenomenological point of view, resorting to effective field theory models.

The pure gauge sector of QCD with N colors, corresponding to infinitely heavy quarks, is well under control in lattice simulations, with a precise prediction for the deconfinement critical temperature, and a good understanding of its thermodynamic behavior [6]. This sector has a global Z_N symmetry associated with the center of the gauge group $SU(N)$. The Polyakov loop ℓ , charged under Z_N , serves as order parameter for the deconfining transition. It is real for $N = 2$, and the Z_2 symmetry breaking deconfinement transition is of second order [7]. For $N = 3$, ℓ is complex and transforms as $\ell \rightarrow e^{2\pi i/3} \ell$ [8]. Accordingly, the expectation value $\langle \ell \rangle$ vanishes at low temperatures, when Z_3 is unbroken, and $\langle \ell \rangle \neq 0$ above the deconfinement critical temperature $T_d \sim 270$ MeV. This transition is verified to be weakly first order [9].

Another theoretically well-studied sector of QCD is the limit of zero and small quark masses, i.e. the chiral limit. In this regime Z_N is always broken, but chiral symmetry is restored above a critical temperature T_c . The order parameter is the quark chiral condensate σ . The transition is believed to be of second order for two [10–12], and of first order for three flavors of massless quarks. For finite quark masses chiral symmetry is explicitly broken. Increasing m_q from zero corresponds to weakening the first-order phase transition. For some

quark mass the first-order critical line turns into a second-order chiral critical point. For even larger m_q the transition becomes a crossover. Effective field theories have been used to analyze this sector [13].

Deconfinement and chiral symmetry restoration are phenomena of very different nature, corresponding to approximate symmetries of the QCD Lagrangian. For realistic quark masses both chiral and Z_N symmetries are explicitly broken, and most likely both transitions are in the crossover domain [14]. The phase diagram illustrating the quark mass dependence of the deconfining and the chiral symmetry restoring transition has been qualitatively investigated in effective theories in [14–16] and on the lattice in [17–19]. In the crossover region neither σ nor ℓ can play the role of a true order parameter. However, lattice simulations show that the susceptibilities associated with these two quantities peak at the same temperature when quarks are in the fundamental representation of the gauge group [20], suggesting that the transitions occur at the same critical temperature. Similar results were obtained also in terms of the chemical potential [21]. Recent, more refined results confirm the coincidence of the two transitions [22], although results using a different method find critical temperatures that differ by ~ 25 MeV [5], showing that this issue is still not settled.

There exists quite a substantial effort to understand the true nature of these transitions using different effective field theories [23–29]. We especially consider Ref. [27], in which the authors provided an explanation, within a generalized Ginzburg-Landau theory, to why for $m_q = 0$ chiral symmetry restoration leads to deconfinement. The analysis was based on the following general idea: the behavior of an order parameter induces a change in the behavior of non-order parameters at the transition [30, 31] via the presence of a possible coupling between the fields, $g_1 \ell \sigma^2$. In [27] it was assumed that $g_1 > 0$, which is in line with recent results from the lattice.

In this paper we extend the analysis of [27] to the case of nonzero quark masses. Within this model we can then not only study the qualitative relationship between deconfinement and chiral symmetry restoration including massive quarks, but also discuss how to provide a few semi-quantitative predictions for the behavior of the condensates and critical temperatures adopting values for the couplings constrained by lattice data.

Lattice QCD results for the temperature dependence of the chiral condensate for three degenerate flavors of massive quarks were reported in [32], and the first results for the behavior of the renormalized Polyakov loop under the same lattice conditions were presented in [33]. Most recent lattice data, using improved lattice actions, provide the temperature dependence of the chiral condensates [34, 35] and the Polyakov loop for 2+1 flavors [35, 36]. They indicate that the chiral and deconfinement transitions coincide but, contrary to the case of the chiral condensate, the flavor and quark mass dependence of the Polyakov loop is contained entirely in the flavor and quark mass dependence of the critical temperature. In [34] the strange quark condensate was studied separately from the light quark condensate, and it was found that chiral symmetry restoration happens together for the quarks of different flavors. Furthermore, it was shown in [35] that, as expected, the strength of the chiral transition decreases while that of the deconfining transition increases with increasing quark mass.

This paper is organized as follows. In Section II we describe the effective theory in which we couple the Polyakov loop to the chiral sector. In Section III we discuss how to relate our model to the recent findings from the lattice in order to constraint the couplings of our model. Here we set up the theory. We will report on the quantitative analysis elsewhere [37]. Section IV contains our summary and outlook.

II. THE EFFECTIVE THEORY

For nonzero quark masses both the chiral and the Z_N symmetries are explicitly broken, so that neither the chiral condensate nor the Polyakov loop play the role of a true order parameter any longer. One can, however, still use these quantities to monitor the approximate transitions, as is done in lattice QCD calculations, where the transition temperatures are determined from the peak position of the corresponding susceptibilities. Using these quantities we can thus construct an effective field theory to study the interplay between the approximate chiral symmetry restoration and the deconfinement of color. In what follows, we write down the most general renormalizable effective Lagrangian that can be built from the chiral field σ and the Polyakov loop ℓ , following the form introduced in [27].

For the chiral part of the potential we chose the $SU(3) \times SU(3)$ linear sigma model

$$V_\chi(\Phi) = m^2 \text{Tr}[\Phi^\dagger \Phi] + \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 + \lambda_2 \text{Tr}[(\Phi^\dagger \Phi)^2] - c(\det[\Phi] + \det[\Phi^\dagger]) - \text{Tr}[H(\Phi^\dagger + \Phi)], \quad (1)$$

where $\Phi = T_a \phi_a = T_a(\sigma_a + i\pi_a)$ is a complex 3×3 matrix containing the scalar σ_a and the pseudoscalar π_a fields, and $H = T_a h_a$ is the explicit symmetry breaking term. Here $T_a = \lambda_a/2$ are the nine generators of $SU(3)$ with λ_a the Gell-Mann matrices, $\lambda_0 = \sqrt{2/3}\mathbf{1}$, and h_a nine external fields. The different symmetry breaking patterns of this theory have been thoroughly investigated in [38] and [39]. We are interested in making a direct comparison with lattice results both for the

$N_f = 3$ degenerate mass flavors, $m_u = m_d = m_s$, and for the more realistic $N_f = 2 + 1$ case, with $m_u = m_d < m_s$.

Here we set up the degenerate $N_f = 3$ case, for which the potential (1) undergoes some simplifications. This particular case corresponds to that named 4a in [38]. Accordingly, $c = 0$, since the $U(1)_A$ anomaly is not taken into account, and $h_0 \neq 0$ is the only symmetry breaking term, since $h_3 = h_8 = 0$. In this case, the scalar and pseudoscalar matrices are diagonal and the masses are given by

$$\begin{aligned} m_\sigma^2 &= m^2 + 3(\lambda_1 + \frac{\lambda_2}{3})\sigma_0^2, \\ m_{f_0}^2 &= m^2 + (\lambda_1 + \lambda_2)\sigma_0^2, \\ m_\pi^2 &= m^2 + (\lambda_1 + \frac{\lambda_2}{3})\sigma_0^2. \end{aligned} \quad (2)$$

In the vacuum, the condensate is $\sigma_0 = \sqrt{2/3}f_\pi$ and $h_0 = \sqrt{2/3}f_\pi m_\pi^2$, thus $\Phi = f_\pi$ and $H = f_\pi m_\pi^2$.

The idea is to write this special case of the $SU(3)$ potential in the form

$$V_\chi(\sigma) = \frac{m^2}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 - H\sigma. \quad (3)$$

This choice is motivated by the fact that the up-down condensate is aligned in the σ direction. Furthermore, without the $U(1)_A$ anomaly the difference between the results obtained for $SU(2)$ and $SU(3)$ in the sigma model are negligibly small. This is clearly illustrated in the melting of the condensates shown on Fig. 6 from [39]. In (3) the coupling is given by $\lambda = \lambda_1 + \lambda_2/3$. The parameters of the model m^2 , λ_1 and λ_2 are determined from the vacuum values of m_σ , m_π and m_{f_0} . In what follows we assume that pions and f_0 's will not play a role, so we can discard them from the outset.

We adopt a mean field analysis, and thus replace the fields by their expectation values. Then, as customary, we choose the vacuum to be aligned in the sigma direction. As a result, all our relevant equations will depend only on $\langle \sigma \rangle$. In the σ direction the potential has the form shown in Eq. (3).

The Polyakov loop potential when the Z_3 symmetry is explicitly broken reads

$$V_{PL}(\ell) = g_0 \frac{\ell + \ell^*}{2} + \frac{m_0^2}{2} |\ell|^2 - \frac{g_3}{3} \frac{\ell^3 + \ell^{*3}}{2} + \frac{g_4}{4} (|\ell|^2)^2. \quad (4)$$

Effective field theories using this potential have been introduced in [40] and further analyzed in [41]. Here we focus only on the real part of the Polyakov loop, since one can always choose the expectation value to be real, at least for $\mu = 0$. In this case, the contributions coming from the Polyakov loop potential simplify to

$$V_{PL}(\ell) = g_0 \ell + \frac{m_0^2}{2} \ell^2 + \frac{g_3}{3} \ell^3 + \frac{g_4}{4} \ell^4. \quad (5)$$

Here, ℓ is a scalar field (with dimensions of mass) proportional to the Polyakov loop (a dimensionless quantity), and m_0 is its bare mass above the transition. The couplings g_i that appear in each piece of the complete potential can be fixed by lattice data as will be shown in the next section.

The interactions between the chiral field and the Polyakov loop which exist when both symmetries are spoiled by the presence of massive quarks yield the following contribution to the total effective potential

$$V_{int}(\ell, \Phi) = (g_1 \ell + g_2 \ell^2) \text{Tr}[\Phi^\dagger \Phi] + (\bar{g}_1 \ell^2 + \bar{g}_2 \ell) \text{Tr}[H(\Phi^\dagger + \Phi)], \quad (6)$$

which simplifies to

$$V_{int}(\ell, \sigma) = g_1 \ell \sigma^2 + g_2 \ell^2 \sigma^2 + \bar{g}_1 \ell^2 \sigma + \bar{g}_2 \ell \sigma. \quad (7)$$

We now expand the fields around their mean values, $\sigma = \langle \sigma \rangle + \delta\sigma$ and $\ell = \langle \ell \rangle + \delta\ell$, and compute the mean-field equations of motion from first variations of the complete effective potential $V = V_{PL} + V_\chi + V_{int}$, obtaining:

$$2\lambda \langle \sigma \rangle^3 - m_\sigma^2 \langle \sigma \rangle + H - \bar{g}_1 \langle \ell \rangle^2 - \bar{g}_2 \langle \ell \rangle = 0 \quad (8)$$

and

$$g_0 + m_\ell^2 \langle \ell \rangle - g_3 \langle \ell \rangle^2 - 2g_4 \langle \ell \rangle^3 + g_1 \langle \sigma \rangle^2 + \bar{g}_2 \langle \sigma \rangle = 0. \quad (9)$$

The mass of the σ field, m_σ , is determined as the second derivative of the potential with respect to σ evaluated at the minimum:

$$m_\sigma^2 = m^2 + 3\lambda \langle \sigma \rangle^2 + 2g_1 \langle \ell \rangle + 2g_2 \langle \ell \rangle^2. \quad (10)$$

Similarly, the mass of the Polyakov loop is given by

$$m_\ell^2 = m_0^2 + 2g_3 \langle \ell \rangle + 3g_4 \langle \ell \rangle^2 + 2\bar{g}_1 \langle \sigma \rangle + 2\bar{g}_2 \langle \sigma \rangle^2. \quad (11)$$

From now on, we neglect higher-order contributions from $\langle \ell \rangle$, but not from $\langle \sigma \rangle$. We have to keep higher-order terms in the chiral field to be able to recover the correct chiral limit discussed in Ref. [27], when quarks are massless and $H = 0$. Moreover, couplings which were included in the interaction potential of the present analysis due to explicit chiral symmetry breaking must vanish for $H = 0$. For this reason, we require these couplings to be linear in $H \sim m_q + O(m_q^2)$. Accordingly, we make the following replacements: $\bar{g}_1 \rightarrow \bar{g}_1 H$ and $\bar{g}_2 \rightarrow \bar{g}_2 H$. The equations of motion thus become

$$2\lambda \langle \sigma \rangle^3 - m_\sigma^2 \langle \sigma \rangle + H - \bar{g}_2 H \langle \ell \rangle = 0 \quad (12)$$

and

$$g_0 + m_\ell^2 \langle \ell \rangle + g_1 \langle \sigma \rangle^2 + \bar{g}_2 H \langle \sigma \rangle = 0. \quad (13)$$

To first order in m_q , we have:

$$\langle \ell \rangle \simeq -\frac{g_0}{m_\ell^2} - \frac{g_1}{m_\ell^2} \langle \sigma \rangle^2 - \frac{\bar{g}_2}{m_\ell^2} H \langle \sigma \rangle \quad (14)$$

and

$$\begin{aligned} \langle \sigma \rangle^3 + \frac{g_1 \bar{g}_2}{2\lambda m_\ell^2} H \langle \sigma \rangle^2 - \frac{m_\sigma^2}{2\lambda} \langle \sigma \rangle \\ + \frac{H}{2\lambda} \left(1 + \frac{g_0 \bar{g}_2}{m_\ell^2} \right) \simeq 0. \end{aligned} \quad (15)$$

Notice that in the chiral limit, i.e. for $H = 0$, we recover the results from Ref. [27]. One can now use (14) and (15) to extract the couplings from lattice results for the renormalized Polyakov loop and the chiral condensate in the presence of massive quarks. In the following we discuss how that can be implemented in a general fashion [37].

III. CONNECTING TO LATTICE DATA

Our aim is to use the above derived equations to describe the corresponding lattice data for the chiral condensate [32] and the Polyakov loop [33] for three degenerate flavors, generated under the same lattice conditions. We can apply the same procedure for the 2+1 flavor most recent lattice data [34–36]. In this case we also study separately the light and strange quark condensates [37].

In order to compare our mean-field results to lattice data, we use the Gell-Mann–Oakes–Renner (GOR) relation

$$m_q \langle \bar{\psi} \psi \rangle = -f_\pi m_\pi^2 \langle \sigma \rangle, \quad (16)$$

which in the vacuum yields $m_q \langle \bar{\psi} \psi \rangle_0 = -f_\pi^2 m_\pi^2$, and define the dimensionless quantity that is measured on the lattice $x \equiv \langle \bar{\psi} \psi \rangle / \langle \bar{\psi} \psi \rangle_0$, where we take $\langle \bar{\psi} \psi \rangle_0 = -2(225 \text{ MeV})^3$. The mass of the sigma meson is given by

$$m_\sigma^2(T, m_q) = m_\pi^2(m_q) + 2\lambda \langle \sigma \rangle^2(T). \quad (17)$$

Furthermore, as customary in a Ginzburg-Landau theory, we assume

$$m_\sigma^2(T, m_q = 0) = aT_c^2 \left(1 - \frac{T}{T_c} \right), \quad (18)$$

below the transition, where a is a dimensionless constant. This is characteristic of a second-order phase transition. For nonzero quark mass the transition is a crossover. We then consider that the temperature and quark mass dependence of the sigma mass are well described by the following ansatz

$$m_\sigma^2(T, m_q) = m_\pi^2(m_q) + aT_c(m_q)^2 \left(1 - \frac{T}{T_c(m_q)} \right). \quad (19)$$

To describe the temperature-dependence of the Polyakov loop mass, a fit to pure SU(3) Yang-Mills results from the lattice [42] was performed in [41]. Accordingly:

$$m_\ell^2(T) = \left(\frac{s(T)}{T} \right)^2, \quad (20)$$

where the temperature-dependent string tension is given by $s(T) = 1.21 \sqrt{s_0^2 - 0.99T^2 \sigma_0 / (0.41)^2}$, with the zero-temperature string tension $s_0 = (440 \text{ MeV})^2$. This parametrization, however, is not valid in full QCD. Therefore we keep the Polyakov loop mass as a free parameter.

Given the conditions above, we can rewrite (15) in a more convenient form:

$$\begin{aligned} x^3 + C_2 \left[\frac{g_1 \bar{g}_2}{m_\ell^2(T)} \right] m_q x^2 - C_1 m_\sigma^2(T, m_q) x \\ + C_0 \left[1 + \frac{g_0 \bar{g}_2}{m_\ell^2(T)} \right] m_q = 0, \end{aligned} \quad (21)$$

where we have defined the following constants: $C_2 \equiv -\langle \bar{\psi} \psi \rangle_0 / 2\lambda f_\pi^2 \simeq 33 \text{ MeV}$, $C_1 \equiv 1/2\lambda f_\pi^2 \simeq 2.9 \times 10^{-6} \text{ MeV}^{-2}$ and $C_0 \equiv -\langle \bar{\psi} \psi \rangle_0 / 2\lambda f_\pi^4 \simeq 3.8 \times 10^{-3} \text{ MeV}^{-1}$.

We can also define the dimensionless quantity

$$y \equiv \frac{\langle \ell \rangle}{(-g_0/m_\ell^2(T))} = 1 - \frac{g_1}{(-g_0)} \langle \sigma \rangle^2 - \frac{\bar{g}_2}{(-g_0)} H(\sigma), \quad (22)$$

to be compared with lattice results for the renormalized Polyakov loop as a function of temperature for different quark masses [33]. Notice that y has the expected behavior in the limit of very high temperatures. In terms of the dimensionless chiral ratio x and the quark mass m_q :

$$y = 1 - \frac{g_1}{(-g_0)} f_\pi^2 x^2 + \frac{\bar{g}_2}{(-g_0)} \langle \bar{\Psi}\Psi \rangle_0 m_q x. \quad (23)$$

The functions $x = x(T)$ and $y = y(T)$ for a few values of m_q , and also the corresponding values of T_c , were computed on the lattice [32, 33]. By fitting equations (21) and (23), one can extract from the data the couplings a , g_0 , g_1 and \bar{g}_2 , to be used in the effective field theory. Our preliminary results indicate that the sign of the coupling g_1 complies with the expectation of [27], assuring that the chiral and the deconfinement transitions coincide. Detailed results with further conclusions are deferred to our upcoming publication [37].

IV. SUMMARY AND OUTLOOK

In this paper we have sketched a systematic method to construct a phenomenological generalized Ginzburg-Landau effective theory describing simultaneously the processes of chiral symmetry restoration and deconfinement in the presence of massive quarks. We devoted special attention to the behavior of the quasi order parameters σ and ℓ with temperature, which can be connected to lattice data. The latter, on the other hand, can be used to provide constraints on the couplings of the effective theory. A detailed analysis of the method, as well as the extraction of the range of physical values of the couplings will be presented elsewhere [37].

The effective theory presented in this paper is very simple. For instance, if one studies the renormalization of Polyakov loops, one is naturally lead to consider effective *matrix* models for the deconfinement transition, which unfolds a rich set of possibilities, especially at large N [43]. In particular, eigenvalue repulsion from the Vandermonde determinant in the measure seems to play a key role [44]. These studies pointed out however, that in the neighborhood of the transition the relevant quantity is the trace of the Polyakov loop, underlying the relevance of our effective theory in this region. The simplicity of our model allows for a direct comparison to lattice data in a way that provides definite constraints on the couplings, after which one can test and predict semi-quantitatively the behavior of the approximate order parameters in different settings.

The full determination of the couplings of an effective model linking confinement and chiral symmetry breaking using reliable lattice data is useful for the study of the phase diagram for strong interactions, and can bring understanding to some of the open questions considered in the introduction. Furthermore, the effective potential derived from this model can be used in the study of the real-time dynamics of the QCD phase transitions, including the effects from dissipation and noise that result from self and mutual interaction of the fields related to the chiral condensate and the Polyakov loop [45]. Results from a real-time study within our effective model will be reported in the future [46].

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