Braneworld with Induced Axial Symmetry

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We take arbitrary gravitational perturbations of a 5d spacetime and reduce it to the form an axially symmetric warped braneworld. Then, we write the filed equations for the linearized gravity perturbations. We obtain the equations that describes the graviton, gravivector and the graviscalar fluctuations and analyse the effects of the Schrödinger potentials that appear in these equations.

I. INTRODUCTION

In the almost all works on the Randall-Sundrum (RS) braneworlds [1], the axial gauge is used to derive the linearized gravity dynamics. In this gauge there are no fluctuations transverse to the brane, and the scenario is axially symmetric. An other important gauge is the harmonic (de Donder) gauge for which, in 5D, the h_{55}-graviscalar and the $h_{5\mu}$-gravivector 4D fluctuations can be non zero, breaking the axial symmetry. However, as pointed out in [2], a new coordinate frame in 5D can be found, where the metric becomes axially symmetric even with $h_{55}, h_{5\mu} \neq 0$. By following [2], we call such coordinate frame the *local frame*. Here, we analyse the field equations for the vacuum fluctuations that arise in the local frame, using the 5D de Donder gauge.

II. AXIALLY SYMMETRIC BRANEWORLD

The 5D metric is expanded as $g_{AB} = \eta_{AB} + h_{AB}$ where $A, B = 0, 1, 2, 3, 5$, $\eta_{AB} = diag(-1, 1, 1, 1, 1)$ and h_{AB} small gravity fluctuations. The 5D line element is

$$dS^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + 2h_{5\mu}dx^\mu dx^5 + (1 + h_{55})(dx^5)^2. \tag{1}$$

The above spacetime take the axially symmetric form

$$ds^2 = g_{(\mu)(\nu)}dx^{(\mu)}dx^{(\nu)} + \varepsilon^2 dy^2 \tag{2}$$

where $x^{(A)} = e_B^{(A)} x^B$, with the $e_B^{(A)}$ given by

$$e_A^{(\mu)} = \delta_A^\mu, \quad e_{(\nu)}^\mu = \delta_\nu^\mu, \quad e_{(5)}^\mu = 0, \quad e_\mu^{(5)} = \varepsilon^{-1}P_\mu, \tag{3}$$

$$e_{(\mu)}^5 = -P_\mu, \quad e_5^{(5)} = \frac{1 + \varphi/2}{\varepsilon}, \quad e_{(5)}^5 = \frac{1 - \varphi/2}{\varepsilon}, \tag{4}$$

where $P_\mu = h_{5\mu}$, $\varphi = h_{55}$, $y = x^{(5)}$ and $\varepsilon^2 = -1, 1$. The physical 4D metric can be given by $g_{(\mu)(\nu)}$, or $\hat{g}_{(\mu)(\nu)} = e^{-2f}g_{(\mu)(\nu)}$. If $f = f(y)$, we have

$$ds^2 = e^{-2f(y)}g_{(\mu)(\nu)}dx^{(\mu)}dx^{(\nu)} + \varepsilon^2 dy^2. \tag{5}$$

Then, we assume that (5) satisfies the action

$$S = \int d^5x[\sqrt{-g}(\kappa^{-2}R + \Lambda_5 + \mathcal{L}_m) + \sqrt{-g_b}\sigma], \tag{6}$$

where $g = \det(g_{(A)(B)})$, $g_b = det(g_{(\mu)(\nu)})$ and $\kappa = M_*^{-3}$. The vacuum solution gives $f'(y)^2 = -\kappa^2\Lambda_5(y)/12$ and $f''(y) = \kappa^2\sigma(y)/12$.

III. LOCAL FRAME GRAVITATIONAL FLUCTUATIONS

We work in the conformal frame where

$$ds^2 = e^{-f(z)}[g_{(\mu)(\nu)}dx^{(\mu)}dx^{(\nu)} + \varepsilon^2 dz^2]. \tag{7}$$

The equations for the gravitational fluctuations are derived with $g_{(\mu)(\nu)} = e_{(\mu)}^A e_{(\nu)}^B g_{AB}$ and, $\bar{\partial}_{(A)}(...) = e_{(A)}^B \partial_B(...)$, with $[\bar{\partial}_{(A)}, \bar{\partial}_{(B)}] \neq 0$. The h_{AB} satisfies $\partial^B h_{AB} = 0$, $h_C^C = 0$, which means that

$$h_\alpha^\alpha = -\varphi, \quad \partial^\alpha P_\alpha = -\varphi', \quad \partial^\alpha h_{\mu\alpha} = -P'_\mu. \tag{8}$$

The the "prime" represents ∂_z. The field equations are

$$\Box\varphi + 12f'^2\varphi + 3f'\varphi' = -\kappa T_{55}^{(m)}, \tag{9}$$

$$\Box P_\mu + 6(f'^2 + f'')P_\mu + 3f'\partial_\mu\varphi - 2[\bar{\partial}_{(\mu)}, \bar{\partial}_{(5)}]f = -\kappa T_{\mu5}^{(m)}, \tag{10}$$

$$\Box h_{\mu\nu} + 3f'(\partial_\mu P_\nu + \partial_\nu P_\mu) - 3f'h'_{\mu\nu} + 12(f'^2 - f'')\varphi\eta_{\mu\nu}$$
$$- 2[\bar{\partial}_{(\mu)}, \bar{\partial}_{(\nu)}]f = -\kappa T_{\mu\nu}^{(m)}, \tag{11}$$

where $\Box = \eta^{AB}\partial_A\partial_B$. The local frame equations depends on the comutator of partial derivatives of the warp function. The equation for the scalar do not changes [3].

Extended KK-gravity: $\Lambda_5 = \sigma = 0$. The system (9)-(11) decouples to

$$\Box\phi = -\kappa T_{55}^{(m)}, \quad \Box P_\mu = -\kappa T_{5\mu}^{(m)}, \tag{12}$$

$$\Box h_{\mu\nu} = -\kappa T_{\mu\nu}^{(m)}. \tag{13}$$

The scenario is an extended Kaluza-Klein gravity with a no compact extra dimension. The gauge conditions enable us to write the 4D tensor $h_{\mu\nu}$ in terms of spin-2, spin-1 and spin-0 fluctuations. The vacuum is flat.

IV. WARPED GEOMETRY FLUCTUATIONS

Graviscalar. Consider $T_{55}^{(m)} = 0$. The φ is re-scaled to $\varphi(x, z) = e^{-3f(z)/2}\tilde{\varphi}(x, z)$ and we look for solutions $\tilde{\varphi}(x, z) = \tilde{\varphi}(x)\psi_s(z)$, with $\partial^\alpha\partial_\alpha\tilde{\varphi}(x) = m_s^2\tilde{\varphi}(x)$. Then, (9) implies

$$[-\partial_z^2 + V_s(z)]\psi_s(z) = m_s^2\psi_s(z), \tag{14}$$

with $V_s(z) = -(\frac{3}{2}f'' - \frac{39}{4}f'^2)$. The vacuum solutions for φ are of no interest, because the compatibility condition between (9) and (10) force us to set $\varphi = 0$ [3].

Gravivector. With $\varphi = 0$, eq. (10) in vacuum gives

$$[-\partial_z^2 + V_v(z)]\psi_v(z) = m_v^2\psi_v(z), \qquad (15)$$

where $V_v(z) = -5(f'^2 + f'')$, for ψ_v defined by $\tilde{P}_\mu(x,z) = \tilde{P}(x)\psi_v(z)$, $\partial^\alpha\partial_\alpha\tilde{P}_\mu(x) = m_v^2\tilde{P}_\mu(x)$, and $\tilde{P}_\mu(x,z) = e^{f(z)}P_\mu(x,z)$. For RS warp, the potential is $V_v(z) = -10k\delta(z)$. Then, we have masive solutions with $m_v^2 = -25k^2$, whereas $m_v^2 = -36k^2$ in the coordinate frame [3].

The compatibility condition between (10) and (11) is

$$[-26(f'^3 + f'f'') + 6f''']\tilde{P}_\mu + (8f'' - 4f'^2)\tilde{P}'_\mu = 0. \qquad (16)$$

If $f = \text{Log}(k|z| + 1)$, we have

$$\left(-10k\delta(z)\text{sgn}(z) + 3k\delta'(z) + 3\frac{k^3\text{sgn}(z)}{(k|z|+1)^3} \right)\psi_v(z)$$
$$+ \left(4k\delta(z) - 3\frac{k^2}{(k|z|+1)^2} \right)\psi'_v(z) = 0. \qquad (17)$$

For $|z| \geq 0$, (17) is satisfied by $\psi_v = 25a_v(k|z|+1)$, where a_v is a constant. At $z = 0$, (17) implies, $a_v = 0$. With the smooth warp, $f(z) = \text{Log}(k^2z^2 + 1)$, eq. (16) becomes

$$\left[\frac{6k^5z^3}{(1+k^2z^2)^2} - \frac{11k^3z}{1+k^2z^2} \right]\psi_v - \left[\frac{3k^3z^2}{1+k^2z^2} - 1 \right]\psi'_v = 0. \qquad (18)$$

At $z = 0$, we have $\psi'_v(0) = 0$. A solution of (15) that satisfy (18), is the massive mode described by

$$\psi_v = c_v\frac{e^{\frac{13}{4}\tanh^{-1}(1/3 + 4k^2z^2/3)}}{(-1 + k^2z^2 + 2k^4z^4)^{5/8}}, \qquad (19)$$

with mass $m_v^2 = -3k^2\left(7 + 25k^2z^2\right)/\left(1 - 2k^2z^2\right)^2$.

The Fig. 1 shows the variation of m_v^2 with the extra coordinate. It is almost constant for small z and diverges for $z \to \pm z^* = \pm 1/(\sqrt{2}k)$. On the $|z| = 0$ 3-brane, $\tilde{P}_\mu(x) = c_\mu e^{\frac{13}{4}\tanh^{-1}(1/3)}e^{ip\alpha x^\alpha}e^{-i\frac{5\pi}{8}}$, where $p^2 = m_v^2(0) = -21k^2$ and, $p^\mu c_\mu = 0$.

Graviton. To obtain the graviton potential we take $c_\mu = 0$ and $T_{\mu\nu}^{(m)} = 0$. Then, the eq. (11) implies

$$[-\partial_z^2 + V_g(z)]\psi_g(z) = m_g^2\psi_g(z), \qquad (20)$$

where $V_g = -(\frac{3}{2}f'^2 - \frac{9}{4}f'')$. This potential reproduces the RSII result for $f(z) = \text{Log}(k|z| + 1)$.

V. CONCLUSIONS

In the no warped scenario, the vacuum fluctuations are described by three independent wave equations which describes the 4D scalar, vector and tensor fluctuations on the $|z|=0$ 3-brane. In warped scenarios, there are no scalar propagation on
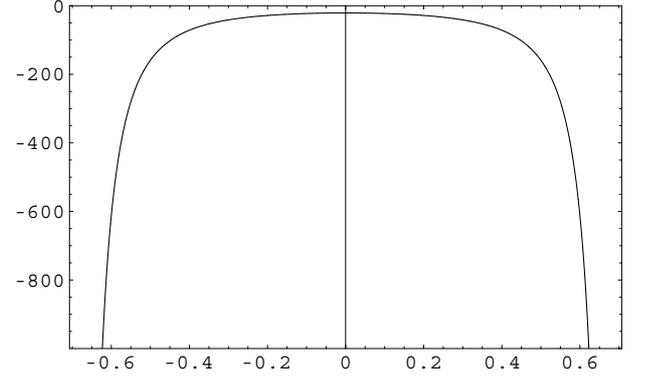

FIG. 1: The squared tachyonic mass, $m_v^2(z)$, in the vertical axis. The horizontal axis is $z^* \leq z \leq z^*$ and $k = 1$.

the 3-brane vacuum. For the RS warp, there are no gravivector on the 3-brane. For the smoothed warped braneworld, we obtain a tachyonic mass solution for the gravivector, that also satisfies the compatibility condition. This solution becomes a massless spin-1 fluctuation if $\Lambda_5 \to 0$.

[1] L. Randall and R. Sundrum, Rev. **83**, 4690 (1999) [hep-th/9906064]; Ibd. 83, 3370 (1999) [hep-ph/9905221].
[2] J. Ponce de Leon, Grav. Cosmol. **8** (2002) 272-284. [gr-qc/0104008]; Int. J. Mod. Phys. D**11**, 1355 (2002) [gr-qc/0105120].
[3] Y. S. Myung, [hep-th/0010208].