Stochastic Dynamics of 2D Electrons in Antidot Lattice in the Presence of an in–Plane Magnetic Field

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The classical dynamics of the two–dimensional electron gas in antidot lattices is studied in the presence of an in–plane magnetic field. We find that the distortion of the Fermi contour induced by the in–plane magnetic field changes the Poincaré section of the electron trajectories in the magnetic field. Comparison with the experimental measurements of the commensurability magnetoresistance oscillations is made.

Progress in lithography has made possible to create devices with size smaller than the ballistic mean free path. Such devices, called electron billiards, allow to study ballistic regime, where it is mainly governed by the shape of the boundary of the sample. For certain irregular shapes of the billiards the motion of electrons becomes completely unpredictable after first few collisions, which means that the chaos develops in the phase space. Within the last few years it has been shown that the two–dimensional array of antidots is a model system, that allows to study the chaotic classical dynamics in condensed matter physics [1]. The general interest in these phenomena is a quantum manifestation of the classical chaos which could create the link between the so-called quantum and classical chaos. In a periodical array of antidots the commensurability peaks in magnetoresistance oscillations have been already explained from classical and quantum points of view [2]. Recently, it has been predicted that the Fermi contour of the two–dimensional electron gas can be distorted by the in-plane magnetic field [3]. In this work, the sensitivity of the electron chaotic dynamics in antidots lattice to the Fermi contour distortion is studied theoretically and experimentally.

The dynamics of a two–dimensional electron gas (2DEG) in periodic antidot array and perpendicular magnetic field is described by the Hamiltonian

$$H = 1/2m(p_x + eBy/2)^2 + 1/2m(p_x - eBx/2)^2 + U(x,y) \quad (1)$$

The antidot lattice can be described by the following model potential

$$U(x,y) = U_0\{\cos(2\pi x/d)\cos(2\pi y/d))^n\} \quad (2)$$

where n controls the steepness of the antidots, d–lattice periodicity. We consider steep potential with $n = 20$. We resolve the equation of motion for Hamiltonian (1) and find electron trajectories for different values of magnetic field. To understand the dynamic of nonlinear systems it is very useful to investigate the motion in phase space by means of Poincaré sections. We calculate Poincaré sections (P_x, x) at $y = 0$. Figures 1a and 2b show 2 Poincaré sections for different values of magnetic field. The ballistic orbit of the electron is bent with the magnetic field and focused into a collecting point. Therefore the geometric resonance in transport phenomena is observed at a magnetic field where an integer multiple of the cyclotron diameter coincides with the periodicity of antidot lattice. Fig.1a and Fig.1b show Poincaré sections for magnetic field, when cyclotron diameter $2R_c$ $((R_c = h/eB(2n_s)^{1/2})$, n_s is electron density) equals d. We see that for these conditions well defined quasiperiodic orbits appear (island), characterized by a cycle in the phase space, surrounding by the chaotic component. These quasiperiodic orbits are responsible for the magnetoresistance peaks. These peaks were interpreted on the basis of pinned classical orbits in a "pinnball model" [1]. In accordance with this model electrons are pinned in orbits around (or between) groups of antidots for a long time and, therefore, do not contribute to the conductivity. Other unperturbated cyclotron orbits are represented by closed loops in the island. An alternative interpretation for the fundamental peaks occurring at magnetic field was proposed in [2], where the peak is due to an enhancement of the diffusion coefficient produced by "running" orbits, which skips regularly from an antidot to the other antidot. These orbits appears in Poincaré sections at lower magnetic field close to the border $x = 7 * 10^{-}7$ m. For low value of magnetic field the region in phase space covered by these quasiperiodic orbits (pinned) is comparable with the size of chaotic orbits, and as B is decreased, only chaotic part remains.

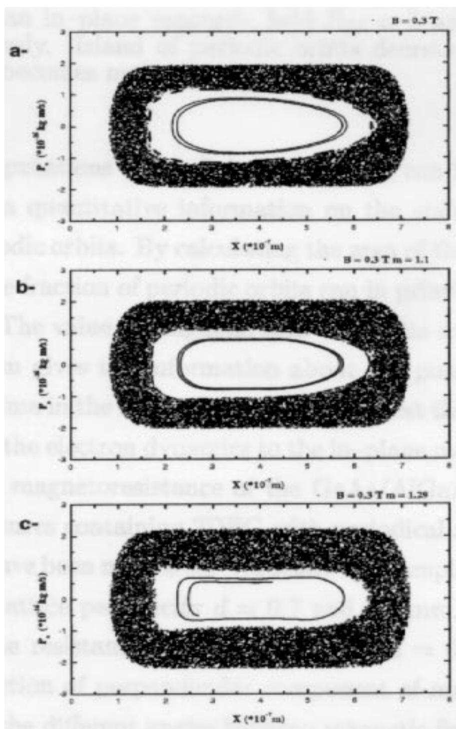

Figure 1. a- Poincaré section for perpendicular magnetic field. Periodic and quasiperiodic orbits are surrounded by a chaotic sea b-, c- Poincaré sections calculated for tilted magnetic fields.

Let us consider now the effect of the in–plane magnetic field, B_{II} on a 2DEG confined in z direction. Assuming for the sake of the simplicity the harmonic confining potential with characteristic energy $h\Omega$. In this case, the electron Hamiltonian has the following form

$$H = 1/2m(p_x/m+\omega_{II}z)^2+p_y^2/2m+p_z^2/2m+m\Omega^2z^2/2m \quad (3)$$

where $w_{II} = eB_{II}/m$ is a cyclotron frequency. The energy dispersion of the electron subband in this case is given as follows

$$E(k) = h^2k_y^2/2m + h^2k_x^2/2m^* \quad (4)$$

where $m^* = m(1 + w_{II}^2/\Omega^2)$ represents the increase of the effective mass induced by the in–plane magnetic field B_{II}. From equation (4) we can see that the Fermi contour has the egg–like shape, instead of circle isotropic shape in the absence of in–plane magnetic field [3]. Such anisotropy of the Fermi contour induce distortion of the electron trajectories and, consequently electron dynamics in antidot lattice. To test the sensitivity of the electron dynamics to the Fermi contour distortion we compute the Poincaré section considering the anisotropy of the effective mass. We substitute into the equation of motion the value of new effective mass calculated in [4]. Figures 1b, 1c, 2b and 2c show Poincaré sections for the same value of the perpendicular B, as in the absence of an in–plane magnetic field. We see that the Poincaré section for the geometric resonance condition is dramatically changed when parallel magnetic field is applied. It is because the stability of the regular orbits are sensitive to the geometry of the experiment. Anisotropy of the effective mass induces anisotropy of the cyclotron diameter and changes resonance conditions in x direction. It means that pinned cyclotron orbits with different initial conditions are absent. Therefore, the chaotic dynamic in the antidot lattice can be used for the study the small change in the shape of the electron Fermi contour under the parallel magnetic field, pressure and other external factors.

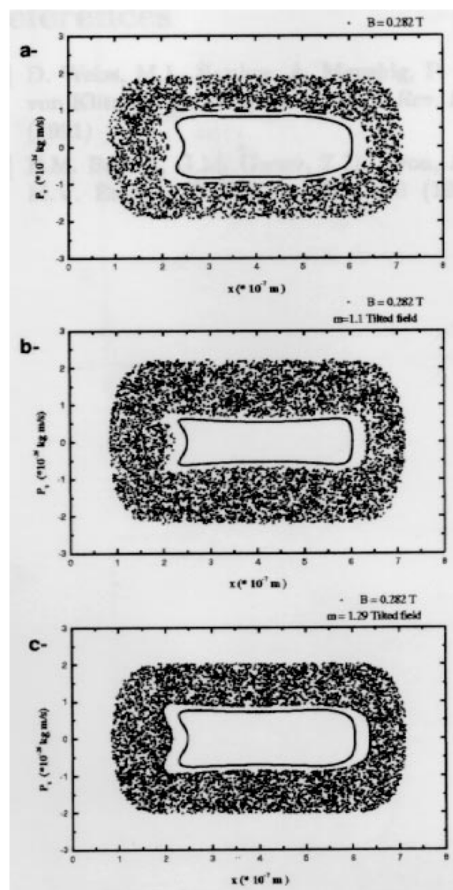

Figure 2. a- Poincaré section for the resonance condition ($d = 2R_c$) b- and c- Poincaré section in the presence of an in–plane magnetic field $B_{II} = 5$ and 10 T respectively. Island of periodic orbits decreases, and motion becomes more chaotic.

Computations of the Poincaré sections can be used to give a quantitative information on the stability of the periodic orbits. By calculating the area of the phase space the fraction of periodic orbits can in principle obtained. The value of magnetic field when this area as a maximum gives the information about the position of the maxima in the magnetoresistance. To test the sensitivity of the electron dynamics to the in–plane magnetic field the magnetoresistance of the GaAs/AlGaAs heterostructures containing 2DEG with periodical antidot lattice have been measured. We study two samples with antidot lattice periodicity $d = 0.7$ and 0.8 mm. Fig.3 shows the resistance of the sample with $d = 0.8$ mm as a function of perpendicular component of magnetic field for the different angles between magnetic field and normal to the surface. At low magnetic field we can see magnetoresistance commensurability peaks. When the magnetic field is tilted away from the vector normal to the substrate, the commensurability peak at high field starts to smear out. At higher magnetic field we observe the negative magnetoresistance, which is assumed to be due to the formation of the rosette–like orbits skipping around antidots. These orbits are localized and do not contribute to the conductivity. We find that the negative magnetoresistance increases in the tilted magnetic field. We suggest that both effects are due to the distortion of the Fermi contour by the parallel magnetic field. From the comparison of the Poincaré sections for $B_{II} = 0$ and $B_{II} = 10$ T we find that closed loops, which are due to the formation of the pinned orbits with different initial conditions, are destroyed in the presence of an in–plane magnetic field. Therefore we can assume that the dynamics of these orbits can be responsible for the negative magnetoresistance in antidot lattice in magnetic fields when $2R_c d$. However for further understanding of the chaotic dynamics in antidot lattice and of the role of the different type of trajectories, numerical calculations of the diffusion coefficient and velocity correlation are necessary.

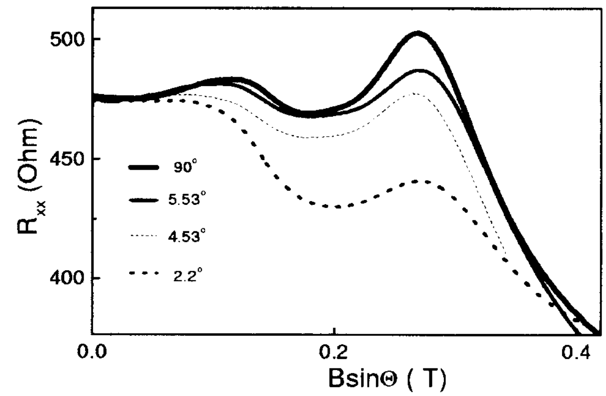

Figure 3. Resistance of the sample with antidot lattice as a function of the perpendicular magnetic field for different tilt angles.

In conclusion we demonstrate theoretically and experimentally that the chaotic dynamics of electrons in antidot lattices is very sensitive to the distortion of the Fermi contour. Therefore, the magnetoresistance of the 2DEG in the antidot lattice can be used for the study of such phenomena as the topological Lifshits transition and small changes in the shape of the electron Fermi contour under the pressure.

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