Non Gravitational Black Holes

V. A. De Lorenci and R. Klippert
Instituto de Ciências Exatas, Universidade Federal de Itajubá,
Av. BPS 1303 Pinheirinho, 37500-903, Itajubá, MG, Brazil

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Analogue models of general relativity have recently been considered with great interest by the scientific community. They connect branches of physics as different as gravitation, condensed matter physics, electrodynamics, acoustics, and quantum field theory. One of the main expectations about such models lies on the possibility of testing in laboratory some aspects of quantum field theory in curved spacetimes. For instance, it seems to be possible to probe the existence of Hawking radiation by means of analogue models in the context of certain condensed matter systems. We briefly report the present day status of this topic of research. Some specific models are considered, particularly those presenting analogue event horizons as solutions. The issue of thermal emission (analogue Hawking radiation) is also discussed.

1 Introduction

The general relativity theory predicts that a bounded region in space from which nothing can scape appears as an external solution for the gravitational field associated with a spherically symmetric configuration for matter. Such exotic object is known as a black hole and it is delimited by a one-way surface called event horizon. However, semiclassical analysis revels that black holes are expect to emit thermal radiation with a Planckian spectrum with temperature $T_H = \hbar c^3/8\pi k_B GM$ [1]. The energy radiated in this process comes from the mass M of the black hole which is thus continuously diminished by a small amount. In other words, black holes evaporate. For a black hole with mass $M = \gamma M_{\odot}$ (where $M_{\odot}$ denotes the solar mass) the Hawking temperature will be

$$T_H \approx \frac{6\times 10^{-8}}{\gamma} K, \tag{1}$$

a temperature which is rather low and unlikely to be measured from astrophysical events ($\gamma \geq 1$).

In 1981 Unruh [2] proposed a hydrodynamical model in which the kinematic properties of a back hole could in principle be verified under laboratory conditions. Since then considerable efforts have been performed in the construction of alternative models where the kinematic properties of gravitation could be reproduced in laboratory. Such models are usually known as analogue models for general relativity [3]. The first work presenting an analogue model was published by Gordon [4] in 1923 where a dielectric medium was described by means of an effective geometry. Before 1981 there were also some other related works, mainly those based on electrodynamics (see Refs. [5-7] for a revision).

Analogue models have been constructed in several branches of physics. It deals with acoustics [2,8-18], optics [19-27], among others [28-35]. These are only representative references.

The experimental construction of analogue black holes represents the most interesting topic in the broad area of analogue gravity, but the effective built-up of such structures is still an open problem. Black holes are considered to be fundamental in the understanding of quantum gravity phenomena, and the examination of analogue models can throw some light on its kinematic aspects (including the issue of thermal emission of radiation near the event horizon). No fundamental dynamics (as Einstein's equations) is expect to apply for analogue geometries, however. Thus, dynamical aspects of gravitation theory (like the entropy of a black hole) are unlikely to be found in analogue models.

We work in Minkowski spacetime employing a Cartesian coordinate system. The background metric is denoted by $\eta_{\mu\nu}$, which is defined by $diag(+1,-1,-1,-1)$. Units are such that $c = 1$.

2 Analogue models from electrodynamics in material media

Inside material media, Maxwell's equations must be supplemented by constitutive laws that relate the electromagnetic excitations and the field strengths by means of quantities characterizing each medium the waves propagate into. In this context, effective geometries were found to describe the modes both for the linear [7, 36, 37] as well as non-linear [19, 20] constitutive relations. The results presented in these references lead us to conclude that electromagnetic waves usually propagate inside material media as if immersed in a curved spacetime. This fact allows one to make an analogy between wave propagation in material media and gravitational phenomena.

2.1 Field equations

The electromagnetic properties of a moving medium are determined by the tensors $\varepsilon^{\alpha}{}_{\beta}$ and $\mu^{\alpha}{}_{\beta}$ which relate the electromagnetic excitation (D^{μ}, B^{μ}) and the field strength (E^{μ}, H^{μ}) by means of the generalized constitutive laws [38]

$$D^{\alpha} = \eta^{\alpha\beta\mu\nu}V_{\beta}u_{\mu}H_{\nu} + \varepsilon^{\alpha}{}_{\beta}(E^{\beta} - \eta^{\beta\lambda\mu\nu}V_{\lambda}u_{\mu}B_{\nu}) \quad (2)$$

$$B^{\alpha} = -\eta^{\alpha\beta\mu\nu}V_{\beta}u_{\mu}E_{\nu} + \mu^{\alpha}{}_{\beta}(H^{\beta} + \eta^{\beta\lambda\mu\nu}V_{\lambda}u_{\mu}D_{\nu}), \quad (3)$$

where V^{μ} represents the velocity 4-vector of an arbitrary observer and u^{μ} the velocity 4-vector of the medium. The Levi-Civita tensor $\eta^{\beta\lambda\mu\nu}$ is defined such that $\eta^{0123} = +1$.

In the absence of free sources, Maxwell's theory can be summarized by the equations

$$\left(V^{\mu}D^{\nu} - V^{\nu}D^{\mu} - \eta^{\mu\nu}{}_{\alpha\beta}V^{\alpha}H^{\beta}\right)_{,\nu} = 0 \quad (4)$$

$$\left(V^{\mu}B^{\nu} - V^{\nu}B^{\mu} + \eta^{\mu\nu}{}_{\alpha\beta}V^{\alpha}E^{\beta}\right)_{,\nu} = 0, \quad (5)$$

together with the constitutive laws (2) and (3).

2.2 Geometrical optics

In order to determine the propagation of waves associated to the electromagnetic field, we consider the eikonal approximation of electrodynamics, making use of the Hadamard method of field discontinuities [39]. We denote the surface where the dynamical equations may present discontinuities by $\mathcal{Z}(t, \vec{x}) = 0$. This surface, except from its border (if any) is denoted by Σ. An arbitrary point $P \in \Sigma$ thus has a neighborhood U which is cut by Σ in three disjoint regions $U = U^{+} \cup U^{-} \cup U^{o}$, with $U^{o} = U \cap \Sigma$, where $U^{\pm}$ respectively contains points $P^{\pm}$ such that $\mathcal{Z}(P^{+}) > 0$ and $\mathcal{Z}(P^{-}) < 0$. We suppose Σ to be an orientable surface, from which the regions $U^{\pm}$ are defined in a globally consistent way. The discontinuity of an arbitrary function $f(t, \vec{x})$ at $P \in \Sigma$ is given by

$$[f(t, \vec{x})]_{\Sigma} := \lim_{\{P^{\pm}\}\to P} \left[f(P^{+}) - f(P^{-})\right]. \quad (6)$$

The electric and magnetic fields derivative are continuous across Σ. Since Maxwell's equations (4)-(5) are of first order, however, the derivatives behave as

$$[E^{\mu}{}_{,\nu}]_{\Sigma} = e^{\mu}K_{\nu}; \quad [H^{\mu}{}_{,\nu}]_{\Sigma} = h^{\mu}K_{\nu} \quad (7)$$

where e^{μ} and h^{μ} represent the discontinuities of the fields across Σ and $K_{\lambda} = \partial\Sigma/\partial x^{\lambda}$ denotes the wave vector. Applying these conditions to the field equations, one obtains the eigen-vector equation

$$Z^{\alpha}{}_{\beta}e^{\beta} = 0, \quad (8)$$

where $Z^{\alpha}{}_{\beta}$ lies in the rest space of the observer V^{μ} (*i.e.*, its 'time' components are zero) and each of its components is quadratic in K_{λ}. The generalized Fresnel equation represents non-trivial solutions of this equation and is given by $\det|Z^{\alpha}{}_{\beta}| = 0$, whose solutions can be written in the form

$$g_{\pm}^{\mu\nu}K_{\mu}K_{\nu} = 0, \quad (9)$$

which is known as the dispersion relation. The $g_{\pm}^{\mu\nu}$ represent the effective geometries associated with the wave propagation, and the symbol $\pm$ indicates that, in general, there will be two possible distinct metrics, one for each polarization mode. For the particular case of electromagnetic waves in vacuum both $g_{\pm}^{\mu\nu}$ reduce to $\eta^{\mu\nu}$, as expected.

2.3 Optical horizon

Let us consider the flow of a dielectric material in a spherically symmetric configuration, with 4-velocity u^{μ} given in spherical coordinates as

$$u^{\mu} = \gamma(1, b, 0, 0), \quad (10)$$

where $\gamma := (1 - b^2)^{-1/2}$ and $F^{01} = E_r$, with $E_r = E_r(r)$ and $b = b(r)$. It can be shown [40] that the radial coordinate of the photon changes with the time coordinate as

$$\frac{dr}{dt} = \frac{nb \pm 1}{n \pm b}, \quad (11)$$

where $n = (\epsilon\mu)^{-1/2}$, which coincides with the refraction index of the medium in the linear limit. By considering matter flowing in-wards ($b < 0$), out-going light apparently freezes ($dr/dt = 0$) at the radius r_{H} such that

$$nb|_{r_H} = -1. \quad (12)$$

Thus r_{H} is identified with the radius of an analogue event horizon (the optical horizon), and the solution will represent an optical black hole [23, 40].

From the above analysis, which agrees with previous calculations [23], we find that either an ultra-relativistic motion of the matter or a highly refringent medium [41, 42], as it is found in Bose-Einstein condensates (see, however, a relevant discussion in Refs. [31, 22]), is needed in order to display such a horizon structure. Highly refringent media are usually quite dispersive, and thus the effective horizon would occur in this case only for a narrow range of frequencies. A practical realization of the analogue black hole configuration requires $b = b(r)$ and possibly $n = n(r)$ also.

Recently, it has been proposed that Hawking temperature is a purely kinematic effect that is generic to Lorentzian geometries containing event horizons [43], and thus being dependent only on the effective metric structure (see however Ref. [22]). For the spherically symmetric solution the Hawking temperature T_H of the analogue black hole would be given by [40]

$$T_H = \left|\frac{\hbar\gamma^2 bn}{2\pi k_B}\left(\frac{\partial b}{\partial r} + \frac{b\epsilon'}{2\epsilon E}\frac{\partial E}{\partial r}\right)\right|_{r_h}, \quad (13)$$

where $\epsilon' = E^{\alpha}\partial\epsilon/\partial E^{\alpha}$. Equation (13) shows that the nonlinearity may significantly contribute to this temperature. It

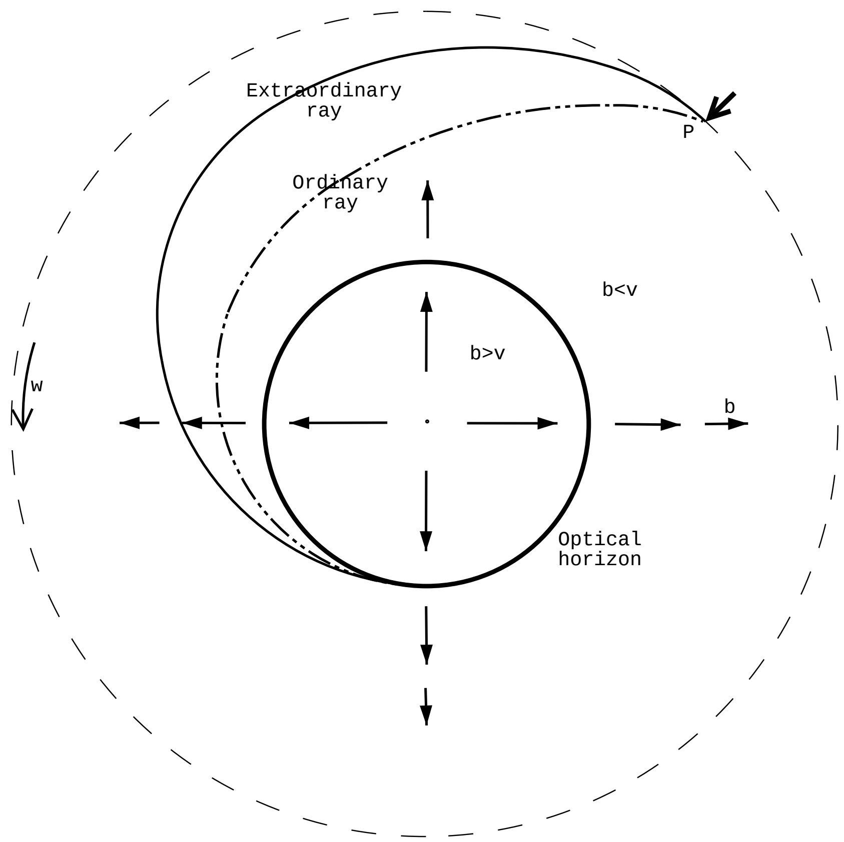

Figure 1. The in-coming light path for both polarization modes are shown for the cylindrical configuration. We assume in the plot a rigid rotation with $w = 10^{-4}$, and the refraction index is $n = 10^3$. The figure corresponds to a dielectric fluid flowing out-wards, with radial velocity $b(\rho) = 10^{-2}/\rho$, and $\mu\epsilon' = 10^{10}$. The unpolarized light ray enters the rotating condenser region (dashed lines) at the point P. The solid line depicts the extraordinary ray, whereas the dotted line represents the ordinary ray. The thicker line corresponds to the optical horizon, and v denotes the light velocity with respect to the medium.

should also be noted, however, that the complete understanding of the physical meaning of the Hawking temperature associated with an analogue black hole is still an open question [31, 22], and it can be settled only after a detailed analysis of the radiation processes at the horizon. It should be stressed that the approximation of geometrical optics becomes unreliable for the modes whose wavelengths are comparable with the size of the horizon, as occurs in Hawking radiation processes. Furthermore, Hawking radiation from analogue black holes should also face the problem of the possible difference between signal velocity and phase velocity.

Experimentally though, it appears to be a rather difficult task to maintain a stationary spherically symmetric and inhomogeneous flow. In order to exhibit a more realistic configuration, we will now focus on the cylindric symmetry. A particular case of such a configuration is the vortex matter flow which was discussed recently [24, 44, 45]. Let us consider a more general situation of a rotating dielectric body subjected to an electric field $E_z(\rho)$ directed along the axis z of rotation (this configuration corresponds to the experimental situation of a rotating condenser). The background metric is then of the form $ds_\eta^2 = dt^2 - d\rho^2 - \rho^2 d\varphi^2 - dz^2$. The electromagnetic field has the only non-zero component $F^{03} = E_z$ and we denote $\epsilon' := E(d\epsilon/dE)$, where $E = |E_z|$. For the matter 4-velocity we have $u^\mu = \gamma(1, b, w, 0)$, with $\gamma = (1 - b^2 - \rho^2 w^2)^{-1/2}$, while w and b being arbitrary functions of the radial coordinate ρ.

In this case, the analogue (cylindric) event horizon is located at $\rho = \rho_h$ defined from

$$\left(nb \pm \sqrt{1 - \rho^2 w^2}\right)\Big|_{\rho_h} = 0. \tag{14}$$

Note that purely vortical motion of the fluid (with $b = 0$) does not produce an analogue event horizon [32], since ρ_h from Eq. (14) would lie beyond the limit of applicability of a rigid body model in special relativity.

We note that $d\varphi/d\rho$ diverges at the surface defined by Eq. (14). As a result, the incoming geodesic light rays spiral towards a horizon radius ρ_h, as can be explicitly demonstrated by the numerical integration of $d\varphi/d\rho$, see Fig. 1. Therefore, we conclude that, for a given stationary flow configuration, the analogue horizon structure is of geometrical nature, as soon as it depends neither on the initial conditions nor on the polarization of the propagating light rays [40].

Summarizing, Eq. (14) demonstrates how an experimental realization of a dielectric analogue horizon might be understood in terms of the parameters n, b, w which describe the dielectric and kinematic properties of the medium. We expect that such horizons can be observed for stationary inhomogeneous kinematic configurations of the dielectric matter flow. The cylindrical configuration is favored due to the combined effect of both the refraction index n and also of the vorticity w of the medium which yield a smaller threshold for the radial velocity b. The horizon structures in both spherical and cylindrical cases are shown to be independent of the presence of nonlinearities in the permittivity tensor.

The present scheme may provide room also for the formation of sonic horizons [2] for the cases in which ultrasonic velocities are achieved.

3 Analogue models from hydrodynamics

The mathematical description of a moving fluid is performed by means of the velocity field $\vec{v}(t, x, y, z)$ and two thermodynamic state function, which are assumed to be the matter density $\rho(t, x, y, z)$ and pressure $p(t, x, y, z)$. The behavior of a perfect fluid is determined by the continuity equation

$$\vec{\nabla} \cdot (\rho\, \vec{v}) + \frac{\partial \rho}{\partial t} = 0, \tag{15}$$

and the Euler equation (in the absence of external fields)

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} p. \tag{16}$$

By considering an irrotational fluid $\vec{v} = \vec{\nabla}\Phi$, the Euler equation leads to the Bernoulli equation

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}v^2 + h(\rho) = 0, \tag{17}$$

where $h(\rho)$ represents the enthalpy density ($h = \int dp/\rho$). Let us now introduce small perturbations of Φ and ρ about the background flow by defining

$$\Phi \to \Phi_o + \delta\Phi := \Phi_o + \phi \tag{18}$$

$$\rho \to \rho_o + \delta\rho := \rho_o + \rho_o \psi. \tag{19}$$

Thus, from Eqs. (15) and (17) the field perturbations lead to

$$\Box_{BL}\phi = 0, \tag{20}$$

where $\Box_{BL}$ is the effective Beltrami-Laplace operator, which can be written as

$$\Box_{BL} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right), \tag{21}$$

with the effective geometry $g_{\mu\nu}$ given by

$$g_{\mu\nu} = \rho_o \begin{pmatrix} 1 - v^2/c_S^2 & v^i/c_S \\ v^i/c_S & -\delta_{ij} \end{pmatrix}, \tag{22}$$

where $c_S := \partial p/\partial\rho$ is the sound velocity in the fluid.

Thus, linearized perturbations of an irrotational flowing fluid can be described by means of massless scalar fields propagating in an effectively curved spacetime whose metric $g_{\mu\nu}$ is determined by the background fluid fields [2].

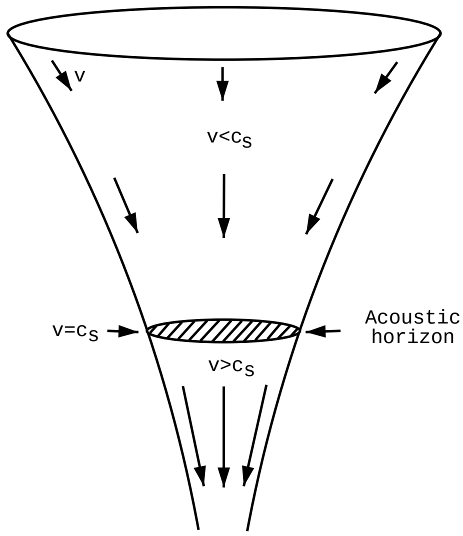

Figure 2. A toy model exhibiting an acoustic horizon. The arrows indicate the fluid flow velocity, with longer arrows for faster flow. The analogue event horizon (acoustic event horizon) occurs when the fluid velocity becomes equal to the sound velocity.

A simple example of this proposal can be understood in the following simplified toy model. Let us consider a fluid flowing through a funnel (Fig. 2) in such way that the fluid velocity increases as the funnel throat gets smaller. Thus, there will be a surface where the fluid velocity becomes equal to the sound velocity, and after it the waves travel with velocity smaller than the fluid velocity. Consequently, this surface will be an one-way surface with respect to the field perturbations (sound) and should be thought as an acoustic event horizon. The acoustic black holes introduced by Unruh [2] are usually called dumb holes.

3.1 Acoustic black-hole

Let us assume the case of a radial flow, with $\vec{v} = v(r)\hat{r}$. By adopting spherical coordinates and applying a coordinate transformation in the time variable as

$$\tau = t + \int dr \frac{v}{c_S^2 - v^2} \tag{23}$$

we obtain from (23) a line element of the form

$$ds^2 = \rho_o \left[\left(1 - \frac{v^2}{c_S^2}\right)d\tau^2 - \left(1 - \frac{v^2}{c_S^2}\right)^{-1} dr^2 - r^2 d\Omega^2\right], \tag{24}$$

where $d\Omega^2$ represents the 2-sphere line element. In the particular case where $v = c_S(r_H/r)^{1/2}$, the metric in Eq. (24)

assumes the form of the Schwarzschild metric (up to the conformal factor ρ_o) with r_H identified with the radius of the acoustic horizon, which makes the role of $2GM$ in the general relativity black-hole.

The quantization of the sound field ϕ can be performed, as usual, by means of the field equation (20), and the same arguments leading to the prediction of radiation emission of a black hole also predict that a thermal spectrum of sound waves should be given out from the acoustic horizon in supersonic fluid flow. Thus, an acoustic event horizon will emit analogue Hawking radiation in the form of a thermal bath of phonons at temperature [2]

$$T_H = \frac{\hbar}{2\pi k_B} \frac{\partial v}{\partial r}\bigg|_{r_H}. \tag{25}$$

It should be stressed that the smooth background flow breakdown at scales of $\sim 10^{-8} cm$, just as for gravity at Planck scale $\sim 10^{-33} cm$. (See however Ref. [8] where the effects of high frequencies on acoustic black hole evaporation were taken into account). Related subjects can be found in Refs. [46, 47, 48]

By assuming a toy model where

$$\frac{\partial v}{\partial r}\bigg|_{r_H} = \frac{c_S}{r_H}, \tag{26}$$

with $c_S \approx 10^2 m/s$ and $r_H \approx 1mm$, the temperature associated with the phonon radiation results $T_H \approx 10^{-7} K$. Due to the small value of the temperature, and the very poor detection technology for sound waves (among several other difficulties associated with classical fluids), the experimental verification of this result seems to be unrealistic. However, as it will be presented in the next section, there exist special kinds of fluids (quantum fluids) where the above results are expected to be verified in laboratory.

4 Analogue models from Bose-Einstein Condensates

A Bose-Einstein Condensate (BEC) corresponds to a configuration in which most of the bosons lie in the same single-particle quantum state. In a dilute gas, the hydrodynamic theory of superfluids in the collisionless regime at zero temperature can be derived from the Gross-Pitaevskii equation [49]

$$i\hbar \frac{\partial \psi(t,x)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(x) + \lambda |\psi(t,x)|^2 \right] \psi(t,x), \tag{27}$$

where $\psi(t,x)$ represents the wave function of the condensate, V_{ext} is an external confining potential, and λ is a parameter related with the scattering length a as $\lambda = 4\pi\hbar^2 a/m$. By means of the Madelung representation for the condensate wave function $\psi = \rho^{1/2} \exp(im\Theta/\hbar)$, the Gross-Pitaevskii equation can be separated in terms of a continuity equation

$$\vec{\nabla} \cdot (\rho \vec{\nabla} \Theta) + \frac{\partial \rho}{\partial t} = 0, \tag{28}$$

and a quantum analogue of the Bernoulli equation

$$\frac{\partial \Theta}{\partial t} + \frac{1}{2} \vec{\nabla}\Theta \cdot \vec{\nabla}\Theta + \frac{\lambda \rho}{m} + \frac{V_{ext}}{m} - \frac{\hbar^2}{2m^2 \sqrt{\rho}} \nabla^2 \sqrt{\rho} = 0. \tag{29}$$

It should be noted that the above set of equations describes an irrotational fluid with velocity $\vec{v} = \vec{\nabla}\Theta$. The quantum potential $V_Q = -\hbar^2 \nabla^2 \rho^{1/2}/2m^2 \rho^{1/2}$ can be neglected when the density profiles become smooth.

From the above results we concluded that the Gross-Pitaevskii equation is equivalent to a hydrodynamic equation for superfluid flow, which (for most situations) is similar to a classical hydrodynamic equation for irrotational fluids with the particular form for the enthalpy density $h = \lambda\rho/m$. In this prescription, the sound velocity is given by $c_S = (\hbar/m)(4\pi a \rho)^{1/2}$. Thus, analogue models from BEC's can be performed in the same way as it is done in the case of classical hydrodynamics. (See Ref. [35] for a more complete treatment of analogue gravity from Bose-Einstein condensates.)

Experimental realizations of BEC's were firstly done in 1995 for alkali atoms [50, 51, 52] and since then several works reporting BEC's in the laboratory have been appeared in the literature.

Some interesting configurations of BEC's were recently proposed in the literature [10, 11, 35, 53] as a way to construct analogue models. In Ref. [11] a ring-shaped configuration and a cigar-shaped configuration (both adjusted by the external potential) were proposed as possible experimental backgrounds where acoustic black/white holes could be constructed (see Fig. 3 for the cigar-shaped configuration). In Ref. [35] it was proposed that the external potential could be replaced by geometrical constraints. The Laval nozzle device was presented as a possible mechanism to produce black/white holes.

In fact, the use of BEC's as the physical systems to construct analogue models for general relativity seems to be very promising. As an example, let us analyse the case of a condensate based on ^{87}Rb. By using the value $a \approx 6 \times 10^{-9} m$ for the scattering length [55], the sound velocity in the condensate results to be $c_S \approx 6 \times 10^{-3} m/s$, which is one of the smaller values for the sound velocity ever found in physical systems. From such results (it was considered a system with an acoustic horizon of order of $1\mu m$), a crude estimate leads to an analogue Hawking temperature $T_H \approx 7nK$. Despite the small value for this temperature, it should be noted that the temperature required to form the condensate is about $T \approx 10^{-8} - 10^{-7} K$.

5 Conclusion

In this mini-survey we have briefly analysed some proposals of constructing non-gravitational systems presenting analogue horizons. We concentrated our attention in electromagnetic and hydrodynamic models.

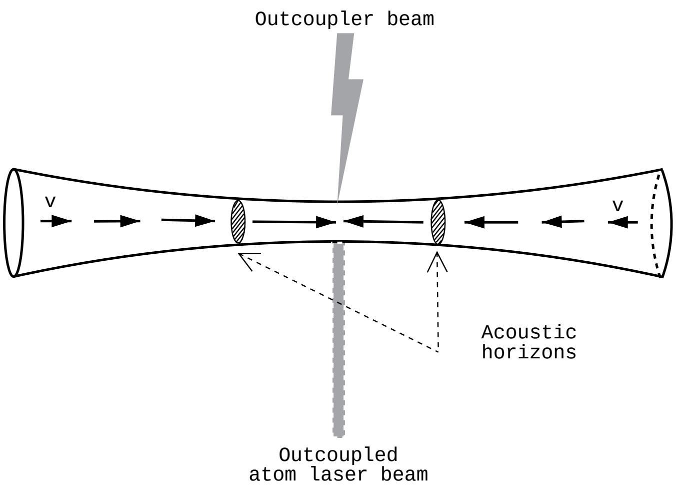

Figure 3. The cigar-shaped configuration [11] with two acoustic horizons. An output coupler is used to obtain an atom laser from the condensate, which leads the condensate to flows towards the 'singularity'. The arrows represent the magnitude of the condensate flow velocity. (See Ref. [54] for outcoupler details.)

We have analysed the wave propagation in moving material media. Two solutions presenting analogue event horizons were examined, the spherically symmetric configuration (with a radial flowing dielectric fluid) and a cylindrically symmetric configuration (with a radial and vortical flowing dielectric fluid). The cylindric configuration is experimentally preferred due to the vortical motion of the fluid, which makes it possible to set smaller thresholds for the radial velocity. Analogue black and white holes can thus possibly be set in moving dielectric media. For optical systems the issue of thermal emission of radiation (analogue Hawking radiation) is not yet well understood.

Analogue models from quantum fluids (in particular, BEC's based on alkali atoms) seem to be the most promising systems exhibiting acoustic event horizons to be built in laboratory in order to investigate the existence of analogue Hawking temperature. Bose-Einstein condensates were experimentally constructed since 1995, and the most attractive feature in such systems lies on the very low condensation temperature. For the case of condensates based on ^{87}Rb the estimated analogue Hawking temperature differs from the typical condensate temperature roughly by only two orders of magnitude. Therefore, it seems to be feasible in the near future to probe in laboratory some interesting aspects of quantum field theory in curved spacetimes by means of analogue models.

Acknowledgments

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