Improvement of Mass Calculations for the Neutron Rich $A \sim 32$ Nuclei

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In previous works anomalies were observed in binding energies for the island of inversion nuclei centered at Z=11, N=21 by the comparison of theoretical calculations and experiment. An attempt was made in this work using an improved term of shell corrections to observe the evolution of these anomalies. This term was added to the macroscopic part of the previous mass formula and binding energy calculations have been carried out. A distribution of these latter on a (N,Z) chart has indicated that the calculated values increase with the increase of N for all values studied as for those of experiment except the value of Z=10. This result shows the presence of these anomalies only in the line Z=10 and their absence in the lines Z=11 and 12. Although this shell correction includes a term representing an interaction between protons and neutrons of valence shells, but it is insufficient to study this deformed mass region.

Keywords: Mass calculations; Anomalies; Island of inversion

I. INTRODUCTION

In previous works, a mass formula was developed by taking the Weizsäcker mass formula as a starting point and basing on the relative neutron excess I [1]. This formula composed of a macroscopic part and shell correction term was applied to calculate binding energies of the $A \sim 32$ nuclei. In this deformed mass region, anomalies were observed for the first time by Thibault et al. [2].

This work is an attempt to improve the macroscopic calculations made in the previous work [1] by improving the shell correction term of the mass formula. The improved term contains the effect of magic numbers and an interaction term between protons and neutrons [3]. The mass formula deduced was employed to calculate binding energies of the $A \sim 32$ nuclei. The calculations show the absence of the anomalies only in the lines $Z = 11$ and 12 of the (N,Z) chart.

II. MASS FORMULA

The mass formula employed is given by

$$E = E_{mac} + E_{shell} \tag{1}$$

Where E_{mac} is a macroscopic term given by [1]

$$E_{mac} = a_v\left(1-k_v I^2\right)A - a_s\left(1-k_s I^2\right)A^{2/3} - a_{c_1}\frac{Z^2}{A^{1/3}} + a_{c_2}\frac{Z^2}{A} \pm \delta \tag{2}$$

And $a_v, k_v, a_s, k_s, a_{c_1}, a_{c_2}$ are parameters to be determined by a fit of experimental binding energies of nuclei [4].

The pairing energy is given by [1]

$$\delta = \begin{cases} +\left(\frac{12}{\sqrt{A}} - \frac{10}{A}\right) - 30\left|\frac{N-Z}{A}\right| , & Z \text{ and } N \text{ even} \\ -\frac{10}{A} - 30\left|\frac{N-Z}{A}\right| , & Z \text{ or } N \text{ odd} \\ -\left(\frac{12}{\sqrt{A}} - \frac{10}{A}\right) - 30\left|\frac{N-Z}{A}\right| , & Z \text{ and } N \text{ odd} \\ -\left(\frac{12}{\sqrt{A}} + \frac{20}{A}\right) - 30\left|\frac{N-Z}{A}\right| , & Z \text{ and } N \text{ odd},\ Z = N \end{cases} \tag{3}$$

The new shell correction is given by [3]

$$E_{shell} = \left(E_{self} + E_{int}\right) G \tag{4}$$

The term E_{self} is a sum of shell effects for each kind of nucleons; it is given by [3]

$$E_{self} = E_s\left[\sum\left(\frac{z_i^2}{D_i^2} + \frac{n_j^2}{D_j^2}\right) + \sum\left(z_i\varepsilon_i + n_j\varepsilon_j\right)\right] \tag{5}$$

Where
$Di = M_{i+1} - M_i$ and $Dj = M_{j+1} - M_j$,
M_i, M_{i+1}, M_j and M_{j+1} are magic numbers.
$\varepsilon_i, \varepsilon_j$ are slopes of the difference $(E_{exp} - E_{mac})$ in function of z_i and n_j respectively.
E_s is a universal factor for all shell effects, its value is: $E_s = 26.004 MeV$ [3].
G is an erosion factor; its value is determined in our work by a fit of experimental binding energies.
E_{int} corresponds to the interaction effects between protons and neutrons of valence shells. To deduce its form, we define:
* The concerned region, $Z_m \le Z \le Z_M, N_m \le N \le N_M, Z_\mu$ *and* N_μ.
* The number of valence particles, p' and n'.
* The function $f(p', n')$ corresponding to this effect.
It has the following form:

$$E_{int} = \sum E_\eta f_\eta\left(\rho_i, \rho_j\right) \tag{6}$$

With

$$E_\eta = E_s\frac{\min\left(Z_\mu - Z_m, Z_M - Z_\mu\right)}{D_i}\frac{\min\left(N_\mu - N_m, N_M - N_\mu\right)}{D_j} \tag{7}$$

Where
$\rho_i = p'/p'_{max}$, $\rho_j = n'/n'_{max}$
p'_{max} and n'_{max} are the maximal values of p' and n' in the region concerned.
$Z_m = \frac{1}{2}\left(M_{i-1} + M_i\right)$, $Z_\mu = M_i$,
$Z_M = \frac{1}{2}\left(M_i + M_{i+1}\right)$,
$N_m = \frac{1}{2}\left(M_{j-1} + M_j\right)$, $N_\mu = M_j$,

$N_M = \frac{1}{2}(M_j + M_{j+1})$.

For magic nuclei, E_{int} is given by:

$$E_{sph} = E_\eta \rho_i^2 \rho_j \left(1-\rho_{i+1}^2\right)\left(1-\rho_{j+1}\right) \tag{8}$$

For neutron rich nuclei, it is given by:

$$E_{rn} = E_\eta \rho_i \rho_j \tag{9}$$

Where $n' = N - N_m,\ p' = Z_M - Z$.

For proton rich nuclei, it is given by:

$$E_{rp} = E_\eta \left(\rho_i - \rho_j\right)^2 \left(1-\rho_{i+1}^2\right)\left(1-\rho_{j+1}\right) \tag{10}$$

III. DETERMINATION OF PARAMETERS

To deduce the parameters of this formula, a fit of experimental binding energies of neutron rich $A \sim 32$ nuclei was carried out [1,3,4].

IV. RESULTS

The parameters deduced are:
$a_v = 14.3507MeV,\ k_v = 1.9001$,
$a_s = 13.3655MeV,\ k_s = 2.4522$,
$a_{c_1} = 0.6834MeV,\ a_{c_2} = 0.4896MeV$,
$\varepsilon_i = -3.1209,\ \varepsilon_j = -1.6325$,
$G = 0.4159,\ E_s = 26.004MeV$.

In Fig. 1, we represent the experimental and calculated binding energies of the neutron rich $A \sim 32$ nuclei versus the number of neutrons N.

The results of binding energies are distributed with those of experiment [4] and those reported in references [1,5] in the (N, Z) chart of Fig. 2.

In Fig. 1, the first observation made on the new calculations (ldm2) is that they follow the same behavior as those of experiment for the three values of Z. They are also situated on the upper side of the three curves but they are very close to experiment in the case of $Z = 12$.

In Fig. 2, the results of binding energies obtained are in agreement with those of experiment [4]. The distribution of these results on a (N, Z) chart shows that the anomalies observed in reference [1] are present only in the line $Z = 10$, and are absent in the lines $Z = 11$ and $Z = 12$ of the (N, Z) chart.

The reason for occurring these anomalies only in the line of $Z = 10$ is probably the great spacing between proton and neutron shells for the isotopes of these nuclei and mainly the Ne isotopes. Another reason is the form of the interaction between protons and neutrons depending only on magic numbers and valence particles, so its application is not important for these nuclei. Adding also the great number of proton holes for the Ne isotopes than for Na and Mg isotopes, so there is a probable interaction between proton holes in the same shell. It is interesting to compare our results with those of Duflo and Zuker [5], where they have developed and employed a purely microscopic mass formula. The results obtained using their formula showed also the presence of anomalies only in the

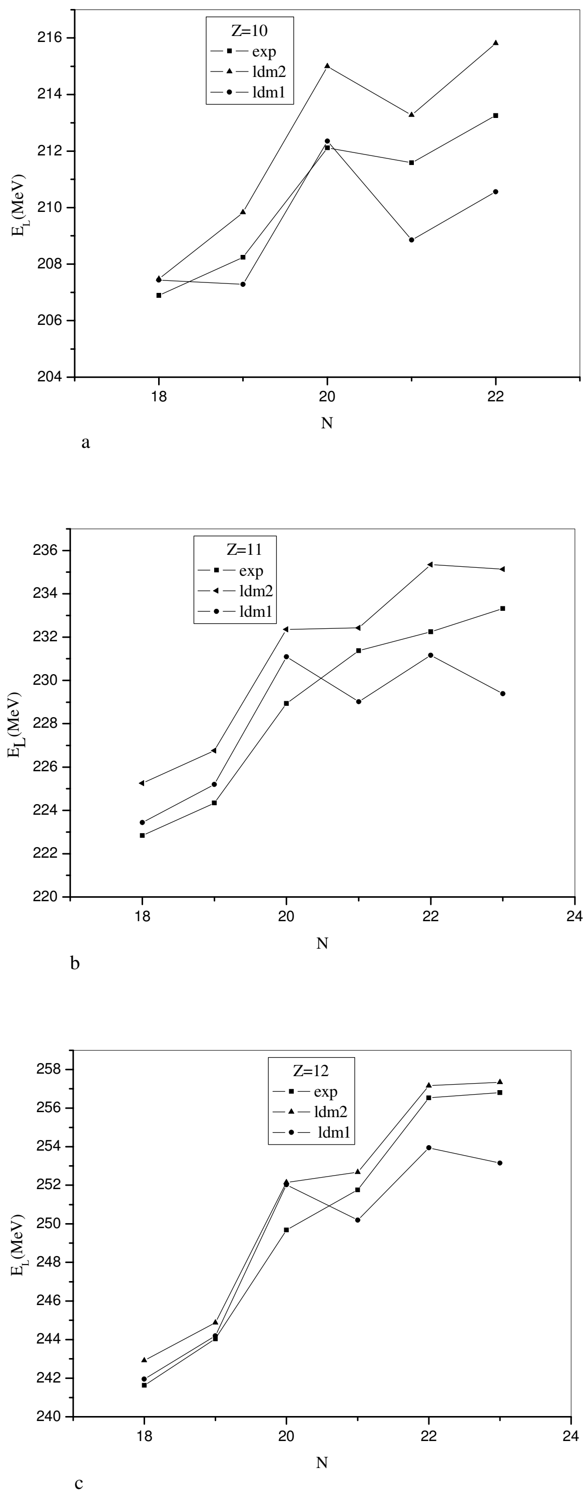

FIG. 1: Representation of experimental and calculated binding energies using the two liquid-drop models (ldm1 [1] and ldm2) versus N. a)- Ne isotopes, b)- Na isotopes, c)- Mg isotopes.

Z		18	19	20	21	22	23
15	DZ	280.07	286.19	294.71	299.25	305.83	309.79
	ldm2	281.51	287.48	297.29	301.59	308.46	312.17
	ldm1	278.28	284.57	294.93	297.03	303.27	306.10
	exp	280.95	287.25	295.62	299.08	305.89	309.73
14	DZ	270.81	275.67	283.32	286.55	292.43	295.10
	ldm2	273.84	278.09	287.44	290.11	296.55	298.72
	ldm1	269.19	273.74	283.76	284.06	289.84	291.10
	exp	271.41	275.89	283.43	285.90	292.10	294.26
13	DZ	254.66	258.89	265.11	267.86	272.27	274.55
	ldm2	254.47	258.25	265.97	268.20	273.11	274.87
	ldm1	254.27	258.33	266.73	266.53	270.75	271.56
	exp	254.99	259.17	264.71	267.17	272.44	274.61
12	DZ	241.39	244.21	249.66	250.98	254.78	255.58
	ldm2	242.91	244.88	252.15	252.68	257.17	257.34
	ldm1	241.96	244.19	252.03	250.18	253.95	253.14
	exp	241.66	244.04	249.85	252.07	256.22	256.97
11	DZ	222.92	225.21	228.98	229.95	232.04	232.60
	ldm2	225.25	226.76	232.36	232.42	235.35	235.14
	ldm1	223.44	225.20	231.09	229.02	231.16	229.39
	exp	222.80	225.17	228.94	230.62	232.85	233.07
10	DZ	207.10	207.75	210.87	210.18	211.81	210.70
	ldm2	207.47	209.83	214.99	213.27	215.81	213.98
	ldm1	207.43	207.28	212.35	208.85	210.56	207.90
	exp	206.92	208.19	211.20	211.54	213.18	212.52
Z / N		18	19	20	21	22	23

FIG. 2: Distribution of experimental and calculated binding energy values on a (N,Z) chart. exp: experimental values, ldm1: values reported in reference [1], ldm2: values of this work, DZ: values reported in reference [5]. The dashed lines surround the island of inversion; the magic number 20 is selected by thick lines.

line $Z=10$, and their absence in the lines $Z=11$ and $Z=12$ of the (N,Z) chart. So the mass formula developed in this work has improved well the binding energies for nuclei with $Z=11$ and 12 and there is not much improvement for the $Z=10$ nuclei.

V. CONCLUSION

In previous works and using a mass formula, anomalies have been observed following the lines $Z=10,11$ and 12 of the (N,Z) chart [1]. In this work we have replaced the shell correction term of the mass formula by a deeper term, containing the effect of magic numbers and a term of interaction between nucleons. The application of this formula has resolved the anomalies problem following only the lines $Z=11,12$ of the island of inversion. The fact of these anomalies is that in these nuclei the last neutrons shell is far from that of protons and the Ne isotopes contain more holes than the Na and Mg isotopes. Consequently the interaction between protons and neutrons is neglected. The weak coupling model discussed in [1] remains the good approximation to study this deformed mass region.

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