Domain Wall, Stiff Matter and Ultra-Relativistic Particles from a Generalized Double-Component Dark Energy Model

El-Nabulsi Ahmad Rami*
Department of Nuclear and Energy Engineering, Cheju National University, Ara-dong 1, Jeju 690-756, South Korea
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This letter discusses a modified Friedmann-Robertson-Walker cosmology in which the equation of state behaves like $p = 3\eta a^m \rho^n - \rho, (\eta, m, n) \in \mathbb{R}$. Many interesting features are revealed in particular the manifestation of domain wall, cosmic string, stiff matter, dust and radiation/ultra-relativistic particles.

Keywords: Super-accelerated expansion, dark energy, domain wall, cosmic string, stiff matter

The recent astronomical observations of the dynamics of galaxies [1,2,3], Type Ia supernovae (SNIa)[4] with redshift $z > 0.35$, the first acoustic peak of the CMB temperature fluctuations or anisotropies [5] and the recent findings of BOOMERANG experiments [6] favour a spatially flat matter universe undergoing a phase of accelerated expansion. These observational facts suggest that the universe is dominated by a mysterious invisible to us and missing energy component dubbed Dark Energy (DE) characterized by a negative Equation of State Parameter (EoSP) $w = p/\rho < 0$, which accounts for about 70% of the total energy content and 30% of dark matter. This ephemeral energy has special effects that have only been detected on the largest scales of our Universe and then only in the past ten years. Questions still linger about the nature of dark matter, especially its distribution in central region of clusters and galaxies. It is worth mentioning that the equation of state parameter (EoSP) for a cosmological constant is $w = p/\rho = -1$, and $-1 < w < -1/3$ for a quintessence fluid, whereas $w < -1$ for a phantom fluid. A large number of theoretical and phenomenological competitive models have been presented to account for its gravitational effects. These models include the ΛCDM model[7], quintessence [8], K-essence[**9**], viscous fluid [10], Chaplygin gas with equation of state $p = -K\rho^{-1}, \rho > 0, K \in \mathbb{R}$ ($p$is the spatially homogeneous pressure and ρ the energy density)[**11**], Generalized Chaplygin gas (GCG) model whose equation of state is $p = -K\rho^{-a}, \rho > 0, 0 < a \leq 1$ [12,13], exotic matters with generalized equation of state$p = A\rho - B\rho^a, (A, B, a) \in \mathbb{R}$ [14,15], Brans-Dicke (BD) pressureless solutions with non-minimal coupling [16] and so on. Other theoretical alternatives include the Modified Gravity (MG) or alternatives theories of gravity sorting from the low energy effective heterotic string theory/$E_8 \times E_8$ M-theory and braneworld scenarios [17-25]. Most of these approaches are accompanied with problems and many difficulties and we still ignore which of these models are the most viable or more realistic than the others. We are grappled with deep cosmological enigmas and many unsolved problems.

In this letter, we will address another simple alternative. Unlike the previous works on dark energy problem, in particular those dealing with scalar field gravity we will deal with a new variable equation of state parameter leading to dark energy and Big Rip. The universe may contain some kind of exotic matters so that energy conditions are violated.[26] The presence of exotic components point toward a need to improve our concepts about the primordial composition of the universe and hence the equation of state needs some generalization. More precisely, the equation of state conjectured here is $p = 3\eta a^m \rho^n - \rho, (\eta, m, n) \in \mathbb{R}$. As already noted, the case where $\eta = 0$ corresponds to the cosmological constant, and that for $m = 0$ corresponds to the case of exotic matters with generalized equation of state $p = -\rho - B\rho^n, (B = -3\eta, n) \in \mathbb{R}$. It will be shown that the equation of state we conjecture in this work will not only result in many suitable analytical solutions but will be the outcome of many interesting cosmological consequences which overwhelmingly will be justified by the latest astronomical observations. As it is believed that the matter density varies with the scale factor of the universe, the equation of state $p = 3\eta a^m \rho^n - \rho$ can be viewed as a generalization of the generalized Chaplygin gas equation of state. This later is a new version of the generalized equation of state for Chaplygin gas recently introduced in literature.[14,15] One normally expects $m < 0$ and $0 < \eta < 1$ that $w > -1$ for the generalized Chaplygin gas but one may also expects new generalized Chaplygin gas models for which m and η can lie outside this range.

We naturally assume that the universe is spatially flat, isotropic and homogeneous at large scales. In a general relativistic context, such a spacetime is described by a four-dimensional manifold and is endowed with a Friedman-Robertson-Walker (FRW) space-time metric $ds^2 = dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]$. Here $a(t)$ is the scale factor of the universe. The effective Einstein's field equation in the presence of the cosmological constant Λ is written in their standard form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi G\left[(p+\rho)u_\mu u_\nu + pg_{\mu\nu}\right], \quad (1)$$

where $R_{\mu\nu}$ is the Riemann's curvature tensor, R is the scalar curvature, $T_{\mu\nu} = (p+\rho)u_\mu u_\nu + pg_{\mu\nu}$ is the stress-energy momentum tensor, $g_{\mu\nu}$ is the metric and G is the gravitational constant. The Friedmann equation in the presence of matter is usually written like[27]:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3}. \quad (2)$$

The first step is to derive the form of the matter density. For this, we make use of the continuity equation $\dot{\rho} + 3H(p+\rho) = 0$ where $H = \dot{a}/a$ is the Hubble parameter. After simple algebraic calculations, we obtain:

$$\rho = \left(C - 9(1-n)\eta\frac{a^m}{m}\right)^{\frac{1}{1-n}}, \quad (3)$$

*Electronic address: nabulsiahmadrami@yahoo.fr

where C is the integration constant. Assuming that at the origin of time $(t=0), a=a_0(\equiv 1) \Rightarrow \rho=\rho_0(\equiv 1)$, we obtain straightforwardly:

$$\rho=\left(1+\frac{9(1-n)\eta}{m}-\frac{9(1-n)\eta}{m}a^m\right)^{\frac{1}{1-n}}. \quad (4)$$

Surprisingly, this equation is similar in form to the matter density obtained within the framework of generalized Chaplygin gas. However, in our framework, the density evolution in our case changes from $\rho \propto a^{m/(1-n)}$ at early times to $\rho=$ constant at late times unless $m/(1-n)<0$. After replacing onto equation (2), we obtain:

$$\frac{\dot{a}^2}{a^2}=\frac{8\pi G}{3}\left(1+\frac{9(1-n)\eta}{m}-\frac{9(1-n)\eta}{m}a^m\right)^{\frac{1}{1-n}}+\frac{\Lambda}{3}. \quad (5)$$

However, equation (4) may lead to useful restrictions on the values of the parameters introduced as is necessary to have a real and positive energy density. This natural and physical fact restricts the parameters to $-1<9(1-n)\eta/m<0$. A desirable solution is obtained now from equation (5) if for instance we assume $9(1-n)\eta=-m$ and $m=-2(1-n)$, i.e. $\eta=2/9, n\neq 1$ for which the Friedmann equation and the generalized equation of state are respectively reduced to:

$$\frac{\dot{a}^2}{a^2}=\frac{8\pi G}{3}a^{-2}+\frac{\Lambda}{3}, \quad (6)$$

$$p=\frac{2}{3}a^{-2(1-n)}\rho^n-\rho, \quad (7)$$

As in this particular case, $m/(1-n)=-2<0$, the density evolution in our case changes from $\rho \propto a^{-2}$ at early times to $\rho=$ constant at late times, whereas in the generalized Chaplygin gas model explored by Bento *et al*[12] with the equation of state $p=-A\rho^{-\alpha}, (A,\alpha)\in\mathbb{R}$, the density evolution in our case changes from $\rho \propto a^{-3}$ at early times to $\rho=$ constant at late times. Of course, other possible solutions for different values of the parameters (η,m,n) may be obtained. The particular solution obtained in Starobinsky model which is based on a semi-classical Einstein equation and in the initial stage and obtained by Mukherjee *et al.*[15] is $p=B\rho^n, n=1/2, B<0$. However, in Starobinsky model, we have no matter and the vacuum energy of the fields act as the source of gravitation. In our case, for $n=1/2$, $p=2a^{-1}\sqrt{\rho}/3-\rho$ different from Starobinsky approach. Obviously, for the critical density $\rho_c=(3a^{2(1-n)}/2)^{1/(n-1)}\propto a^2$, the pressure vanishes and singularity will occurs. Apparently, in our framework, $\rho\propto a^{-2}$; hence $p=-\rho/3$ and accordingly the universe is dominated in our approach by cosmic strings. This case gives straightforwardly $n=1$.

The analytic solution of the differential equation (6) in closed form is [**26**]:

$$a(t)=\sqrt{\frac{2\pi G}{\Omega\Lambda}}\left[e^{\sqrt{\Lambda/3}(t-t_0)}-\Omega e^{-\sqrt{\Lambda/3}(t-t_0)}\right]. \quad (8)$$

Here

$$\Omega=\left(\sqrt{(8\pi G a_0^{-2}+\Lambda)/3}-\sqrt{\Lambda/3}\right)\Big/\left(\sqrt{(8\pi G a_0^{-2}+\Lambda)/3}+\sqrt{\Lambda/3}\right)$$

with $0<\Omega<1, \Lambda>0$ and $a_0=a(t_0=0)\equiv 1$ for mathematical simplicity. Accordingly, equation (8) is simply rewritten like:

$$a(t)=\sqrt{\frac{2\pi G}{\Omega\Lambda}}\left[e^{\sqrt{\Lambda/3}t}-\Omega e^{-\sqrt{\Lambda/3}t}\right], \quad (9)$$

where

$$\Omega=\left(\sqrt{(8\pi G+\Lambda)/3}-\sqrt{\Lambda/3}\right)\Big/\left(\sqrt{(8\pi G+\Lambda)/3}+\sqrt{\Lambda/3}\right)$$

On the other hand, as $t\to\infty$, the solution gives asymptotically a de Sitter universe, even if the equation of state is not characteristic of the vacuum state $p=-\rho$. It is noteworthy that in case where $\Omega\approx 1$ or $\Lambda<<1$, equation (9) is reduced to $a(t)\propto \sinh(\sqrt{\Lambda/3t})$ which is also reduced at late time for the asymptotically de Sitter universe. Nevertheless, for $m=-3(1-n)$ and $\eta=1/3$, equation (4) is reduced to $\rho=a^{-3}$ and accordingly, equation (2) gives: $\dot{a}^2/a^2=Aa^{-3}+B, (A=8\pi G/3, B=\Lambda/3)$ and the analytical solution in closed form is $a(t)=\sqrt{A/4B\Omega}[e^{3\sqrt{B}(t-t_0)}-\Omega e^{-3\sqrt{B}(t-t_0)}]^{2/3}$. As $t\to\infty$, the solution gives asymptotically a de Sitter-like universe $a(t)\approx\sqrt{A/4B\Omega}e^{2\sqrt{B}(t-t_0)}$ which is faster than the solution given in equation (9) which is $a(t)\approx\sqrt{A/4B\Omega}e^{\sqrt{B}(t-t_0)}$ even if the density of matter behaves like matter ($\rho=a^{-3}$). Finally, note that for $m=-4(1-n)$ and $\eta=4/9$, equation (2) gives: $\dot{a}^2/a^2=Aa^{-4}+B, (A=8\pi G/3, B=\Lambda/3)$ and the analytical solution in closed form is $a(t)=\sqrt{A/4B\Omega}[e^{3\sqrt{B}(t-t_0)}-\Omega e^{-3\sqrt{B}(t-t_0)}]^{1/2}$ and hence as $t\to\infty$, the asymptotic solution is a de Sitter-like with $a(t)\approx\sqrt{A/4B\Omega}e^{3\sqrt{B}(t-t_0)/2}$.

If $t\to 0$ is the Big Bang time moment, the pre-Big Bang occurs at $t<0$ and the scale factor evolves correspondingly like

$$a(-t)=\sqrt{\frac{2\pi G}{\Omega\Lambda}}\left[e^{-\sqrt{\Lambda/3}t}-\Omega e^{\sqrt{\Lambda/3}t}\right]. \quad (10)$$

It is noticeable that $a(t) = a(-t)[e^{\sqrt{\Lambda/3t}} - \Omega e^{-\sqrt{\Lambda/3t}}]/[e^{-\sqrt{\Lambda/3t}} - \Omega e^{\sqrt{\Lambda/3t}}]$ and hence at the origin of time $a(t) = a(-t)$. This scenario is similar to the Pre-Big Bang framework without singularity [28].

The Hubble expansion parameters at t and $-t$ are respectively:

$$H^{+} \equiv H(t) = \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{\Lambda}{3}}\frac{1+\Omega e^{-2\sqrt{\Lambda/3}t}}{1-\Omega e^{-2\sqrt{\Lambda/3}t}}, \tag{11}$$

$$H^{-} \equiv H(-t) = \frac{\dot{a}(-t)}{a(-t)} = \sqrt{\frac{\Lambda}{3}}\frac{\Omega+e^{-2\sqrt{\Lambda/3}t}}{\Omega-e^{-2\sqrt{\Lambda/3}t}}, \tag{12}$$

and therefore $H^{+}(t=\infty) = H^{-}(t=\infty) = \sqrt{\Lambda/3}$and $H^{+}(t=0) = -H^{-}(t=0) = \sqrt{\Lambda/3}(1+\Omega)/(1-\Omega)$. If for instance, $\Omega = \pm 1$, then $H(t) = H(-t)$. There is hence a probability that the universe accelerates from negative times towards the Big Bang and then decelerates just after the Boom.[28] It is noteworthy that for the case of a non-flat Universe modeled by the Friedmann equation

$$\frac{\dot{a}^2}{a^2} = \left(\frac{8\pi G}{3} - k\right)a^{-2} + \frac{\Lambda}{3}, \tag{13}$$

where $k = -1, 0, +1$ is curvature parameter for open, lat and closed spacetime, the analytic solution is given by:

$$a(t) = \sqrt{\frac{3}{4\Xi\Lambda}\left(\frac{8\pi G}{3} - k\right)}\left[e^{\sqrt{\Lambda/3}t} - \Xi e^{-\sqrt{\Lambda/3}t}\right], \tag{14}$$

where

$$\Xi = (\sqrt{(8\pi G+\Lambda)/3-k} - \sqrt{\Lambda/3})\Big/(\sqrt{(8\pi G+\Lambda)/3-k} + \sqrt{\Lambda/3}),\ 0<\Xi<1, \Lambda>0$$

and $k < 8\pi G/3$.

Nevertheless, in case where $\Xi \approx 1$ or $\Lambda << 1$, equation (14) is reduced again to $a(t) \propto \sinh(\sqrt{\Lambda/3t})$which is also reduced at late time for the asymptotically de Sitter universe.

Finally, in return to equation (4), an interesting point may arise if for instance $(n = 1/2, \eta \neq 0, m \in \mathbb{R})$with $-1 < 9\eta/2m < 0$. Hence:

$$\begin{aligned}\rho &= \left(1+\frac{9\eta}{2m} - \frac{9\eta}{2m}a^m\right)^2 = \left(1+\frac{9\eta}{2m}\right)^2 \\ &\quad -\frac{9\eta}{m}\left(1+\frac{9\eta}{2m}\right)a^m \\ &\quad +\left(\frac{9\eta}{2m}\right)a^{2m} = \rho_1+\rho_2+\rho_3,\end{aligned} \tag{15}$$

$$\begin{aligned}p &= -\left(1+\frac{9\eta}{2m}\right)^2 + \left(3\eta + \frac{27\eta^2}{2m} - \frac{9\eta}{m} - \frac{81\eta^2}{2m^2}\right)a^m \\ &\quad -\left(\frac{27\eta^2+9\eta}{2m}\right)a^{2m} = p_1+p_2+p_3,\end{aligned} \tag{16}$$

and it is an easy task to identify the following three-components:

1. The cosmological constant which may account for the dark energy problem

$$p_1 = -\rho_1 = -\left(1+\frac{9\eta}{2m}\right)^2. \tag{17}$$

2. The second equation of state

$$\begin{aligned}p_2 &= \left(1 - \frac{9\eta m+2m^2}{27\eta+6m}\right)\rho_2 = \frac{(9\eta+2m)(3-m)}{3(9\eta+2m)} \\ \rho_2 &= \frac{3-m}{3}\rho_2.\end{aligned} \tag{18}$$

3. The third equation of state

$$p_3 = -(1+3\eta)\rho_3. \tag{19}$$

As $-1 < 9\eta/2m < 0$, then $p_1 = -\rho_1 < 0$ as it is obvious from equation (17). Furthermore, for positive m, $-2m/9 < \eta < 0$, and accordingly $p_3 = -(1+3\eta)\rho_3 > -\rho_3$. Furthermore, notice that for $m = 3$, $p_2 = 0$. As phantom field are not allowed in our framework, it is required to have $0 < m < 6$ and consequently, $-12 < -2m < 9\eta < 0$. If, for instance, $m/(1-n) = -2$, then $1 < n < 4$.

For this special case, the Friedmann equation is:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(1+\frac{9(1-n)\eta}{m} - \frac{9(1-n)\eta}{m}a^m\right)^2 + \frac{\Lambda}{3}, \tag{20}$$

with the restrictions

$$-12 < -2m < 9\eta < 0, 1 < n < 4, 0 < m < 6. \tag{21}$$

To illustrate, we choose $n = 5/2$, $\eta = -1/4$ and $m = 3$. Accordingly, equations (17)-(19) give straightforwardly:

$$p_1 = -\rho_1 = -\frac{25}{64}, \tag{22}$$

$$p_2 = 0, \tag{23}$$

$$p_3 = -\frac{5}{8}\rho_3, \tag{24}$$

and the energy density behaves amazingly like $\rho \propto a^{-2}$ at late time dynamics. This case corresponds for cosmic string-like which could have important consequences on galaxies formation. It is noteworthy that the equation of state parameter which results from equation (15) and (16) is:

$$w = \frac{p}{\rho} = \frac{p_1 + p_2 + p_3}{\rho_1 + \rho_2 + \rho_3}. \tag{25}$$

For the particular case $n = 5/2$, $\eta = -1/4$ and $m = 3$, equations (22)-(24) give:

$$w = \frac{p}{\rho} = -\frac{\frac{25}{64} + \frac{5}{8}\rho_3}{\frac{25}{64} + \rho_2 + \rho_3}, \tag{26}$$

and as a result, $w > -1$ if, for instance, $\rho_2 + 3\rho_3/8 > 0$.

Of course, there exist different values of the parameters and dissimilar restrictions which give as well interesting scenarios. If, for instance, $m/(1-n) = -1$, then $\rho \propto a^{-1}$ and thus the cosmological scenario behaves as a domain wall. For this special case, we find for $0 < m < 6$, $1 < n < 7$. As a simple illustration, we choose $m = 1$, $n = 2$ and $\eta = -1/9$. However, for this special case, we find

$$p_1 = -\rho_1 = -\frac{1}{4}. \tag{27}$$

$$p_2 = \frac{2}{3}\rho_2, \tag{28}$$

$$p_3 = -\frac{2}{3}\rho_3. \tag{29}$$

whereas equation (5) results into:

$$\frac{\dot{a}^2}{a^2} \approx \frac{\Lambda}{3}a^2 - \frac{4\pi G a}{3}, \tag{30}$$

which approximates a de-Sitter like solution at late-time dynamics. The equation of state parameter which results now from equation (15) and (16) is therefore:

$$w = \frac{p}{\rho} = \frac{-\frac{1}{4} + \frac{2}{3}(\rho_2 - \rho_3)}{\frac{1}{4} + \rho_2 + \rho_3}, \tag{31}$$

and as a result, $w > -1$ if, for instance, $5\rho_2 + \rho_3 > 0$. Supplementary different solutions (stiff matter, exotic matter, dust, radiation/ultra-relativistic particles) may exist depending on the choice of the parameters.

In conclusion, the acceleration of the universe presents one of the greatest problems in theoretical physics today. This problem has been attacked head on, but no compelling, well-developed and well-motivates solutions have yet merged. While much work in literature has focused on the search for new matter sources that yield accelerating solutions to general relativity, more recently complementary approach of examining whether new gravitational physics might be responsible for cosmic acceleration was developed. The work done in this letter is just a kind of the modified gravity theory to yield accelerating expansion. In this letter, we have discussed a particular cosmological scenario dominated by a particular generalized equation of state $p = 2a^{-2(1-n)}\rho^n/3 - \rho$. This particular form combines the cosmological constant and dark energy models and provides a possible mechanism to allow for a currently accelerating universe. Our analysis yields additional information beyond the standard generalized Chaplygin gas recently explored in literature. The idea ansatz introduced through this work is interesting that deserves further investigation.

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