Fundamental Decoherence in Quantum Gravity

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A recently introduced discrete formalism allows to solve the problem of time in quantum gravity in a relational manner. Quantum mechanics formulated with a relational time is not exactly unitary and implies a fundamental mechanism for decoherence of quantum states. The mechanism is strong enough to render the black hole information puzzle unobservable.

I. INTRODUCTION

The "problem of time" in quantum gravity arises largely due to the presence of constraints in the theory, in particular the Hamiltonian constraint (see [1] for a review). If one could eliminate the constraints almost all the conceptual problems with the problem of time can be eliminated. In particular, one can implement the quantization proposed by Page and Wootters [2] in which one promotes all variables to quantum operators and chooses one of these quantum variables to be the "clock". One then computes conditional probabilities for other variables to take certain values when the "clock" variable is at a certain "time".

We have recently introduced [3] an approach to general relativity that eliminates the constraints. The main idea is to approximate the theory by a discrete theory, much like it is done in particle physics when one approximates a field theory on the lattice. The novelty of our approach is that the discrete theories we construct are constraint-free, yet they can approximate general relativity. As a consequence, one can complete the Page–Wootters quantization of the discrete theories and introduce a relational time [4].

An immediate consequence of having a "quantum clock" variable in quantum mechanics is that the evolution is not unitary [5, 6]. Both the clock and the system under study evolve unitarily and under the usual rules of quantum mechanics in terms of a fiducial background time n (we use the letter n to emphasize that we are working in a discrete formulation, though this is not central to the points discussed in this paper). This time is an inaccessible variable, we could only measure it if we had a perfectly classical clock. What we can measure are the dynamical variables of the problem, in particular t, the variable that describes the clock. This variable is represented by a quantum operator and it will have an expectation value and a dispersion. Upon evolution, the dispersion will increase. One can show that if one prepares the clock initially in a state in which t is highly peaked around a given value of the fiducial time n, the quantities under study (let us call them O) will evolve according to an approximate Heisenberg equation, but there will be corrective terms that imply that pure states evolve into mixed states.

This kind of discussion is relevant to the black hole information puzzle [7]. The puzzle arises because one could have a pure state that undergoes gravitational collapse to form a black hole. The black hole will eventually evaporate leaving behind outgoing thermal Hawking radiation. It will therefore be in a mixed state, so somehow the initial pure state evolved into a mixed state. This is a problem in ordinary quantum mechanics. In quantum mechanics with a relational time, since pure states decohere anyway, one should evaluate if the rate of fundamental decoherence is slower or faster than the process of black hole evaporation. If the state would have become totally mixed anyway due to fundamental decoherence by the time the black hole evaporates, then the puzzle is unobservable.

We have carried out some preliminary calculations to yield an estimate of the fundamental decoherence and compare it to the black hole evaporation rate [8]. The rate of decoherence is related to "how classical" the clock one uses is. Therefore we addressed the question of what is the optimal clock that one can construct. Following arguments of Salecker and Wigner and further discussions by Ng and Amelino-Camelia [9], we established that the optimal clock one can find is a black hole. The accuracy with which a black hole can measure time is given by the frequency of its (quasi)normal modes, which scales as the inverse of the black hole mass. One would like therefore to have a black hole of small mass as a clock in order to have more accuracy. But there is a limit to this, if the mass is too small, the black hole-clock will evaporate too quickly for one to observe the physics of interest. Therefore one has an optimal accuracy one can achieve given a lapse of time that one wishes to measure. This limit on the accuracy is clearly only theoretical, in practice there will be other environmental factors that will affect the accuracy of the clock. Even if one isolates the system from the environment, there are quantum uncertainties (for instance, in the position of the clock) that need to be taken into account. Here we will only concentrate on the above mentioned effect, since it is one of the effects of fundamental nature and can be viewed as an ultimate limit on the accuracy of a clock.

The organization of this paper is as follows. In section II and III we review results ([6, 8]) on the evolution of conditional probabilities and the estimate of an optimal clock, respectively. In section IV we present a novel way of calculating explicitly the evolution of the conditional probability in closed form for the black hole information puzzle scenario. This generalizes previous work in which calculations were carried out to first non-trivial order in an expansion [8].

II. EVOLUTION OF THE CONDITIONAL PROBABILITY

To give a more quantitative version of things, let us compute the time evolution of the conditional probability of measuring an observable O as a function of a real clock t (for further detail see [6]).

We start by considering the conditional probability defined we mentioned above for an observable O to take a certain value o when the clock variable takes a value t,

$$\mathcal{P}(o \in \Delta o | t \in \Delta t)_\rho = \frac{\sum_n \mathrm{Tr}\left(P_o(n) P_t(n) \rho P_t(n)\right)}{\sum_n \mathrm{Tr}\left(P_t(n)\rho\right)}, \tag{1}$$

and one could have omitted the last $P_t(n)$ using the cyclicity of the trace since we are assuming that P_t and P_o commute. (If the observables (O and t) have continuous spectra, the projectors in the above expression should be understood as integrated over the interval, i.e. $P_o(n) = \int_{\Delta o} P_{o'}(n) do'$ and similar for P_t.) The sum over all possible fiducial times n is due to the fact that we do not know for which value of n the variable t takes the value we want, and as the clock disperses there will be several values of n that correspond to t. We now introduce the hypothesis that the clock and the rest of the system interact weakly and write explicitly the evolution of the projectors in the step parameter n to get,

$$\begin{aligned} &\mathcal{P}(o \in \Delta o | t \in \Delta t)_\rho = \\ &= \frac{\sum_n \mathrm{Tr}\left(U_2^\dagger(n) P_o(0) U_2(n) U_1^\dagger(n) P_t(0) U_1(n) \rho_1 \otimes \rho_2\right)}{\sum_n \mathrm{Tr}\left(P_t(n)\rho_1\right) \mathrm{Tr}\left(\rho_2\right)} \\ &= \frac{\sum_n \mathrm{Tr}\left(U_2^\dagger(n) P_o(0) U_2(n) \rho_2\right) \mathrm{Tr}\left(U_1^\dagger(n) P_t(0) U_1(n) \rho_1\right)}{\sum_n \mathrm{Tr}\left(P_t(n)\rho_1\right) \mathrm{Tr}\left(\rho_2\right)}. \end{aligned} \tag{2}$$

Here we have assumed the system breaks into two subsystems, the clock (system 1) and the variables under study (system 2) and that the density matrix for the total system is a direct product $\rho = \rho_1 \times \rho_2$ and $\rho_{1,2}$ are evolved in the fiducial time n by unitary evolution operators $U_{1,2}$.

From this expression, using the cyclic property of the trace, we can identify the expressions of the density matrix evolved in relational time. We start by defining the probability that the measurement t corresponds to the value n,

$$\mathcal{P}_n(t) \equiv \frac{\mathrm{Tr}\left(P_t(0) U_1(n) \rho_1 U_1^\dagger(n)\right)}{\sum_n \mathrm{Tr}\left(P_t(n)\rho_1\right)}, \tag{3}$$

and notice that $\sum_n \mathcal{P}_n(t) = 1$.

We now define the evolution of the density matrix,

$$\tilde{\rho}_2(t) \equiv \sum_n U_2(n) \rho_2 U_2^\dagger(n) \mathcal{P}_n(t), \tag{4}$$

and noting that

$$\mathrm{Tr}\left(\tilde{\rho}_2(t)\right) = \sum_n \mathcal{P}_n(t) \mathrm{Tr}\left(\rho_2\right) = \mathrm{Tr}(\rho_2) \tag{5}$$

one can equate the conditional probability (2) with the usual expression for a probability in quantum mechanics,

$$\mathcal{P}(o|t)_\rho \equiv \frac{\mathrm{Tr}(P_o(0)\tilde{\rho}(t))}{\mathrm{Tr}(\tilde{\rho}(t))}, \tag{6}$$

where the projector is evaluated at $t = 0$ since in the Schrödinger representation the operators do not evolve. From here on we drop the subscript 2 on the density matrix, since it is understood that we are discussing the density matrix of the system under study.

It should be noted that all the sums in n, due to the assumption that the time variable is semiclassical, are only nontrivial in the small interval $\Delta_t n$ since outside of it, probabilities vanish. Something else to notice is that when we introduced the projectors, there was an integral over an interval. Therefore in the above expression for the evolution of the density matrix, this has to be taken into account. Since the interval Δt is arbitrary, one can consider the limit in which its width tends to zero, apply the mean value theorem in the integrals, and the interval in the numerator and denominator cancel out, yielding an expression for $\tilde{\rho}(t)$ that is independent of the interval, and involves the non-integrated projector $P_t(0)$.

We have therefore ended with the standard probability expression with an "effective" density matrix in the Schrödinger picture given by $\tilde{\rho}(t)$. In its definition, it is evident that unitarity is lost, since one ends up with a statistical mixture of states associated with different n's. We also notice that probabilities are conserved, as can be seen by taking (6) and integrating over x. We recall that $\tilde{\rho}$ is not the normalized density matrix; the latter can be easily recovered dividing by the trace.

We will assume that $\mathcal{P}_n(t) \equiv f(t - t_{max}(n))$ with f a function that decays quite rapidly for values of t distant of the maximum $t_{\max}$ which depends on n.

To manipulate expression (4) more clearly, we will assume we are considering a finite region of evolution and we are in the limit in which the number of steps in that region is very large. We denote the interval in the step variable n as going from zero to N with N a very large number. We define a new variable $v = \varepsilon n$ with dimensions of time such that $N\varepsilon = V$ with V a chosen finite value. We can then approximate expression (4) by a continuous expression,

$$\tilde{\rho}(t) = \int_0^V dv f(t - t_{max}(v)) \rho(v). \tag{7}$$

In this expression $t_{\max}(v) \equiv t_{\max}(n = v/\varepsilon)$ and

$$\rho(v) = U_2(n = v/\varepsilon) \rho U_2^\dagger(n = v/\varepsilon). \tag{8}$$

In all the above expressions, when we equate $n = v/\varepsilon$ it should be understood as $n = \mathrm{Int}(v/\varepsilon)$, which coincide in the continuum limit. (Notice that strictly speaking we should write $\rho(v/\varepsilon)$ to keep the same functional form as for $\rho(n)$, but we will drop the ε to simplify the notation.) To simplify things further we will assume that we chose a physical variable as our clock that has a linear relation with v, i.e. $t_{\max}(v) \sim v$. In practice this is really not possible, there will be departures from this linearity and this is another effect that should be taken into account and will probably lead to further decoherence.

III. AN OPTIMALLY CLASSICAL CLOCK

We now need to make some assumptions about the clock. As we argued above, we use the ideas of Salecker–Wigner and Ng-Van Dam and Amelino-Camelia [9] to suggest that the "most classical" clock one can build is a black hole. Briefly described, the argument goes as follows: if one considers an ideal isolated clock, it will lose accuracy as its wavefunction spreads. To try to diminish this spread, one can increase the mass of the clock. This process cannot continue indefinitely because eventually one will have enough mass to produce a black hole (trying to keep the density low by making a larger clock does not work since then one has to take into account that matter is elastic, etc). The black hole is therefore the most accurate clock from this point of view and it is also attractive as a fundamental clock given its fundamental nature (it is made of spacetime itself). The accuracy of such a clock is given by the quasinormal frequencies of the black hole, which scale as the inverse of the mass. That is, to have a more accurate black hole clock, one needs it to have a small mass. But a black hole of small mass evaporates due to Hawking radiation quickly. This creates a tension between these two requirements that leads to a formula that determines the best accuracy one can achieve in the measurement of a time $T_{\max}$,

$$\delta t \sim t_P \sqrt[3]{T_{\max}/t_P} \tag{9}$$

where t_P is Planck's time and from now on we choose units where $\hbar = c = 1$.

The reader might question why the spread of the wavefunction of the clock limits its accuracy. After all, presumably the clock is interacting with an environment which prevents the wavefunction from spreading. We ignore this effect, since this interaction is further source of inaccuracy in the clock (effects like this have been studied in [10]) and wish to concentrate on the spread, which is an effect of fundamental nature, unrelated to the environment.

IV. APPLICATION TO THE BLACK HOLE INFORMATION PUZZLE

We need to make a quantum model of the black hole in order to study its decoherence. Here we will make a very primitive model. We assume the black hole horizon's area (or equivalently its energy) is quantized. This is usually assumed in quantum black hole studies and in particular it is predicted by loop quantum gravity. We choose a basis of states for the black hole labeled by the energy (area). The problem has some resemblance to the problem of an atom that is in an excited state and emits radiation to reach its fundamental state. If one considers the physical system under study to be the atom plus the radiation field, its evolution is unitary. One would expect a similar situation to hold for the black hole interacting with the gravitational and matter fields surrounding it. Here is where the paradox lies, since the evaporation process leads to loss of unitarity for the total system. Our model will include information about the black hole and the surrounding fields such that it starts its evolution in a pure state, and we will study its evolution according to the formalism developed in section II. We consider the system as described by a density matrix,

$$\rho = \sum_{ab} \rho_{ab} |\psi_a(t) >< \psi_b(t)|, \tag{10}$$

where

$$|\psi_a(t) >= |E(t) + \varepsilon_a, E_0 - E(t) > \tag{11}$$

and where the first entry in the bra (ket) represents the energy of the black hole at instant t, which changes with time in an adiabatic fashion, the constant E_0 represents the mean value of the total energy of the system (which is conserved) and $E_0 - E(t)$ is the energy of the field at instant t. We consider the state to be a superposition of states of the black hole that differ in energy from $E(t)$ by ε_a. To simplify the analysis we consider only a pair of levels of energy that are separated by an energy proportional to the temperature, as one would expect for an evaporating hole. Concretely, the characteristic frequency for this energy is given by

$$\omega_{12}(t) = \frac{1}{(8\pi)^2 t_P} \left(\frac{t_P}{T_{\max} - t} \right)^{1/3} \tag{12}$$

with $T_{\max}$ the lifetime of the black hole (how long it takes to evaporate) and the subscript 12 denotes that it is the transition frequency between the two states of the system. Although this model sounds simple-minded it just underlies the robustness of the calculation: it just needs that the black hole have discrete energy levels characterized by a separation determined by the temperature of the black hole. It is general enough to be implemented either assuming the Bekenstein spectrum of area or the spectrum stemming from loop quantum gravity [11]. We assume that we start with the black hole in a pure state which is a superposition of different energy eigenstates (there is no reason to assume that the black hole is exactly in an energy eigenstate, which would imply a stationary state with no radiation being emitted; as soon as one takes into account the broadening of lines due to interaction one has to consider a superposition of states within the same broadened level with a time dependent separation with a similar behavior). Therefore the density matrix has off-diagonal elements.

We now compute the evolution of the two level model for the black hole using the formulas we developed in section II. We consider the off-diagonal matrix element of the density matrix in an energy eigen-basis, ρ_{12}. Its time evolution is given by,

$$\rho_{12}(t) = \rho_{12}(0) \int_0^V dv \mathcal{P}_v(t) \exp\left(i \int_0^v \omega_{12}(T) dT \right). \tag{13}$$

We can now compute the integral in the exponent,

$$\varphi_{12} = \int_0^v \omega_{12}(T)dT = -\frac{3}{2\left(8\pi\right)^2 t_P} \times \left\{ \left[t_P\left(T_{\max}-v\right)^2\right]^{1/3} - \left[t_P T_{\max}^2\right]^{1/3} \right\}. \quad (14)$$

To compute the evolution we need to provide a model for $\mathcal{P}_v(t)$. We will assume it takes the form,

$$\mathcal{P}_v(t) = \Theta_{\tau(t)}(v-t)\frac{1}{\tau(t)} \quad (15)$$

where the function Θ is one if $|v-t|<\tau/2$ and zero otherwise, that is, a step function of width τ centered at t. As we shall see the determination of the decoherence of the state does not depend on the particular form of the width as a function of t, one only needs to recall that the final width is given by the limit for the accuracy of the clock computed in equation (9).

$$\rho_{12}(t) = \frac{\rho_{12}(0)}{\tau(t)}\int_{t-\tau(t)/2}^{t+\tau(t)/2} dve^{i\varphi_{12}}. \quad (16)$$

To compute the integral, and evaluate it for the value at evaporation time $T_{\max}$ we make the variable transformation $(T_{\max}-v)/t_P = u^3$, and write,

$$\rho_{12}(T_{\max}) = \frac{\rho_{12}(0)}{\tau(T_{\max})}\exp\left(\frac{3i}{2\left(8\pi\right)^2}\left(\frac{T_{\max}}{t_P}\right)^{2/3}\right) \times \int_{-U}^{U} duu^2 t_P \exp\left(\frac{3iu^2}{2\left(8\pi\right)^2}\right) \quad (17)$$

with limits of integration $U = \sqrt[3]{-\tau(T_{\max})/(2t_P)}$. For a Solar sized black hole $T_{\max}/t_P = (M_{\rm Sun}/M_P)^3$, and therefore the integration limits are large. The integral can be evaluated in closed form in terms of Fresnel integrals, but it is more instructive to write the asymptotic form. The modulus of the integral behaves asymptotically as $\sqrt[3]{M_{\rm Sun}/M_P}$. One therefore has an estimate for the modulus of the density matrix element behavior,

$$|\rho_{12}(T_{\max})| \sim |\rho_{12}(0)|\left(\frac{M_P}{M_{\rm Sun}}\right)^{\frac{2}{3}} \sim 10^{-28}|\rho_{12}(0)|. \quad (18)$$

So for astrophysical black holes the puzzle is unobservable. One could still ask what is the situation for black holes that are smaller. We should recall that we have neglected several effects that further imply decoherence, so it is likely that the effect is larger than the estimate we present here.

V. DISCUSSION

The calculation we have carried out here differs from those of our previous papers. In [12] we made a first estimate of the decoherence in the context of the black hole information puzzle. In that first estimate we did not use an optimal clock and used a cruder model of the spectrum of the black hole (temperature independent). This calculation yielded that the quantum state did not decohere entirely by the time of evaporation, though it decohered in a significant amount. In [8] we used an optimal clock and an improved model of the spectrum of the hole, but used only the first order expansion of the evolution equation for the state as valid throughout the whole evolution, namely,

$$\frac{\partial\rho(t)}{\partial t} = -i[H,\rho] - \sigma(t)[H,[H,\rho]], \quad (19)$$

where $\sigma(t)$ encodes the information about the quantum fluctuations of the clock and it is related to the width $\tau(t)$ by $\sigma(t) \equiv \frac{d}{dt}\tau^2/24$. In [8] we did an explicit assumption for the form of $\tau(t)$, defining $\tau^2(t)$ as the difference for the spread of the clock in the interval $[0,T_{\max}]$ minus the spread in the interval $[t,T_{\max}]$, that is

$$\tau^2(t) = t_P^2\left[\left(\frac{T_{\max}}{t_P}\right)^{2/3} - \left(\frac{T_{\max}-t}{t_P}\right)^{2/3}\right], \quad (20)$$

One can then determine $\sigma(t)$ in the expansion to be,

$$\sigma(t) = t_P/36\left(\frac{t_P}{T_{\max}-t}\right)^{1/3}. \quad (21)$$

The calculation in [8] yielded the remarkable result that it erased completely the information by the time evaporation occurs. In this paper we have integrated the full evolution equation and again one finds that there is a large level of decoherence by the time of evaporation.
In the case of laboratory-like experiences our previous approach is justified as roughly an expansion in $w_{12}\tau(t)$, where w_{12} is taken as a the natural energy gap of the system. In order to see this we can integrate now (16) for a time independent spectrum obtaining,

$$\rho_{12}(t) = 2\frac{\rho_{12}(0)}{\tau(t)}e^{i\omega_{12}t}\frac{\sin(\omega_{12}\frac{\tau(t)}{2})}{\omega_{12}} \quad (22)$$

Expanding now in $\omega_{12}\tau(t)$,

$$|\rho_{12}(t)| = 2|\rho_{12}(0)|\frac{sin(\omega_{12}\frac{\tau(t)}{2})}{\omega_{12}\tau(t)} \sim |\rho_{12}(0)|(1-\frac{1}{24}\omega_{12}^2\tau^2(t)\ldots) \quad (23)$$

Noticing that if one is using the optimal clock for a total interval T, according to (9) $\tau(T) = t_p^{2/3}T^{1/3}$ and therefore,

$$\omega_{12}^2\tau^2(t) = t_p^{4/3}\omega_{12}^2 T^{2/3}, \quad (24)$$

we can immediately compare with a similar calculation with a two level system, up to first order, obtained by using (19) in [8], where the level of fundamental decoherence is[14],

$$\log\left(\frac{\rho_{12}(T)}{\rho_{12}(0)}\right) = -\frac{1}{24} t_P^{(4/3)} T^{(2/3)} \omega_{12}^2. \qquad (25)$$

Both expressions are in agreement and the formalism is consistent. The effect is too small to be observed in the lab, unless one can construct a system with a significant energy difference between the two levels. The most promising candidate systems would be given by systems of "Schrödinger cat" type. Bose–Einstein condensates could in some future provide a system where the effect could be close to observability [5, 13]. On the other hand, one could design an experiment where the effect is intentionally large, i.e. choosing a clock that does not behave semi-classically, as a proof of principle. In the case of a Black Hole, where the spectrum is explicitly time dependent, the calculation in [8] can be also seen as an approximation of the exact result in the case of astrophysical black holes. Even though strictly speaking it is not a valid expansion for $t = T_{\max}$, one can show that it is a good approximation to the exact result provided $\tau(t) << T_{\max} - t$, region in which the phase in (16) can be expanded in series. This condition translate into $t_{\max} << T_{\max} - t_P^{2/3} T_{\max}^{1/3}$ and therefore covers a good portion of the $[0, T_{\max}]$ interval for $M_P/M << 1$.

Several caveats are in order. To begin with, it is clear that we have taken a very crude model for the black hole and a more detailed calculation is needed before one can completely write off the black hole information puzzle as an observable effect, but the present calculation provides good hope that the problem can indeed be solved. A realistic calculation seems somewhat beyond the state of the art. For instance, it is clear that the calculation should model quantum mechanically the black hole but also the fields it interacts with in a detailed way in the context of a theory of quantum gravity. There is also the issue that in these calculations we are neglecting the elaborate space-time structure of the black hole and we are treating it as a "star that disappears" in the sense that it occupies a finite region of space and time. This is in order to have a definite "evaporation time" to use in the calculation. In reality, a black hole that evaporates will imply a change in the causal structure of space-time that is yet to be understood. Properties like information loss should eventually be properly framed in a space-time context. This paper can only be viewed as a further step towards understanding how the imperfection in clocks can yield to loss of coherence in quantum mechanics.

Summarizing, we have shown that unitarity in quantum mechanics only holds when describing the theory in terms of a perfect idealized clocks. If one uses realistic clocks loss of unitarity is introduced. We have estimated a minimum level of loss of unitarity based on constructing the most accurate clocks possible. The loss of unitarity is universal, affecting all physical phenomena. We have shown that although the effect is very small, it may be important enough to avoid the black hole information puzzle.

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[14] The result in [8] was computed with an estimated $\sigma(t) \sim \left(\frac{t_P}{T_{\max}-t}\right)^{1/3}$. The factor 1/36 included here comes from explicitly computing $\sigma = \dot{b}$, where $b = \int_{t-\tau/2}^{t+\tau/2} dv \frac{v^2}{2}(\mathcal{P}_v(t) - \delta(v-t)) = \tau^2/24$. See reference [6] for details.