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Countering the ideology of certainty through reflective knowledge in mathematical modeling

Contrapondo a ideologia da certeza por meio do conhecimento reflexivo na modelagem matemática

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Abstract

This article intends to reveal the overlap between certainty in mathematics and the development of a critical view. For this, we use the ideology of certainty concept, which can be understood as a tendency to consider mathematics always right, being the final statement of arguments and applicable in all circumstances, without exception. Additionally, to describe the students' critical posture, we used reflective knowledge. This qualitative analysis was based on the improvement of the steps suggested by Ole Skovsmose for the development of reflective knowledge. The mathematical modeling activities had topics of interest to the students and were carried out in groups. We found that reflective knowledge tends to improve after the production of a first model and that the development of this knowledge is directly associated with the destabilization of the ideology of certainty. On the other hand, many students found it difficult to delve deeper into reflective knowledge and, consequently, have a greater distance from the ideology of certainty. Thus, we realize that it is necessary that assistance be given to students so that they can improve reflective knowledge and counter the ideology of certainty.

Keywords: Mathematical Modeling. Reflective Knowledge. Critical Citizenship. Differential Equations. Higher Education.

Resumo

Através deste artigo pretende-se revelar as imbricações entre certeza na matemática e o desenvolvimento de uma visão crítica. Para isso, utilizamos o conceito de ideologia da certeza, que pode ser compreendido como uma tendência de considerar a matemática sempre certa, sendo a declaração final de argumentos e aplicável em todas as circunstâncias, sem exceção. Adicionalmente, para descrever a postura crítica dos alunos, utilizamos o conhecimento reflexivo. Esta análise qualitativa se respaldou no aprimoramento dos passos sugeridos por Ole Skovsmose para o desenvolvimento do conhecimento reflexivo. As atividades de modelagem matemática tiveram temáticas de interesse dos alunos e foram realizadas em grupos. Constatamos que o conhecimento reflexivo tende a melhorar após a produção de um primeiro modelo e que o desenvolvimento desse conhecimento está diretamente associado à desestabilização da ideologia da certeza. Por outro lado, muitos alunos apresentaram dificuldade em se aprofundar no conhecimento reflexivo e ter, consequentemente, um distanciamento maior da ideologia da certeza. Assim, percebemos que é necessário que um auxílio seja dado aos estudantes para que eles possam aprimorar o conhecimento reflexivo e contrapor-se à ideologia da certeza.

Keywords: Modelagem Matemática. Conhecimento Reflexivo. Cidadania Crítica. Equações Diferenciais. Ensino Superior.

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1 Introduction

In the context of mathematics education, critical citizenship is related to understanding the ways in which mathematics is used in society, evaluating and critically reflecting its use in social, financial, and political structures (KOLLOSCHE; MEYERHÖFER, 2021). The importance of this critical posture has become more evident after the emergence of COVID-19 (MAASS *et al*., 2022), and it is of paramount importance to use mathematics education to contribute to the formation of critical citizens, capable of assimilating and using mathematics in their daily lives (SKOVSMOSE, 1994; KOLLOSCHE; MEYERHÖFER, 2021; MAASS e*t al*., 2022).

To achieve this understanding of mathematics as a daily tool, it is necessary to establish reflections on the role of mathematics, which may involve the following questions (BARWELL, 2013): what are the facts? What is valued? Who benefits from private versions of events? Which decisions are possible or not? What actions can we take?

Maass *et al.* (2022) investigated which skills could help enable students to develop critical citizenship, and recognized that this question led to another, namely: what skills do students need when reflecting on problems in the mathematics classroom? We understand that one of the main skills is reflective knowledge. The use of reflective knowledge (or reflexive knowledge) in the production of a mathematical model involves rescuing the modeled phenomenon, addressing problems and uncertainties in the transposition of real situations to mathematical models and vice versa (SKOVSMOSE, 1994), as well as implying in what ways the model may be formatting reality, that is, recognizing that mathematics influences society and that the production of models can be affected by private values and interests.

Moreover, for the development of critical citizenship, it is necessary that mathematics is not considered as neutral in intentions, nor given the authority of certainty in arguments (SOUZA; ARAÚJO, 2022). Thus, it is important to question the certainty and rationality of mathematics, because to believe that mathematics is always right, is neutral, and does not serve private interests is to be persuaded by the ideology of mathematical certainty which, by the way, is reinforced in everyday life, since that the media constantly uses the supposed guaranteed certainty of mathematics.

Therefore, we agree with Rodrigues, Silva and Silva (2021) when they state that we must rethink the way mathematics is perceived in our society and the implications of this, that is, we must not only identify the presence of the ideology of certainty, but understand how it works, and “looking for ways to overcome it [is] an essential part of this process”

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(RODRIGUES; SILVA; SILVA, 2021, p. 437). Corroborating the aforementioned authors, Araújo (2012, p. 854) recognized that one of the objectives of critical mathematics education is to find ways to “question the ideology of certainty in mathematics”, and it is worth mentioning Straehler-Pohl (2017, p. 12), who considers that “a complete and total identification with the ideology of certainty is an insult to the autonomous, self-conscious, and rational subject”.

Seeking to contribute to the formation of students who become critical citizens, in view of this context of reflective knowledge and ideology of certainty, we carried out an empirical study composed of mathematical modeling activities which were developed in order to stimulate the manifestation of reflective knowledge by students when interpreting the produced models. We intend to study the following question: how can the manifestation of reflective knowledge to validate and interpret a mathematical model help to oppose the ideology of certainty?

2 Theoretical Aspects

2.1 Certainty in Mathematics and its Implications

In this section, we present the conception of the ideology of certainty, exemplifying its action. Mathematics is considered, by common sense, apolitical, and non-ideological, as it is guided by logical arguments and does not aim to meet specific interests. Borba and Skovsmose (1997) attest that mathematics is supported by an ideology of certainty, being in a position of superiority – above everything and everyone – as a non-human device acting to control human imperfection. Thus, the ideology of certainty is the result of the assumption that mathematical knowledge is true, timeless, and neutral, as mathematical, numerical, or statistical arguments are used in order to ensure certain thought, reaffirming positions, and occupying the role of the unquestionable truth and the definitive reply. We are taught to believe in the certainty of numerical confirmation, as numbers do not lie.

Borba and Skovsmose (1997) point out two main meanings of the ideology of certainty. The first is that mathematics is perfect, does not need empirical study and does not serve any private interest, whether political, social, ideological, or otherwise. The second is that math is essential and applies to any situation. A negative consequence that comes from these ideas is the overvaluation of the numerical component and the undervaluation of the intrinsic qualities of the object, which are no longer considered in the face of the supremacy of mathematics.

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The ideology of certainty is born from three premises (SKOVSMOSE, 2019): (1) mathematics is seen as neutral, as it is not subject to influence and does not serve private interests; (2) mathematics is objective, as it presents phenomena as they really are, without subjectivities; (3) because it has a rational character, its certainty is guaranteed. The consequences of this ideology are negative, as decisions can be seen as important and necessary (i.e., right), neutral and objective.

Mathematics occupies a position of power and authority in the media and in school and academic environments (BORBA, 2001), and many believe that it is pure and does not suffer influences, and can be applied in any situation, which camouflages private and political interests. As an example of this, we can mention the fact that we find, in society and in the media, expressions such as: "it has been mathematically proven" and/or "the numbers speak for themselves" (BORBA, 2001, p. 129). Thus, we understand that the ideology of certainty is not only routinely corroborated by the media, but also the teaching and research institutions themselves perpetuate this ideology by separating pure and applied mathematics, without questioning it (SILVEIRA; CALDEIRA; WAGNER, 2019).

As an example, we can mention climate issues (BARWELL, 2013; HAUGE *et al*., 2015; STEFFENSEN, 2021), which involve complex issues that imply uncertainties, conflicts, risks, and values, covering, for example, aspects such as time, central authority, who causes it, who resolves it and the direction of limited actions (STEFFENSEN, 2017). The measurement of temperature involves uncertainties, since it varies in time and space and, precisely for this reason, strata of approximations and simplifications are made at a global level when obtaining the temperature of a location. In this way, uncertainty is minimized, although there is also no guarantee of *certainty*. We can see that, in this context, mathematics is not neutral either, as political discourses on climate issues are uncertain and even contradictory, in addition to the possibility of hidden private interests that direct decision-making to resolve this issue (BARWELL, 2013; HAUGE; BARWELL, 2017). Thus, due to these simplifications of the climate model and these biased discourses, mathematics becomes non-objective.

The opposition to the ideology of certainty must presuppose that mathematics does not always offer the final (or ideal) answer. It is necessary to allow the space for questioning in order to dismantle the ideology of certainty. Thus, it is convenient to allow certainty, in mathematics, to be disturbed by doubt, bearing in mind that the "result can be contested" (BORBA, 2001, p. 137). Critical distance allows us to understand that we are in a mathematized world and, thus, recognize that we are influenced by existing mathematical apparatuses.

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Furthermore, in order to overcome the ideology of certainty, dialogic development and reflective knowledge are essential. Such knowledge can occur when we consider mathematical knowledge as a human production, contingent like others, in which certain values are privileged while others are neglected. Para uma emancipação, a matemática deve proporcionar um ambiente onde o conhecimento matemático abra as portas para um conhecimento reflexivo. Mathematical modeling (MM) is not only an alternative to approach mathematical content, but also to provide opportunities for students to connect their knowledge to reality, making use of reflective knowledge to act critically, thus moving away from of the ideology of certainty and disconnecting from the "exercise paradigm", that is, from the routine mathematical exercises based on an example model and that have no connection with reality (CIVIERO; OLIVEIRA, 2020, p. 174).

2.2 A modeling perspective in favor of critical citizenship

Critical mathematics education is also related to the social and political issues of mathematics learning, involving critical citizenship, both inside and outside the classroom (SKOVSMOSE, 2019). Maass *et al.* (2022) showed that it is possible to associate MM with education in favor of critical citizenship not only in theoretical terms, but in practical situations. In order to carry out this association, one of the objectives must be to involve the MM in the sociocritical perspective, that is, "learn to critically understand your world", connecting modeling with the real world (MAASS *et al.*, 2022, p. 135). We agree with Zapata-Cardona and Martínes-Castro (2021) when they state that the development of critical citizenship is one of the objectives of mathematical modeling from a sociocritical perspective, because in this way students build a model and can use the results obtained to reflect on their surroundings and transform it.

It is important to debate the nature and role of mathematical models, as they do not describe the world in a neutral way and contain information that is not provided to people in general (BARBOSA, 2006). Thus, the mathematical model is both a tool that makes it possible to deepen knowledge and one that enables the development of critical citizenship. In addition, according to the review made by Silveira, Caldeira, and Wagner (2019), one of the targets of the MM, from a sociocritical perspective, has been the ideology of certainty, and the MM has focused on deconstructing the position of unquestionability that the MM mathematics occupies in society.

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Abassian *et al.* (2020) reviewed the literature in order to characterize and present the foundations of the five main international perspectives on MM, including the sociocritical perspective. Following the ideas of this review, we briefly explain this MM perspective.

The main objectives of this modeling conception involve developing MM skills in order to make decisions in society, with the main skill in focus being the development of critical thinking. The competency to think critically involves not only the ability to apply mathematics, but also to reflect on the application made, examining the role of mathematics in society (LOPES, 2023). Thus, the mathematical model is a translation, through mathematics, of a relevant situation.

Thus, the tasks, in addition to involving a social context, need to focus on the development of specific mathematical notions, which is why research in this perspective proposes the use of mathematics by students, so that they can critically understand society, giving sense and meaning to the world that surrounds them. As an example of key researchers in this perspective, we have Jonei Barbosa and Ole Skovsmose, in addition to the great contribution made by Ubiratan D'Ambrósio.

Despite its relevance, according to Gibbs and Park (2022) there is little guidance in the literature on how to design and plan modeling activities that encourage reflective knowledge in practice.

3 Methodological Aspects

3.1 Theoretical-Analytical Structure

Skovsmose (1994) makes a distinction between three types of knowledge in mathematics, and, in a classroom, these three forms of knowledge can coexist and be interconnected. *Mathematical knowledge* relates to the ability to use mathematical skills, such as performing calculations and procedures, developing justifications and mathematical proofs; *technological knowledge* refers to the ability to use mathematics in the context of technology, that is, it involves the competence to produce and use a technology, such as building and applying algorithms that drive technology, for example; *reflective knowledge* refers to the ability to analyze circumstances and their impacts on social issues of mathematical and technological knowledge, based on a broad sphere of interpretation.

Thus, for students to develop the reflective capacity, it is necessary, more than mathematical or technological skills (SKOVSMOSE, 1994), to reflect on social issues, norms,

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and values. Furthermore, the debate on certain issues may involve reflection on ethical, political, and economic aspects, as is the aforementioned case of climate change (HAUGE; BARWELL, 2017; STEFFENSEN; HERHEIM; RANGNES, 2021). In this way, we understand that, in order to act as critical citizens, reflective knowledge becomes essential.

The discussion held in a classroom can be examined through Skovsmose's (2014) six steps of reflection, given that such steps help in the development of reflective knowledge. In short, the steps are reflections that refer to the following questions: (i) Are the calculations performed correctly? (ii) Was a suitable algorithm used? (iii) Is the mathematical approach taken *reliable*? (iv) Would it be feasible to solve the problem without using formal mathematics? (v) How does the mathematical *approach contaminate* the specific circumstance of the problem? (vi) Would it have been possible to have reflected in *another way*?

The six steps mentioned refer to reflective knowledge, which is fundamental for the formation of critical citizenship, and refer to the stages of reflection that are carried out when students solve problems. As an example, Hauge *et al.* (2015) focused on steps (i) and (ii) to analyze the reflective knowledge of graduate students in mathematics education when using/interpreting graphs that predicted temperature change. Unlike Hauge *et al.* (2015), we focus on other steps, considering that we are carrying out MM activities with the aim of investigating a possible distance from the ideology of certainty.

Furthermore, we added a seventh step, which can be summarized in the following question: *what are the possible social implications of this mathematical approach and, in view of this, what actions could be taken?* With this last step (vii), we seek to establish a synchrony with reflective knowledge, as such knowledge is the "necessary competence to be able to take a justified position" (SKOVSMOSE, 1994, p. 35). We therefore intend to improve the development of reflective knowledge.

The first two steps refer to reflection within the scope of mathematics and mathematical knowledge, which are not our focus. The other steps refer to a more comprehensive assimilation of mathematics which includes, by the way, its social function. Therefore, we chose steps (iii), (v), (vi) and (vii) for our analysis. We consider these reflections, which arise during the validation/interpretation of a model, as being essential for the development of reflective knowledge, making it possible to destabilize the ideology of certainty and, consequently, contribute to the development of critical citizenship.

3.2 Contextualization

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The field research was carried out at a federal university in Minas Gerais, between April and July 2020. The Differential Equations (DE) classes were taught by the research professor in two classes. The curriculum covered the study of first-order and higher-order ordinary differential equations (ODEs) and systems of ODEs. The participants were 117 students from nine different engineering courses: environmental engineering, computer engineering, control and automation engineering, electrical engineering, materials engineering, mechanical engineering, mobility engineering, production engineering, and health and safety engineering. These students had already taken the Differential and Integral Calculus course, and most were taking the ED course for the first time. Due to the conditions imposed by the COVID-19 pandemic, classes were taught in synchronous meetings through video conferences with the help of *Google Meet*. The class was planned to adapt to the current curriculum and develop mathematical modeling activities.

3.3 Mathematical Modeling Tasks

The modeling activities were carried out in two blocks, each of which contained two activities. The first block involved modeling using 1st order ODEs. In the second block, the modeling was related to higher order ODEs. Each group chose one topic per block, totaling two activities per group. Thus, similar to Gibbs and Park (2022), the teacher suggested themes to be selected by the groups. The topics are:

1st block: 1A) Alcohol absorption and the risk of accidents
1B) Planning a diet
2nd block: 2A) Consumer purchasing behavior
2B) Spread of an epidemic

Groups of four to six members were formed, chosen by the students themselves. In the first class (T1), the 52 students formed nine groups and in the second (T2), the 65 students formed eleven groups. In the first block, as a mathematical modeling theme, eleven groups chose 1A and nine chose 1B. In the second block, twelve groups chose theme 2B, while eight selected theme 2A. After the end of each modeling block, each group presented a summary with the development made in a synchronous meeting, via videoconference. Finally, students responded to a final evaluation questionnaire that served as an extra tool for personalized and individual analysis. The modeling tasks began in synchronous meetings, through video conferences, and were completed outside of class, asynchronously, with the tasks developed by the groups and the responses to the questionnaire being recorded on the *Moodle* platform.

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MM from a sociocritical perspective is seen as a “learning environment” in which students, gathered in small groups, can examine a real problem through mathematics (BARBOSA, 2006, p. 294). Rather than describing how the modeling cycle occurs, this perspective “simply describes the mathematical model as a mathematical representation of a real-world situation” (ABASSIAN *et al.*, 2020, p. 61). Thus, we detail how we condutect the modeling activities, in the research reported here.

We divided each modeling activity into three stages, aiming to manifest the three types of knowledge: In the first stage (a), students had contact with the problem and debated its formulation; in stage (b), they solved the equation found and sketched graphs of the solution function and others for a better understanding of the problem; in the last stage (c), the aim was to interpret the results found. Mathematical knowledge was used to carry out the first two stages and technological knowledge was necessary to use computer programs to sketch the graphics in the second stage. In turn, reflective knowledge was manifested already in the first stage, when starting to discuss the problem and choosing the main variables. Despite this, reflective knowledge had its greatest manifestation in stage (c), when students discussed the results obtained and connected them with reality, and this last stage is the focus of our analysis.

We illustrate, in Table 1, the first proposed modeling activity. The book by Burghes and Borrie (1981) served as inspiration for this topic. We started with this MM task, as it was the one that presented the least mathematical complexities, so the expectation was that students would focus mainly on the critical discussion of the phenomenon.

Problem: how to develop a mathematical model that equates the risk of a person being involved in a traffic accident after drinking an alcoholic beverage?

(a)

(a. 1) Identify the variables involved in the problem. What is the rate of variation to be analyzed? What are the relationships between the variables?

(a. 2) A mathematical model can be formulated to relate the risk of having a car accident, R, and the blood alcohol level, b. The relative accident risk equation is as follows:

$$\frac{dR}{db} = kR$$

Note that the equation presents a proportionality constant k. Do you think this model is reasonable to describe the phenomenon in question (accident risk x alcohol intake)?

(b)

(b. 1) Solve the problem equation. Consider that at $b = 0$ (no alcohol consumption), the risk of an accident is 1%, that is, $R_0 = R(0)= 1$.

(b. 2) Considering the solution obtained, determine the constant k knowing that we have R = 20% at b = 0.14%. What happens at R=100? What do these values mean?

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(b. 3) Sketch the solution function obtained from the phenomenon. What change would there be in the graph with constant *k* variation?
(c)
(c. 1) What did you think of the model obtained: Does it fit reality? What points do you consider positive and/or negative?

Table 1 – Topic 1-A: alcohol absorption and the risk of accidents
Source: prepared by the author (2022)

After completing all the tasks, we administered the questionnaire, in which we obtained 102 respondents. We asked three questions: (1) How do you perceive the relationship between school mathematics and people's daily lives? (2) How do you use mathematics (basic or higher level) in your daily life? (3) How did modeling activities contribute to giving new meaning to your mathematical knowledge?

The data were examined using content analysis (BARDIN, 2016), through categories determined a posteriori. To simplify, as we have several groups in two classes (T1 and T2), we adopted the notation T1G1 and M1A to indicate, respectively, group 1 of class 1 and topic 1-A, and similarly for the other groups and topics. We also adopted fictitious names to replace the students' real names.

In the next section, we present the three topics evaluated: (1) reflective knowledge in groups, (2) reflective knowledge in a second mathematical modeling activity, and (3) individual reflective knowledge.

4 Results and Discussions

4. 1 Reflective Knowledge in the Groups

In the first modeling stage, it was assumed that students would initiate critical discussion around the chosen topic, but few reported their opinions. We therefore found that they were not familiar with this type of mathematics activity, thus requiring an external incentive to articulate (ANHALT *et al*., 2018). Consequently, some questions were asked in order to help them at the beginning of the discussions, for example: what should be the main variables of this phenomenon, and which are the least relevant? What physical principle or type of behavior is involved? What can increase the risk of a traffic accident? Then, they completed the mathematical knowledge part, as shown in Table 1. Using 1st order ODE resolution techniques, they found the solution function and then sketched the graph of the obtained function.

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In the third stage, the expectation was that the groups would validate and interpret the model and make connections with reality. We can quote the statements from the T2G4 group:

> *From the general solution we obtained in the differential equation, we see that there is a dependence between the variables. This model is best explained by the increase in the exponential curve generated in the graph* (T2G4 Group Statement, 2020).

> *The risk of a person being involved an accident after drinking is greater with increased alcohol consumption, as there is an increase in the blood alcohol level* (T2G4 Group Statement, 2020).

We observed that the T2G4 group managed to interpret the phenomenon in mathematical terms, but there was no reflection on the reliability of the mathematical approach and how this approach could contaminate the circumstances of the problem or, even, whether another reflection would be possible. Furthermore, there was no critical analysis that related the conclusions to social issues, as expected. Some other groups (groups T1G1 and T2G1) approached the phenomenon in a similar way, mathematically interpreting the results obtained and describing the behavior of the sketched graph without, however, taking a critical approach.

On the other hand, other groups developed analyzes beyond mathematical interpretation, that is, they understood the phenomenon and identified points for improvement, such as variables to include, for example. This was the case for the T2G8 group:

> *Although there are some positive points – such as numerically demonstrating the risks that exist when driving after drinking alcohol – the model does not adapt to reality, as the number of existing variables is greater, such as: weight, gender, type of drink, age, and time elapsed after consumption* (T2G8 group thought process, 2020).

We realized that the T2G8 group was able to identify that the model produced *was not perfect*, as they pointed out adjustments and improvements that could be made to improve it and realized that the problem could have been solved in another way, exposing the phenomenon in another way.

The T1G6 group also identified other aspects of improvements:

> *The model obtained is suitable for identifying risk and accidents, however, aiming for a better developed model, other variables must be considered (such as the person's weight, road conditions, etc.). More variables would make the model more complex and more realistic. It is also worth mentioning that drinks have different levels of alcohol, and this can interfere with the model, potentially representing another variable* (T1G6 group thought process, 2020).

The T1G6 and T1G7 groups identified that problems on the track could directly affect the model, and, by being able to identify improvements, we recognize that these groups (T2G8, T1G6 and T1G7) reflected critically. In addition to these, three other groups (T1G3, T2G6, T2G2) recognized aspects that could be improved; however, they did not identify any relationship with social issues, nor did they suggest possible actions considering what was analyzed.

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On the other hand, some groups did more than just mention the inclusion of other variables that could be relevant to describe the phenomenon: they identified social issues involved and/or suggested action to be taken. As an example, we can mention the T2G9 group, which recognized:

> *[...] the risk of someone being involved in a car accident after drinking alcohol is high, and it is absolutely necessary to raise awareness among the population so that they do not do so* (Reflection of the T2G9 group, 2020)

Analyzing this comment, we realized that the T2G9 group understood that it is extremely important to carry out a social action to raise awareness among the population. The T2G3 group also recognized that the model could be improved by including a variable that would open the range for specifying types of drinks with different alcoholic strengths. Furthermore, he added:

> *Observing the graphs, it became possible to notice the coherence of the severity applied by the Dry Law, since, no matter how small the volume of alcoholic beverage intake, some of the skills required for safe driving the vehicle may be compromised by the effects of drunkenness, such as such as: the reduction of motor and cognitive capabilities, in addition to reflexes and, mainly, quick responses to decision-making* (Reflection of the T2G9 group, 2020).

We noticed that there was an association between what was analyzed and the existence of a law, that is, the T2G9 group exceeded the mathematical discussions and suggestions for a more efficient model by presenting a relationship between the model and social issues.

In short, we found that – fortunately – few groups focused only on the mathematical discussion of the phenomenon. However, although many were able to identify points of improvement, contributing to an improved mathematical approach, few related the situation under study to any social issue and/or made suggestions for solving identified problems. We can say, therefore, that many groups took a step against the ideology of certainty, since our analyzes showed that this was not so ingrained in many students who interpreted the models produced, since the objectivity of mathematics was questioned when they realized that the modeled phenomenon undergoes changes through the choices and simplifications made. Despite this, few demonstrated a greater distance, as expected, as we hoped them to present and discuss the implications and social consequences of the model, indicating and, additionally, some action/resolution.

4.2 Reflective knowledge in a second mathematical modeling activity

For topic 2-B, we used the article by Catlett, (2015) and the book by Burghes and Borrie (1981) as reference. We did not detail the steps, as we did in Table 1 for M1A, due to the

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limitations imposed on an article, but the problem proposed in M2B was: how to model the evolution of an epidemic?

We used the results of the production of this model in order to investigate the manifestation of reflective knowledge in the modeling of the second block. Due to the new coronavirus pandemic, experienced at the time of modeling activities (in the first half of 2020), several groups chose to carry out M2B to produce a model of the evolution of COVID-19. For the development of the model, aiming to streamline discussions carried out by the groups, it was suggested that a city be chosen to determine the constants involved in the phenomenon's EDO system.

In the second block, unlike the first, all groups presented some type of discussion beyond the mathematical content, even if in a generalist way. The T2G10 group, for example, stated:

> *The negative point is that the model produced does not fully adapt to reality, as it lacks certain specifications of variables and constants, in addition to not being versatile* (T2G10 group statement, 2020).

Although the group recognized the need for changes and realized that the model could be improved, it was not specified *how* this could be done. Still, we consider an improvement – even if not very significant – in reflective knowledge, in addition to a slight departure from the ideology of certainty. Likewise, the T1G5 group presented general considerations.

On the other hand, let us focus on the statements made by the T2G6 group.

> *The trajectory of diseases is predictable and uniform and, therefore, an extremely useful and reliable tool [...]. Prevention depends on factors that go beyond financial resources* (T2G6 group statements, 2020).

We observed, through the excerpt, that the T2G6 group did not specify in what sense the model would be an *extremely useful tool* or to what extent *predictable*, why it would be reliable, in addition to lacking further explanations about why prevention depends on factors that *go beyond financial resources*. Thus, we noticed that in groups T1G10, T1G5, and T2G6 there was no inclusion of any consideration about the model's relationships with social issues or any indication of agency.

We note, however, that many groups demonstrated competence in reflective knowledge, although some considerations they presented prevented them from advancing further. This was the case of the T2G5 group, which mentioned that the justifications for government actions and the data presented by it may differ from reality when stating that.

> *[...] the number of infected people in the country could be six times higher than that reported by the authorities* (T2G5 group statement, 2020).

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Although they criticized the government, this group mentioned statistical data without any reference. This type of practice may indicate media influence and society's tendency to use mathematical terms and estimates without basis or reference. Thus, despite having addressed a relevant sociopolitical issue, there was a lack of basis to give credibility to the statement.

The T2G8 group confirmed the validity of the model they produced, considering it updated, and compared the results obtained to those arising from a model widely used and publicized by the media. Considering that the model of propagation of an epidemic used is old and was produced under different circumstances (CATLETT, 2015), this group relied on the objectivity and neutrality of mathematics by not debating the current nature of the model in question, that is, they trusted the mathematical approach without questioning it.

On the other hand, the T2G9 group stated that.

> *[...] mathematical models are supporting the decision-making of governments in developed countries that care about their citizens* (T2G9 group statement, 2020).

By realizing the relationship between the use of models and decision-making by governments, this group made a relevant sociopolitical observation in the COVID-19 pandemic, despite having only considered the situation in developed countries, leaving room for the consideration that governments in underdeveloped countries might not be efficient in this regard. When the students carried out this MM activity, the quarantine had already been declared in the country and people in need were receiving assistance from the government, as well as low-income students were able to request assistance from the university. By not discussing these and other issues, we consider that this group did not delve deeper into reflective knowledge.

The T1G3 group, on the other hand, recognized that another approach would be possible by suggesting the inclusion of the age group variable in the model, since, according to the group,

> *[...] the age group is decisive for the onset of the disease in that individual* (T1G3 group statement, 2020).

However, despite the contribution, the T1G3 group did not go deeper and/or develop a more elaborate speech about the suggestion made. In this way, we recognize that the groups T2G5, T2G8, T2G9, and T1G3 showed a certain depth in reflective knowledge and distanced themselves, even if not much, from the ideology of certainty.

On the other hand, five other groups stood out by moving further away from the ideology of certainty, presenting a deepening of reflective knowledge and, some of them, indicative of action. Everyone went beyond the first levels of discussions and mathematical interpretations,

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and, except for the T2G2 group, everyone suggested aspects to improve the mathematical model.

The T2G4 group mentioned that producing a model is not something precise, as people's behavior varies and not everyone wears masks and maintains social distancing, for example. They also recognized:

> *[...] the behavior of the infected curve is a powerful tool for the government to act with preventive and corrective measures* (T2G4 group thought process, 2020).

Likewise, the T1G2 group recognized the importance of

> *[...] making predictions and outlining protective measures in relation to the health situation of the population and the economy* (T1G2 group thought process).
>
> *The importance of such models nowadays lies in the fact that they can be used both for a government to control the spread of the virus with protective measures (such as quarantine and the like), as well as to know when to interrupt/resume the traffic between countries. Furthermore, alarming people about the danger of the virus can encourage everyone to take necessary health measures to prevent the virus from spreading* (T1G2 group thought process).

Let's look at the excerpt from the T1G7 group:

> *[An effective model helps to] reduce the probability that the numbers of infected people reach the mathematical projection", [as] protective measures (such as social isolation and use of masks) can be taken according to the mode of transmission of the disease and the severity, helping to control the spread until a vaccine is developed* (T1G7 group thought process, 2020).

Finally, the T2G2 group mentioned that the model would be better formulated with

> *[...] data obtained from health departments, [therefore, such government bodies must] always disclose the data, [as this] will make it possible to carry out mathematical analyzes consistent with reality* (T2G2 group thought process, 2020).

Thus, students in this group understand that.

> *[...] the model is important not only to alert the population about the risk of contagion, but also to show those responsible for public health the direction they should follow, [although the published data may] cause despair in the population, as people will be worried about the numbers* (T2G2 group thought process, 2020).

We noticed that these groups realized that mathematics can be used for private interests and to support political decisions (BORBA; SKOVSMOSE, 1997; SKOVSMOSE, 2019). In addition to suggesting improvements to the model, demonstrating that the model is not objective and that another approach is possible, some indicated actions (such as constant dissemination of data) to develop a model closer to reality, in addition to suggesting the use of preventive measures in for population health. Thus, we understand that such groups exceeded mathematical knowledge and presented an improvement in reflective knowledge when debating the objectivity and neutrality of mathematics, as they moved reasonably away from the ideology of certainty. Comparing the results of the second block with the ones of the first, we noticed

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that, even with the presence of more relevant mathematical difficulties in the second block, there was a greater deepening of reflective knowledge and a greater distance from the ideology of certainty.

4.3 Reflective Knowledge Individually

In relation to reflective knowledge individually, ascertained through the questionnaire, we noticed that the attachment to mathematical certainties was greater individually, that is, we can say that the level of criticality was lower. The greater degree of criticality in group modeling activities is probably associated with the fact that the exchange of ideas favors more complex discussions and significant contributions (CIVIERO; OLIVEIRA, 2020).

We found several students who did not develop reflective knowledge in the questionnaire, as expected. Here we have the highlights given by the following students:

> *Sara: [...] the question arises 'how to live in a world without mathematics?' It would truly be IMPOSSIBLE.*
> *Rodrigo: [...] mathematics is involved in all areas, and they depend on it to formulate solutions to every problem. Mathematics is essential in our lives* (Individual statements from research subjects, 2020).

These comments show that these students see the world from the perspective of the essentiality of mathematics and, more importantly, they do not question whether the approach taken is even appropriate. As they did not present any suggestions for action, we conclude that these students did not develop reflective knowledge and considered mathematics as objective, reliable, and applicable in all situations, that is, we can say that they did not distance themselves from the ideology of certainty.

On the other hand, other students advanced in reflective knowledge, as explained in the following excerpts:

> *Alice: [...] mathematics teaching in public schools is not of high quality and mathematics is not taught with a focus on real applications.*
> *Luciano: The mathematics taught in schools is far from the students' daily routine reality. Much of the past theory is not applicable to students' reality, causing a lack of interest on their part. I believe that it is necessary to bring the student to the field, as this way they will be able to really understand the applicability of what is learned in the classroom* (Individual statements from research subjects, 2020).

Furthermore, although some have presented criticisms of education, they did not delve deeper into this issue, that is, they did not explore the critical bias mentioned, including the implications of the use of mathematics, for example.

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Some students additionally recognized that MM provides a different learning environment. Let's look at their statements.

> *Kelvin: [...] the production of models diversified, bringing changes to the exercise routine, list, and questionnaire.*
> *Levi: [...] before we worked with exercises in which the equations did not have an applicability In modeling, we transform an everyday problem into equations, which end up giving more meaning to what we are working on* (Individual statements of research subjects, 2020).

We therefore noticed that these students recognized that MM activities presented an opportunity to distance themselves from the exercise paradigm and apply the content to a real problem (BORBA, 2001; CIVIERO; OLIVEIRA, 2020). Although some others made similar comments, the acceptance of the MM was not unanimous, as two students, for example, cited their preference for the traditional approach:

> *Antônio: [...] traditional, with lectures and lists of exercises* (Individual statement of research subject, 2020).

Some students went deeper into developing reflective knowledge and some presented suggestions for actions. Students like Fernanda and Lorena recognized that traditional teaching does not apply much to reality and perceived a connection between MM and its contribution to a critical perspective by stating that the activities were an opportunity to develop their criticality, as seen in the examples.

> *Pedro: [...] I believe that modeling makes us reflect and research, which contributes to more critical and grounded training on the subjects.*
> *Eduardo: [...] we need to know how a brand exerts social, economic, and political influence in a region.*
> *Júnia: [...] a government, nation, individual or company can adapt a model to their liking, taking advantage of the analyses. The model is a projection and may be wrong or even outdated. We see that mathematical modeling is supporting the actions taken by governments and countries* (Individual statements of research subjects, 2020).

Before we begin the discussions and final considerations, it is worth pointing out some limitations regarding task application. We found that the fact that students discussed little about the chosen topic may be related to task formulation. We realized that a more *open* and *free* formulation of tasks can encourage student expression. Therefore, seeking to resolve this point, a new application could reformulate the tasks, causing the initial part (a) to be presented to students containing more questions that encourage them to reflect and discuss the phenomenon under study. In this reformulation, we can include, for example, a suggestion for students to present an equation for the problem.

5 Discussions and Final Considerations

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Returning to the objective of the article, we intend to clarify the overlap between certainty in mathematics and the development of a critical vision. To this end, we investigated how the manifestation of reflective knowledge to validate and interpret a mathematical model can help to counter the ideology of certainty.

We noticed that many groups demonstrated some reflective knowledge and a certain degree of departure from the ideology of certainty, especially in the second block of modeling. This may be an indication that MM activities can help in the formation of individuals who question the position of certainty occupied by mathematics. In the first block, the demonstration of reflective knowledge and the departure from the ideology of certainty occurred to a lesser extent and few suggested agency, despite the groups considering the models in this block easier to implement.

In individual terms, few presented and discussed social issues and indicated some action. We noticed that many students did not demonstrate reflective knowledge and were attached to the ideology of certainty. Those who presented social issues were, for the most part, concise in their comments, and identified problems related to the education system, especially the public one. The set of MM activities enabled a different learning environment in which reflective knowledge could be developed. This environment offers an alternative to the checklist paradigm and provides an opportunity to distance oneself from the ideology of certainty, as well as providing an occasion for investigations. (SKOVSMOSE, 2014).

These results seem to indicate that more activities that promote reflective knowledge should be developed with students, because, as some research indicates, the individual's criticality can improve over time (CEVIKBAS; KAISER; SCHUKAJLOW, 2022). We agree that it is necessary to allow time and opportunities for students to experience being critical in a constructive way (STEFFENSEN; HERHEIM; RANGNES, 2021). Thus, although we do not intend to assume that two MM activities are sufficient for the development and improvement of reflective knowledge and for an effective distancing from the ideology of certainty, we found signs of an improvement in reflective knowledge and a distancing from the ideology of certainty through MM.

Skovsmose (2014) considers reflective knowledge to be fundamental for the development of critical citizenship and departure from the ideology of certainty, and the ability to question the production of the model and the reasons for the choices involved in its production is fundamental (BARWELL, 2013; HAUGE *et al*., 2015). Furthermore, recognizing the biased nature of a model is relevant, as an inadequate model can lead not only to an incorrect

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reading of a given situation, but also to incorrect management (BARBOSA, 2006; SKOVSMOSE, 2019).

In relation to the manifestation of reflective knowledge, both at the individual level and in groups, we noticed that it was more noticeable in the presentation/exposure of a social issue than in the complexity of the argumentation of the issue in question. We noticed, for example, that there was a lack of critical depth in relation to government actions and the use of mathematical data and, therefore, we infer that assistance is needed for students to develop their critical skills, that is, students must not only be presenting numerical data for discussion, but also debating it and, according to Barbosa (2006, p. 298), stimulating discussions "should be a primary objective" of teachers.

Just as we use questions at the beginning of MM activities, we suggest that groups of questions be applied throughout the modeling process about "the assumptions embedded in mathematics and on the role of mathematics" (HAUGE; BARWELL 2017, p. 33), in order to help students not only express themselves, but also develop reflective knowledge (ANHALT *et al*., 2018; LOPES, 2022). However, it is important to be cautious in order to avoid possible impositions of values, particular views or biased questions that direct students' responses (GIBBS; PARK 2022).

Furthermore, it is important that current and future teachers support and encourage their students to participate in critical discussions, as this way they will develop their critical competence. It is the role of the educator to "initiate and facilitate such opportunities, to provide the external stimulus necessary for critical reflection to occur" (WRIGHT, 2021, p. 161), since this incentive can enhance discussions regarding the role of models in society.

However, it is important to highlight that we do not want to say that the way to become a critical individual is to abandon mathematical and scientific results, as this could be dangerous and lead individuals to believe in *conspiracy theories and fake news* (SOUZA; ARAÚJO, 2022). However, enabling students to recognize uncertainties involved in mathematics can strengthen science (STEFFENSEN; HERHEIM; RANGNES, 2021). Critical mathematical education highlights critical citizenship, and activities that promote reflective knowledge, such as mathematical modeling, can be relevant in the development of a critical citizen.

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