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ARTICLE

Problem Posing: understandings

Proposição de Problemas: entendimentos

Janaína Poffo **Possamai**[*]
ORCID iD 0000-0003-3131-9316
Norma Suely Gomes **Allevato**[**]
ORCID iD 0000-0001-6892-606X

Abstract

The problem posing has gained prominence in curriculum documents and current research, in which many questions are still open, and are not manifested clearly and coherently in school practices. This article aims to identify and establish understandings for the terms formulation, elaboration, creation, and problem posing to support the context of Brazilian research and practice in the field of Mathematics Education. To this end, a literature review is presented, resuming historical aspects and constituting a theoretical reference about the implications on the context of mathematics learning and students' comprehensive education. Then, based on the results obtained from a systematic study review and moving forward in the constitution of a theoretical body knowledge, we present the meanings attributed to these expressions in several Brazilian and international studies and then present the definitions we assume: we understand that problem creation involves the processes of problem formulation and elaboration and it is inserted in the problem posing which advances to present the created problem to a potential solver. The relevance of these definitions covers both contexts of educational practices that contemplate the association with problem solving and research in Mathematics Education, with conditions and reflections on the collection and analysis of data, in order to deepen the understanding of specific aspects related to problem posing to be considered in future studies.

Keywords: Elaboration/Formulation of problems. Problem solving. Mathematics Teaching

Resumo

A proposição de problemas tem ganhado destaque em documentos curriculares e pesquisas atuais, em que muitas questões estão ainda em aberto, e não se manifestam clara e coerentemente nas práticas escolares. Este artigo tem como objetivo identificar e estabelecer entendimentos para as expressões formulação, elaboração, criação e proposição de problemas com vistas a subsidiar o contexto de pesquisas e práticas brasileiras, no âmbito da Educação Matemática. Para tanto, apresenta-se uma revisão da literatura retomando aspectos históricos e constituindo um referencial teórico acerca das implicações no contexto da aprendizagem matemática e da educação integral dos estudantes. Então, a partir dos resultados obtidos de um estudo de revisão sistemática realizado, e

[*] Pós-doutorado em Ensino de Ciências e Matemática pela Universidade Cruzeiro do Sul (UNICSUL), Doutora em Engenharia de Produção pela Universidade Federal de Santa Catarina (UFSC). Professora e pesquisadora da Universidade Regional de Blumenau (FURB), Blumenau, SC, Brasil. E-mail: janainap@furb.br.
[**] Doutora em Educação Matemática pela Universidade Estadual Paulista Júlio de Mesquita Filho (UNESP). Professora e pesquisadora do Programa de Pós-graduação em Ensino de Ciências e Matemática da Universidade Cruzeiro do Sul (UNICSUL), São Paulo, SP, Brasil. E-mail: normallev@gmail.com.

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avançando na constituição de um corpo teórico, apresenta-se os sentidos atribuídos a essas expressões, em diversos estudos brasileiros e internacionais, para, então, serem apresentadas as definições por nós assumidas: entendemos que a criação de problemas envolve os processos de formulação e elaboração de problemas e está inserida na proposição de problemas, que avança para apresentar o problema criado para um potencial resolvedor. A relevância dessas definições explicitadas abrange tanto contextos de práticas educativas que contemplem a associação com a resolução de problemas, quanto a pesquisa em Educação Matemática, com condicionamentos e reflexos sobre a coleta e análise de dados, com vistas ao aprofundamento das compreensões acerca de aspectos específicos ligados à proposição de problemas a serem considerados em estudos futuros.

Palavras-chave: Elaboração/formulação de Problemas. Resolução de Problemas. Ensino de Matemática.

1 Introduction

Problem Posing has been recognized (Cai; Hwang, 2020) as an emerging research demand in Mathematics Education, especially in studies associated with Problem Solving, as well as due to the strong appeal to be incorporated into school curricula and practices as mathematical task (Brazil, 2018; NCTM, 2020).

This search to make problem posing a present activity in Mathematics classes is accompanied by the argument that students are asked to solve problems, but few opportunities are offered for them to invent their own problems, despite the relevance of this activity (Brown; Walter, 2005; Kilpatrick, 1987).

George Polya (1988, p. 68, author's emphasis), considered the father of Problem Solving, already highlighted in his famous book *How to Solve It*, that "the mathematical experience of the student is incomplete if he never had an opportunity to solve a *problem invented by himself*".

The creation or invention of new problems, whether by students or teachers, is often called problem posing in international studies (Koichu; Kontorovich, 2013; Li et al., 2020; Silver, 1994a; Singer; Ellerton, Cai, 2013; Zhang; Cai, 2021), as we will discuss in more depth in the next sections.

In Brazil, Possamai and Allevato (2022) present a systematic review study of 24 dissertations and theses, revealing that the terms elaboration, formulation, and posing are used to designate activities linked to the creation of problems by students, but without a consensus or clear positioning about their meanings, understandings and forms of implementation.

We consider that the literal translation of the English expression *problem posing* does not reflect the diversity of social actions and practices related to these terms. Furthermore, formulation, elaboration, and posing are expressions whose understanding transcends dictionary records, as we have already seen, and which give rise to different portraits of the school's sociocultural context. The meanings of these terms in educational practices involve

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both the teacher creating and presenting problems and the students being asked to do them and, therefore, defining these terms involves a context of complexity, the challenge of which we dare to take on and record in this text.

In this aspect, Branco and Maia (2016, p. 214, our translation) emphasize that "[...] if language represents the world symbolically, the messages exchanged between individuals have information circumscribed in the set of ideals of each society, since the linguistic codes reveal the idiosyncrasies of each cultural context".

It is within this scope that the research described in this article was developed, aiming at identifying and establishing an understanding of the terms formulation, elaboration, creation, and posing of problems aiming at supporting the context of Brazilian research and practices.

To socialize the results of this research, theoretical elements related to the theme of this study are presented below, revisiting historical aspects and discussing educational implications; the methodological characterization of the research; the understandings and definitions assumed by us, based on the studies we carried out; and, finally, final considerations and references.

2 Theoretical scenario

In the book *Evolution of Physics*, Einstein and Infeld (1938) already portrayed that the formulation of problems marks the real advance in Science, stating that it "[...] is often more essential than its solution, which can be just a matter of mathematical or experimental ability [...]" (Einstein; Infeld, 1938, p. 95, our translation). Portraying the difference between formulating and solving problems, through an analogy, the authors discuss the difference between a scientist and a detective, indicating that the scientist/researcher is responsible for defining and investigating (formulating) the problem, while the crime is given to the detective, the problem is posed, and it is up to him to find the criminal (solve the problem).

Another important book in the history of Science, especially Mathematics, *How to Solve It: a New Aspect of Mathematical Method*, written by George Polya (whose first edition dates back to 1945), already indicated some ways of modifying problems after obtaining their solution, thus generating new problems.

However, almost 35 years later, Getzels (1979, p. 170) lamented that problem posing was a neglected area in research, warning that "[...] there are differences well worth study between the act of thought in problem solving that begins with an already formulated problem and one that must begin with discovering or creating the problem itself".

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Although writing creatively, and inventing stories, is part of students' school experiences as an important form of expression for language development, teachers rarely encourage them to write creatively in Mathematics by inventing problems, despite Mathematics also being considered a language (Ellerton, 1986).

Kilpatrick (1987) also took a position on the problem formulation and highlighted:

> [...] almost all of the mathematical problem that a student encounters have been proposed and formulated, by another person - the teacher or textbook author. In real life outside of school, however, many, if not most, must be created or discovered by the solver (Kilpatrick, 1987, p. 124).

These arguments, constructed throughout history and portraying the importance of problem posing as a possibility to expand and deepen the work with problem solving in teaching and learning mathematics, have also been reflected in the constitution of curricular documents.

In several countries, curricular documents guide that the problems to be solved are posed not only by teachers but also by students. The National Statement on Mathematics for Australian Schools (Australian Education Council, 1991, p. 47), when discussing teaching strategies related to problem solving, expresses that “Students should engage in extended mathematical activities which encourage problem posing, divergent thinking, reflection and persistence. They should be expected to pursue alternative strategies, and to pose and attempt to answer their own mathematical questions”.

The *National Council of Teachers of Mathematics* (NCTM), which is the largest Mathematics teaching organization in the world and which has influenced curriculum reforms in several countries, especially in the United States and Canada, also brings this guidance to the discussion of the curriculum for School Mathematics. The document *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989, p. 138) explicitly states that students must “[...] have some experience recognizing and formulating their own problems, an activity that is at the heart of doing Mathematics [...]”.

Next, the document *Principles and Standards for School Mathematics* highlights that it is natural for young children to pose problems involving curious situations (How long does it take to count to a million?) and everyday situations (How much money do I still need to be able to buy the toy?), and indicates that it is up to parents and teachers to encourage them to pose mathematical problems in their worlds (NCTM, 2000). Furthermore, it emphasizes that “teachers should regularly ask students to formulate interesting problems based on a wide variety of situations, both within and outside mathematics” (NCTM, 2000, p. 258).

More recently, the document *Catalyzing Change in Early Childhood and Elementary Mathematics* emphasizes that:

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> [...] When children cannot ask 'Why?' or share their doubts, their interest and engagement with Mathematics become compromised. However, when children are encouraged to ask questions and pose their own mathematical problems, beliefs about what mathematics is and how it can be approached can be powerfully affected [...] (NCTM, 2020, p. 49, our translation).

Also, in other countries, problem posing has been incorporated into school curriculum documents associated with problem solving. In the Chinese curriculum, one of the objectives is that "[...] students to change their learning styles from passive to active through being engaged in problem posing and problem solving" (Cai et al., 2016, p. 6). In the secondary school curriculum in Cuba, it is indicated that Mathematics teaching takes place through the posing and solving problems (Ramirez, 2006). In Singapore, the curriculum for secondary education associates problem solving with Mathematical Modeling, highlighting the importance of students getting involved in solving problems associated with real situations, placing as one of the topics for student assessment in Mathematics, that they develop the "[...] ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; [...] (Singapore, 2019, p. 38).

Following this movement, in Brazil, the National Common Curricular Base (Brazil, 2018) indicates that students not only solve problems but also formulate/elaborate problems, strongly indicating this association in a growing number of skills involving these activities to be developed throughout Basic Education.

However, despite the discussion on the importance of problem posing being long-standing, and being explicitly in current curricular documents, it constitutes a new demand, showing that:

> [...] problem posing needs to pervade the education systems around the world, both as a means of instruction (meant to engage students in genuine learning activities that produce deep understanding of mathematics concepts and procedures) and as an object of instruction (focused on developing students' proficiency in identifying and formulating problems from unstructured situations) with important targets in real-life situations. (Silver; Ellerton; Cai, 2013, p. 5)

These indications are evident from and within Mathematics Education's researches. In several countries, studies that place students as those who pose mathematical problems are being developed, associated with the analysis of educational impacts. It is worth mentioning the books entitled *Mathematical Problem Posing: from Research to Effective Practice* (Singer; Ellerton; Cai, 2015) and *Posing and Solving Mathematical Problems: Advances and New Perspectives* (Felmer; Pehkonen; Kilpatrick; 2016), which incorporate the work of researchers from more than 20 countries dealing with this topic.

Silver (1994a) highlights that problem posing has the potential to personalize and

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humanize Mathematics in the classroom, especially through more open activities in which students are invited to express their lived experiences, reflecting personal values and commitments. The author reports that, in this type of activity, students make fewer arithmetic errors, which are trivial when solving problems due to carelessness or lack of attention, as they feel like owners of that production and are more committed to the problem and, consequently, to the Mathematics involved in the resolution.

There are several reasons to believe that the association of problem posing with problem solving can have a positive impact on student learning. Generating subsidiary problems, such as problems with smaller numbers, for example, to solve a given problem, improves understanding and aids the resolution process (Polya, 1988; Cai; Brook, 2006). In fact, students' performance in problem solving is highly correlated with their performance in problem posing (Cai; Hwang, 2003; Silver; Cai, 1996). Problem posing, as an activity in which students are asked to create problems, "[...] is therefore not only likely to foster student understanding of problem situations, but also to nurture the development of more advanced problem-solving strategies" (Cai *et al*. 2013, p. 60).

Furthermore, when students solve or ask others to solve their problems, they have the opportunity to evaluate and improve the quality of the posed problems (Koichu; Kontorovich, 2013), including improving the processes of reading and writing mathematical problems (Cai *et al*. al., 2013). Bonotto (2013, p. 51) corroborates these positions by highlighting:

> [...] With regard to the problem-solving phase, this appears to be important and helpful in allowing a better understanding of the initial situation, fostering quality control of the problems created by the students themselves, and giving a starting point for analyzing the structure of problems.

In this aspect, Koichu and Kontorovich (2013) and Koichu (2020) indicate that problem posing should not be an isolated activity, in which students propose a problem as an exercise that the teacher requested to be carried out, as opportunities for enhanced learning. The author emphasizes that problem posing is an implicit objective in another activity, being a powerful vehicle for teaching for and through problem solving.

It is worth noting that posing problme is also an important component in evaluating student learning, allowing teachers to perceive conceptual flaws and difficulties that need to be worked on (Cai et al., 2013; Tichá; Hošpesová, 2013, Xu et al., 2020). English (2020, p. 3) highlights that "encouraging teachers to adopt a critical and inquiry-oriented approach in their mathematics teaching could establish a foundation for their teaching through problem posing".

Corroborating this idea, Chica (2001, p. 169, our translation) emphasizes that:

> [...] For the teacher, formulating problems is an assessment tool at all times, as it

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> provides evidence of whether or not students are mastering mathematical concepts. Using the data obtained, the teacher can plan the new teaching actions that he wants to develop with his students.

In this same vein, Cai and Hwang (2020, p. 3) highlight the problem posing as an activity that allows teachers to better understand students' thinking:

> One of the potential benefits of including problem posing in mathematics classrooms is the capacity for problem-posing tasks to reveal useful insights about students' mathematical thinking. The more information that teachers obtain about what students know and think, the more data they have to inform their efforts to create effective learning opportunities for all of their students. Thus, teachers' knowledge of students' thinking has a substantial impact on their classroom instruction, and, hence, on students' learning.

Also, aspects related to mathematical creativity have been studied through and in the context of problem posing, by examining the variety of problems that a student can present from an indicated starting point (Bonotto; Santo, 2015; Harpen; Srinaman, 2013; Leikin; Elgrably, 2020; Silver, 1994b; Singer; Voica, 2015; Teixeira, 2019).

In this aspect, it is worth highlighting the work of Andrade (1998, 2017), who explores the problem posing in the research he carries out, developing studies in which he associates this practice with the development of creativity, exploration, and problem solving, and recording, in the research that he guides, classroom experiences in which he considers multi-contextuality. Thus, it highlights the need to promote a critical and reflective environment in class, which transcends the specificities of learning mathematical content, and which meets the integral formation of the social subject, as we have already highlighted and is recommended as a principle in current guiding documents and school curricula. (Brazil, 2018; São Paulo, 2019)

Furthermore, some research (Cai *et al*., 2013; Silver, 1994b) indicates that students' involvement in problem posing is conducive to them connecting Mathematics with their own interests, stimulating interest in Mathematics. And these connections provide opportunities for students to interpret and critically analyze reality, since:

> [...] (1) the students have to discern significant data from immaterial data; (2) they must discover the relations between the data; (3) they must decide whether the information in their possession is sufficient to solve the problem; and (4) they have to investigate if the numerical data involved are numerically and/or contextually coherent. (Bonotto, 2013, p. 40)

In other words, several contributions are related to activities in which students are involved in mathematical problem posing, including aspects of integral education, as portrayed by Chica (2001, p. 169, our translation):

> [...] the main objective of formulating problem texts is the formation of an autonomous individual in the face of problems, capable of facing obstacles and developing their skills of argumentation, observation, deduction, and, mainly, their critical spirit. We want our students to be agents of their learning, to become readers and writers in mathematics, and to produce something that has meaning and use for them.

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It is worth noting, however, that these different contributions of problem posing, highlighted in the studies of different researchers, depend on the objective established for the activity and also on the starting point that is provided to students. There is a range of possibilities for working with the problem posing in the classroom: they can arise naturally in or from students' mathematical activities; be generated from problems in textbooks or other materials, by modifying or reformulating their characteristics; or be part of problem-solving activities (Stoyanova; Ellerton, 1996). The situations that trigger the problem posing are classified by Stoyanova and Ellerton (1996) and Stoyanova (1998) as free, semi-structured, and structured.

The authors considered *free,* those activities in which information or a situation is provided about which students must generate problems without any restrictions. *Semi-structured* situations are intended for students to pay attention to the structure of the problem and its resolution. Also, in problem posing based on *structured situations*, students are involved in reformulating problems they have already solved or are asked to vary the conditions of a problem. Table 1 shows this difference by presenting some problem posing situations that can be used in these categories.

Problem-posing categories:	**Problem-posing situations:**
Free	Problems written for a friend; Problems from data; Problems I like; Problems which involve the use of a specific concept or mathematical method, etc.
Semi-structured	Problem posing situations based on a specific problem structure: Problems which fit given computations; Problems which are similar to a previously solved problem; Open-ended problems; Mathematical investigations, etc. Problem posing situations based on a specific solution structure: Problem posing which involves the use of a specific mathematical method within a given problem structure, etc
Structure	Problem-posing situations based on a specific problem: Problem variations; Reformulations, etc. Problem-posing situations based on a specific solution: Restating a problem on the basis of its solution, etc

Table 1 – Problem-posing situations
Source: Stoyanova (1998, p. 180)

A more or less structured problem posing activity can direct towards the development of formative or cognitive aspects, some activities with more potential than others for specific objectives, which need to be considered by the teacher in relation to the design and organization of the development of the activity in the classroom.

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The clear delimitation of these objectives allows for better guidance and conduction of the actions of the teacher and students in class, based on the more appropriate incorporation of the elements that make up the problem posing (Allevato; Possamai, 2022; Allevato; Possamai, 2023; Possamai; Allevato, 2023; Possamai; Allevato, 2024).

Thus, the interest of researchers, as well as efforts to include problem posing by students in curricular prescriptions, indicates that it is a prominent activity in Mathematics classes. And, in this context, the present study aims to establish a broad set of ideas about the conceptions and objectives of activities that involve the problem posing by students, as discussed below.

3 Methodological characterization

This article portrays and derives from a systematic review study started in 2021. In the carried out review, the results relating to Brazilian research involving 24 dissertations and theses on problem posing, were recorded by Possamai and Allevato (2022). It discussed the references adopted by Brazilian researchers in studies involving problem posing practices, some of which are considered and revisited here. The Brazilian part of the review specifically involved the analysis of definitions and descriptions related to educational practices implemented by researchers. In those practices, students were asked to create problems, due to this, our focus remained especially on the expressions adopted to designate this action.

Among the results constructed in the study, we highlight the "[...] weaknesses in the constitution of a theoretical body, in the research analyzed [...], that would allow guiding a more consistent methodological path for implementing problem-creating practices in classes of Mathematics [...]" (Possamai; Allevato, 2022, p. 1). Therefore, we continued the investigation, considering that a more in-depth search for international studies could support us on this path.

Mendes and Pereira (2020, p. 8, our translation) emphasize that the systematic literature review "[...] consists of systematizing aspects of interest contained in the literature taken as reference, in order to follow an organization that highlights what was done to, subsequently, be able to point out directions for investigations [...]".

Therefore, advancing in the deepening and expansion of the systematic review (Galvão; Ricarte, 2020), the inclusion criteria adopted for the review of international literature involve the inclusion of the works of Jinfa Cai and Edward A. Silver - important researchers on problem posing, with an outstanding volume of published works and theoretical advances - in addition to those of his collaborators and other studies frequently cited in his research.

It is worth highlighting that, considering the central objective of the study recorded in

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this article, which is to identify and establish an understanding of the expressions formulation, elaboration, creation, and posing problem with a view to supporting the context of Brazilian research and practices, this study is part of the scope of qualitative research, providing explanations for the phenomenon under analysis (Galvão; Ricarte, 2020).

Therefore, below we discuss some research, especially, but not only, international, which supported us in the construction of the definitions constituted from these studies that we carried out, definitions that will also be explained below.

4 Definition and understandings

We begin this section by highlighting that the terms problem and context, which are explained below, express understandings, which are assumed in this research. When we use the term problem we are indicating that it refers to "[...] everything that we don't know how to do, but that we are interested in solving [...]", and that "[...] the problem is not an exercise in which the student applies, in an almost mechanical way, a formula or a certain operational technique [...]" (Onuchic, 1999, p. 215, our translation).

In this aspect, it is worth highlighting that, while in teaching through problem solving the focus relies on the relationship between the solver and the problem during the resolution process, in teaching through problem posing[1] "[...] learning takes place during the process of students' posing of mathematical problems and classroom discussion of posed problems" (Zhang; Cai, 2021, p. 962).

It is also worth highlighting, in relation to problem posing as an educational practice aimed at student learning, that "[...] a well-formulated problem is at once a result of knowledge, a stimulus to more knowledge, and is itself knowledge" (Getzels, 1979, p. 169).

Still, two aspects need to be mentioned: one about the nature of the action and the other about the characteristics of the executor. The first can be either the creation of a problem based on a situation, motivation, or information, or changes made to an existing problem; and in each of these cases, there are different starting points (Possamai; Allevato, 2023). When it comes to the executor, it can be either the student, being positioned as the one who creates or makes changes to an existing problem, or the teacher. We emphasize that the current demand for

[1]Once again, we would like to emphasize that in this study the Portuguese expressions 'proposição de problemas' and 'formulação de problemas' are translated as problem posing and problem formulating, respectively, in order to capture the meanings attributed by the researchers. In the English-language studies presented, the terms used by the researchers are maintained.

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research on problem posing considers placing the student as the executor of the action.

Although it is not the objective of this article to discuss the implementation of problem posing practices in the classroom, we clarify that, to plan this type of activity, the teacher needs to establish the intended pedagogical objective, and then determine trigger elements – initial elements provided, in general, by the teacher, which constitute the starting point for problem posing –, and a prompt – a request, a "command" that transmits to the student what is expected of him –, as it is discussed in more detail in Cai (2022) and Possamai and Allevato (2024).

We also emphasize the understanding attributed here to the term *context*, which reflects both situations within Mathematics itself and situations that incorporate real-life phenomena of interest to students or relationships with other areas of knowledge involved in the problem posing.

Having clarified these aspects, we begin the discussion by presenting the different conceptions verified in the national and international literature review carried out to date, as well as the terms used by the authors of these studies considered.

In the systematic review study that we carried out (Possamai; Allevato, 2022), 24 Brazilian dissertations and theses that investigated students' problem posing were analyzed. The analyzed studies do not show a broader understanding or a specific definition for problem posing by students and use the terms elaboration, formulation, or posing, interchangeably, in most cases, to refer to the set of possibilities for educational practices that can be developed.

From this Brazilian research, we highlight the doctoral dissertation by Andreatta (2020, p. 157, our translation) who states that "[...] in this work, we use the expression 'Problem Elaboration' to designate the construction of problem texts by students who are part of the search". In the same vein, Fonteque (2019) and Silva (2019) use the term elaborate to refer to the expression, the textual production of the problem created by students.

This positioning is in line with what is shown in one of the Brazilian references, which is often cited, Chica (2001, p. 147, our translation), which expresses its understanding by indicating that "[...] when the student creates their own texts problems, he needs to organize everything he knows and prepare the text, giving it adequate meaning and structure so that he can communicate what he intends [...]".

After analyzing Brazilian research, we move on to international research. The studies we have carried out so far are partially recorded in Possamai and Allevato (2022, 2023, 2024) and in Allevato and Possamai (2022, 2023) and allow us to look through the existing literature, which deals with problem posing, to establish parameters for our understandings about elaborating, formulating, creating and posing problems.

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Table 2 presents definitions of *problem posing*, explained by international researchers that we considered in our studies.

Author	Definition
Bonotto (2013, p. 39)	The term *problem posing* has been used to refer both to the generation of new problems and to the reformulation of given problems
Cai e Hwang (2020, p. 3)	(a) Students *pose mathematical problems* based on given problem situations which may include mathematical expressions or diagrams, and (b) students *pose problems* by changing (i.e., reformulating) existing problems.
Chen e Cai (2020)	Teachers ask students to *pose mathematical problems* based on a given numerical expression.
Elgrably e Leikin (2021, p. 894)	*Problem posing* is a broad concept, usually related to the creation of a new problem in response to a requirement to create a problem or a set of problems.
Singer e Voica (2013, p. 10)	[...] PP involves extracting/identifying a (new) problem from the multitude of data or information available to the proposer.
Stoyanova e Ellerton (1996, p. 518)	*Problem posing* will be defined as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems.
Xu *et al.* (2020)	*Problem posing* refers to both the creation of new problems and the reformulation of certain problems.
Cai (2022, p. 37)	*Problem posing* is defined as the following specific intellectual activities: (1) Students pose mathematical problems based on given problem situations which may include mathematical expressions or diagrams; (2) students pose problems by changing (i.e., reformulating) existing problems; (3) teachers generate mathematical problem-posing situations for students to pose problems; and (4) teachers predict the kinds of problems that students can pose based on given problem situations.

Table 2 – *Problem posing* definitions
Source: created by the authors (2023, emphasis added).

As can be seen, there is no clear convergence of aspects that characterize the problem posing in international research. It is, in fact, a broad concept, as expressed by Elgrably and Leikin (2021).

In particular, we highlight, in addition to the previous ones, the definition presented by Silver (1994a, p. 19, emphasis added) which is adopted and referenced in several national and international studies: "*Problem posing* refers to both the generation of new problems and the re-formulation, of given problems. Thus, posing can occur before, during, or after the solution of a problem".

The author attract attention to the formulation problems associated with the problem-solving process:

> When solving a nontrivial problem a solver engages in this form of problem posing by recreating a given problem in some ways to make it more accessible for solution. Problem formulation represents a kind of problem posing process because the solver transforms a given statement of a problem into a new version that becomes the focus of solving. *Problem formulation is related to planning*, since it may involve posing problems that represent subgoals for the larger problem. [...] The operative question that stimulates this form of posing is: how can I formulate this problem so that it can be solved? (Silver, 1994a, p. 19-20, emphasis added)

We highlight, in this quote, the clear association that the author makes of the posing

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with problem solving and we understand that the formulation, as presented by Silver (1994a), is not effectively defined in a problem that is expressed in written form, but constitutes a *mental process*, of thinking about how to make the problem you want to solve simpler so that you can understand the mathematical concepts/procedures necessary to obtain its solution. This becomes more evident when Silver (1994a, p. 20, author's emphasis) differentiates the problem posing that occur disconnected from the process of solving a problem:

> Not all problem posing occurs within the process of solving a complex problem. Problem posing can also occur at times when the goal is not the *solution* of a given problem but the *creation* of a new problem from a situation or experience. Such problem posing can occur *prior* to any problem solving, as would be the case if problems were generated from a given contrived or naturalistic situation. This type of problem generation is also sometimes referred to as problem formulation, but the process being described here is different from that described above.

Trying to capture the mental processes that involve the creation of a problem, Koichu and Kontorovich (2013, p. 75, emphasis added) question participants in their research with questions like: "How did you start? How did the *idea* of the problem emerge? Which *ideas* did you abandon during the posing and why? How did you *decide* that the problem is complete?". In this aspect, we reaffirm the premise that the creation of problems goes through a process, sometimes cyclical, of constituting ideas and expressing them, recording them. Capturing and analyzing these ideas allows us to understand how to move towards posing better-quality problems, and creating teaching strategies that contribute to this process.

Silver (1994a) highlights that the posing problems can also happen after problem solving, with the aim of analyzing new problems that can be generated when examining the conditions of the one that was solved. This strategy is called "looking back" by Polya (1988) and by Brown and Walter (2005) "What if not?", and its purpose is for students to generate new problems considering variations in the conditions or objectives of a given scenario. problem previously resolved.

We emphasizes the definition presented by Zhang and Cai (2021, p. 962, emphasis added): "mathematical problem *posing* is the process of *formulating and expressing* a problem within the domain of mathematics". Ratifying this definition, Zhang *et al*. (2022, p. 2, author's emphasis) present their understanding of the structure of the cognitive processes of problem posing, with a view to mathematical communication:

> (1) Information input, also called '*understanding* the task', involves the process of understanding the context of the problem-posing task; (2) information processing, also called '*constructing* the problem', refers to selecting and determining which elements will be used and recognizing the relationships between the elements that have been selected to construct a new problem space […]; and (3) information output, also called '*expressing* the problem', refers to organizing the language (i.e., making use of syntax, tenses, etc.) to express the problem space that has already been constructed in the

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information-processing stage.

Analyzing, both these understandings related to the expressions *problem posing* – as a broad concept (Elgrably; Leikin, 2021) – and *problem formulating* – associated with the constitution of ideas (Silver, 1994a) – attributed to international researchers on this topic, as well as the meanings verified in the work of Brazilian researchers, we assume the challenge and complexity of assigning meanings to the *posing, creation, formulation and elaboration* of problems, considering that this is a field that lacks clear positioning, definition and structure. We considered that reducing the problem to the translation of terms in English does not take into account the constituent elements and social contexts associated with the use of these words in educational practices involving the problem posing. This is not, therefore, our personal perception, but is the result of a detailed review of academic literature, from which we adopted "theoretical partners" whose concepts we consider appropriate to the Brazilian context.

Furthermore, it is worth highlighting the relevance of these understandings for research in Mathematics Education, guiding or directing data collection and analysis and, consequently, results, depending on the objectives established for the studies.

Thus, at this point in our research, we express our understandings by attributing the following meanings to these expressions:

- *Problem formulation:* this is the initial stage when the activity is proposed by the teacher and the student forms the first set of ideas about the possibilities of the mathematical context and elements that he intends to relate to the problem, or its structure. This phase, of ideas, constitutes an important part, in which the student produces meaning and establishes connections. The formulation can also happen during the resolution of a problem when another problem is thought of by the student with the intention/idea of minimizing the complexity of a larger problem that he wants or needs to solve.
- *Problem elaboration*: constitutes the stage in which the student begins to record these ideas, expressing the problem and articulating the data so that Mathematics connects with the intended elements and with the proposed starting point, producing a written text, an oral narrative, a drawing, or other form. It is the phase in which ideas advance from preparation to production/expression of the problem.

Thus, when *creating problems*, the student goes through *formulation* and *elaboration*. In the *formulation* he organizes or builds the first mathematical ideas and the structure in which

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he intends to constitute the problem; he thinks, reflects, envisions, remembers, plans, and establishes connections. Following, or simultaneously, through the *elaboration* of the problem, he expresses the *text* of the problem orally or through written recording; he elaborates the statement, uses different forms of representation, and organizes and structures the problem with native and mathematics languages. Formulation and elaboration constitute the process of *creating* the problem, which can also happen in cycles of (re)formulation and (re)elaboration, allowing the student to produce meaning for and improve their construction, and be able to communicate what they intend.

Thus, when problem posing, a student can create a problem that does not necessarily constitute, for him or herself, a problem; perhaps it is for a potential solver. But what makes the activity challenging for the creator is the construction of the problem itself, the organization of ideas, and the expression through words within a mathematical domain (Possamai; Allevato, 2023).

It is worth confirming that the *formulation* takes place mentally, but can, however, be associated with oral or written manifestations. When students work in groups to create a problem, for example, the ideas and planning that constitute the formulation stage are socialized and constructed together, based on discussion, debates, and sharing of ideas. Also, when we refer to *elaboration*, we are intrinsically assuming that it could happen together with formulation; however, we are referring to the expression of the problem that will be presented, either orally (especially with young children) or in written form.

Figure 1 illustrates this set of definitions that we are currently assuming and that we have explained up to this point in this article:

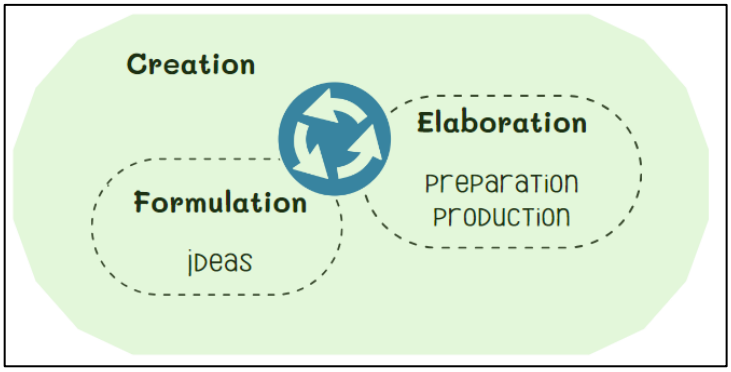

Figure 1 – Problem creation by students
Fonte: created by the authors (2023)

Moving on to other constituent elements of the problem posing, we assume that it is important to associate the *resolution* process with the *creation* of the problem, giving the student

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the opportunity to reformulate their ideas and produce an improvement in the problem created based on emerging perceptions of the problem resolution, improving the process of writing and expressing the problem within the mathematical domain. In this aspect, in agreement with Silver (1994b, p. 76), we found, based on empirical studies that we carried out (Duarte, 2020; Gieseler et al., 2021; Possamai; Souza, 2022; Vieira; Possamai; Allevato , 2023), which is in the "[...] of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity".

In this way, the problem posing advances and also includes the *presentation*, when the problem created - by the teacher, by other authors (of textbooks or other materials) or even by the students – is submitted to the potential solver. In other words, we define *problem posing* as a process involving the *creation* of problems and progressing to their *presentation* to a potential solver, who may be the person who created the problem, a colleague, or another person, associating the *formulation* and *elaboration* processes. Figure 2 presents a synthesis of this understanding of *problem posing*, highlighting the possibilities of its association with problem solving.

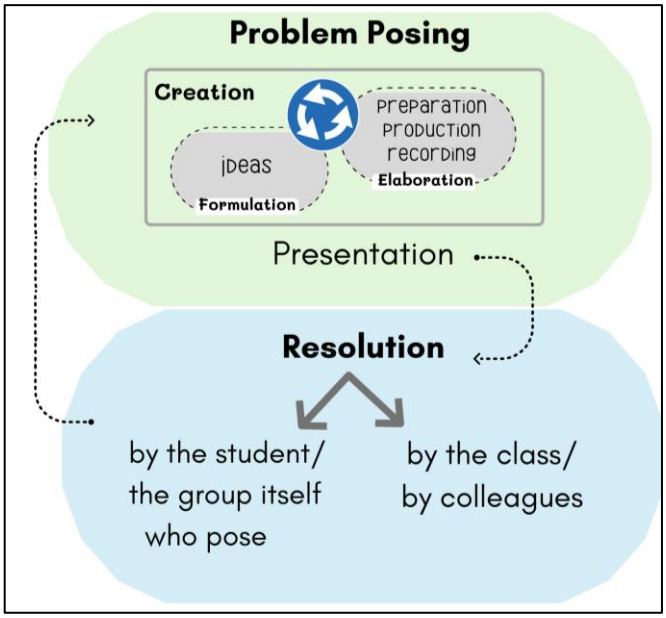

Figure 2 – Problem posing
Source: created by the authors (2023)

Figure 2 shows us that there is no hierarchy between these actions, but a relationship of inclusion between some of them, in such a way that, when related, they constitute what we understand as Problem Posing.

The association of *problem posing* with problem solving has been recommended in curriculum prescriptions. In the National Common Curricular Base – BNCC (Brazil, 2018), the expression "solve and elaborate problems" is indicated in an increasing number of skills to be

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developed throughout Basic Education, starting with one skill in the 1st year of Elementary School and reaching nine skills in 7th and 8th years. And there is the clarification that:

> [...] In this statement, it is implicit that the aim is not only to solve the problem but also for students to reflect and question what would happen if some data in the problem were changed or if some conditions were added or removed. From this perspective, it is intended that students also formulate problems in other contexts (Brasil, 2018, p. 277, our translation).

The document shows that the terms elaboration and formulation are used as synonyms, referring to activities that involve modifying the conditions of existing problems (Allevato; Possamai, 2023).

However, this context of associating problem posing with problem solving can also be expanded as a way of improving or evaluating the problems created by students. It should be noted that some problems naturally require students to move from posing to solving. For example, when a problem is requested to be created that has a certain answer, resolution is necessary to check whether there is compatibility between the posed problem and the result requested. On the other hand, there are problem posing activities – such as creating a problem from a provided image, for example – which, in general, lead to the creation of problems that do not require resolution for validation.

It is therefore noteworthy that the problem posing associated with problem solving allows students to move towards improving creative processes; writing, reading, and interpreting problems; and also, advance the quality of the problems created concerning the Mathematics involved. For example, when students are asked to pose problems for their colleagues to solve, creators become involved in posing better-structured problems, both in terms of the texts produced and in relation to Mathematics (Silver, 1994a). Or, when students are asked to create a difficult problem for themselves to solve, weaknesses in mathematical content may be revealed that need to be discussed. There is also evidence that when students are asked to propose problems for themselves to solve, they are less demanding, mathematically, as they are intimidated by the need to solve them.

Therefore, the strategy adopted by the teacher to associate the problem posing with problem solving, in which the students are those who pose the problems, will depend on the teacher's intentions regarding the objectives of mathematical learning. English (2020, p. 2) highlights that “if problem posing fails to achieve this goal, then it becomes no different to mathematical exercises where opportunities for teachers to gain insights into students’ thinking and learning are minimal”.

By analyzing the educational practices portrayed in Brazilian research, it seems that it

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is necessary to advance in the methodological structuring of this association of the problem posing with problem solving, exploring the different possibilities and associated objectives. To assist in this advancement, several works resulting from the studies we have developed have already been published.

5 Final considerations

This paper was constituted as a result of a research trajectory that began with concerns about the meanings of terms related to Problem Posing, a theme that has been at the top of national and international research developed within the scope of and related to Problem Solving. Following this trend, recent official curricular documents from several countries – including Brazil (Brazil, 2018) – and publications providing guidance for teaching, learning, and teaching practice within the scope of Mathematics Education have made explicit indications for the inclusion of problem posing in classrooms.

However, there is a diversity of terms – formulation, elaboration, creation, posing – that have been used in these researches, documents, and publications, and which, often, do not present themselves with clearly assumed meanings, which can lead to doubts and insecurities in implementing practices in the classroom, in teacher training and, even, in research.

Considering the importance of research as the foundation of teaching training and practice, after a detailed investigation of problem-posing practices by students, recorded in Brazilian dissertations and thesis and, subsequently, having advanced in the knowledge of international research production, we constitute the present study, which aims to identify and establish understandings for the terms formulation, elaboration, creation and problem posing to support the context of Brazilian research and practices. In an attempt to capture the complexity of these terms in educational practices, we define them beyond searches in dictionaries or literal translations, we do not even assume the terms indicated in the curricular prescriptions as positions.

It is worth noting that this research, still in development, began in April 2021, with the first author's post-doctoral internship, supervised by the second author of this article. Between comings and goings, studies, reflections, and broad analysis of a national and international theoretical body, the definitions we are adopting aim to guide both the discussion of a methodology that guides educational practices in the classroom and future research related to this theme.

Methodologically, it was initially guided by an in-depth analysis of national and

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international studies and research on Problem Posing, in which the aforementioned terms are present, whose significance was presented as a theoretical scenario in this article. This scenario shows, as we have already highlighted, a lack of clarity or even consensus regarding what is the posing (or the formulation or the elaboration or the creation) of problems and highlights a pressing demand for objectification concerning the forms of implementation in the classroom of math class. We continue, relying on some of these studies, chosen for their cohesion with linguistic aspects related to the social uses of these terms in Brazil, after also consulting experts in this area, and we dare to constitute a set of understandings and definitions, the name of the session in which they are presented in the present work.

Therefore, aware of the provisionality of the results of all scientific research and, also, of a wide range of aspects to be explored with Problem Posing in Mathematics Education, at this moment, we are assuming the use of the expression *problem posing* to denote the entire set of ideas that constitute the processes involving the creation and presentation of problems. *Creation* begins with the organization and construction of the first mathematical ideas and the structure of the problem (*formulation*) and progresses to the expression of the problem, in which the statement is established, associating the native and mathematics language (*elaboration*); this is then followed by its *presentation* to a potential solver.

With this differentiation, we do not intend to constitute absolute meanings for the terms, since it does not correspond exactly with the conceptions of other researchers (as we tried to clarify in this study) or with those expressed in other publications. But we have in mind, especially: (1) establishing a language standard that, as a language, encompasses its meanings, allowing us (and, perhaps, also other researchers and trainers) to provide greater clarity to the approaches we intend to develop in the contexts of practices in Mathematics classrooms, teacher training and the research we continue to develop on Problem Posing; and (2) provide those interested in this topic with concerns and positions linked to the understanding we built in our research, that Problem Posing involves specific aspects linked to thought – *formulation*; to language(s) – *elaboration*; these two associated elements – *creation*; and the transfer of the created problem to the solver – *presentation*, the set of which we are calling *problem posing*; and which is preferably followed by the *resolution* of the problem. We consider it extremely important that research and practices involving the problem posing take into account, if not this specific terminology, the particularities, conditions, and constraints resulting from these aspects.

Furthermore, with these definitions, in research, it is important to reflect on the data collected when wanting to evaluate the implications of problem posing tasks on mathematical

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learning or the skills, mathematical or general, developed (or not) with students. In other words, when analyzing the record of studies related to the problem presented, the elaboration of the problem is assessed, which does not necessarily show evidence of the formulation process. However, when students are questioned, obtaining their explanations and justifications, it is possible to understand the mental processes that led them to formulate problems, and weaknesses, insecurities, advances, and thinking styles, among other important aspects, can be revealed.

Furthermore, we have collected evidence that the posing, which, as we understand, includes presenting the problem to a potential solver, also involves relevant aspects, to be considered in research, related to the motivations and conditioning that lead students to create *worse or better problems*, for example, regarding the structure of the problem or of the Mathematics involved, or the teaching objectives outlined for the activity.

With these possibilities, a broad panorama of the potential of problem posing has been revealed, both for learning and evaluating learning in Mathematics and for the integral education of students, from the development of skills to the treatment of affective issues related to this curricular component.

To uncover these potentialities, several works resulting from the studies we have developed have already been published and were cited in this article. But there is still a long way to go. It is suggested, against the backdrop of the definitions presented here, that future research be carried out, envisioning educational and teaching practices to be implemented and analyzed based on the different aspects relating to formulation, elaboration, creation, presentation, posing, the posing associated with problem solving...

We still intend to move forward and answer so many open questions involving this issue of Problem Posing, some of which are suggested here; but others may arise. May this work support teachers and other researchers in the search for answers.

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