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Effect of Hall current on the velocity and temperature distributions of Couette flow with variable properties and uniform suction and injection

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Abstract. The unsteady hydromagnetic Couette flow and heat transfer between two parallel porous plates is studied with Hall effect and temperature dependent properties. The fluid is acted upon by an exponential decaying pressure gradient and an external uniform magnetic field. Uniform suction and injection are applied perpendicularly to the parallel plates. Numerical solutions for the governing non-linear equations of motion and the energy equation are obtained. The effect of the Hall term and the temperature dependent viscosity and thermal conductivity on both the velocity and temperature distributions is examined.

Mathematical subject classification: 76D05, 76M25.

Key words: Fluid Mechanics, nonlinear equations, Hall effect, numerical solution.

1 Introduction

The flow between parallel plates is a classical problem that has important applications in magnetohydrodynamic (MHD) power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Hartmann and Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between

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two infinite parallel stationary and insulating plates. The problem was extended in numerous ways. Closed form solutions for the velocity fields were obtained [2-5] under different physical effects. Some exact and numerical solutions for the heat transfer problem are found in [6, 7]. In the above mentioned cases the Hall term was ignored in applying Ohm's law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable [5]. Under these conditions, the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force. Tani [8] studied the Hall effect on the steady motion of electrically conducting and viscous fluids in channels. Soundalgekar et al. [9, 10] studied the effect of Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed either to be constant [9] or varying linearly along the plates in the direction of the flow [10]. Abo-El-Dahab [11] studied the effect of Hall currents on the steady Hartmann flow subject to a uniform suction and injection at the bounding plates. Attia [12] extended the problem to the unsteady state with heat transfer.

Most of these studies are based on constant physical properties. It is known that some physical properties are functions of temperature [13] and assuming constant properties is a good approximation as long as small differences in temperature are involved. More accurate prediction for the flow and heat transfer can be achieved by considering the variation of the physical properties with temperature. Klemp et al. [14] studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb [7] studied the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity in the presence of a uniform magnetic field. Later Attia [15] extended the problem to the transient state. The influence of Hall current and variable properties on unsteady Hartmann flow with heat transfer was given in [16]. The influence of variations in the physical properties on steady Hartmann flow was studied by Attia without taking the Hall effect into considerations [17]. The effect of variable properties on Couette flow in a porous medium was done by Attia [18].

In the present work, the unsteady Couette flow of a viscous incompressible electrically conducting fluid is studied with heat transfer. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. The fluid is flowing between two electrically insulating plates and is acted upon by an exponential decaying pressure gradient while a uniform suction and injection is applied through the surface of the plates. The upper plate is moving with a constant velocity while the lower plate is kept stationary. An external uniform magnetic field is applied perpendicular to the plates and the Hall effect is taken into consideration. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected [4]. The two plates are kept at two constant but different temperatures. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. Thus, the coupled set of the equations of motion and the energy equation including the viscous and Joule *dissipation terms* becomes non-linear and is solved numerically using the finite difference approximations to obtain the velocity and temperature distributions.

2 Formulation of the problem

The fluid is assumed to be flowing between two infinite horizontal plates located at the $y = \pm h$ planes. The two plates are assumed to be electrically insulating and kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$. The upper plate is moving with a constant velocity U_o while the lower plate is kept stationary. The motion is produced by an exponential decaying pressure gradient $dP/dx = -Ge^{-\alpha t}$ in the x-direction, where G and α are constants. A uniform suction from above and injection from below, with velocity v_o, are applied at $t = 0$. A uniform magnetic field $\mathbf{B}_o$ is applied in the positive y-direction. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The Hall effect is taken into consideration and consequently a z-component for the velocity is expected to arise. The viscosity of the fluid is assumed to vary exponentially with temperature while its thermal conductivity is assumed to depend linearly on temperature. The viscous and Joule dissipations are taken into consideration. The fluid motion starts from rest at $t = 0$, and the no-slip condition at the plates implies that the fluid velocity has neither a z nor an x-component at $y = \pm h$. The initial temperature of the fluid is assumed to be equal to T_1. Since the plates are infinite in the x and z-directions, the physical quantities do not change in these directions and the problem is essentially one-dimensional.

The flow of the fluid is governed by the Navier-Stokes equation

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p + \nabla \cdot (\mu \nabla \vec{v}) + \vec{J} \wedge \vec{B}_o \tag{1}$$

where ρ is the density of the fluid, μ is the viscosity of the fluid, $\vec{J}$ is the current density, and $\vec{v}$ is the velocity vector of the fluid, which is given by

$$\vec{v} = u(y,t)\vec{i} + v_o\vec{j} + w(y,t)\vec{k}$$

If the Hall term is retained, the current density $\vec{J}$ is given by the generalized Ohm's law [4]

$$\vec{J} = \sigma\left[\vec{v} \wedge \vec{B}_o - \beta\left(\vec{J} \wedge \vec{B}_o\right)\right] \tag{2}$$

where σ is the electric conductivity of the fluid and β is the Hall factor [4]. Using Eqs. (1) and (2), the two components of the Navier-Stokes equation are

$$\rho \frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = G e^{-\alpha t} + \mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y} - \frac{\sigma B_o^2}{1+m^2}(u + mw) \tag{3}$$

$$\rho \frac{\partial w}{\partial t} + \rho v_o \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial \mu}{\partial y}\frac{\partial w}{\partial y} - \frac{\sigma B_o^2}{1+m^2}(w - mu) \tag{4}$$

where m is the Hall parameter given by $m = \sigma \beta B_o$. It is assumed that the pressure gradient is applied at $t = 0$ and the fluid starts its motion from rest. Thus

$$t = 0 \colon u = w = 0 \tag{5a}$$

For $t > 0$, the no-slip condition at the plates implies that

$$y = -h \colon u = w = 0 \tag{5b}$$

$$y = h \colon u = U_o,\ w = 0 \tag{5c}$$

The energy equation describing the temperature distribution for the fluid is given by [19]

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_o \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma B_o^2}{1+m^2}(u^2+w^2) \tag{6}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is thermal conductivity of the fluid. The last two terms in the right-hand-side of Eq. (6) represent the viscous and Joule dissipations respectively.

The temperature of the fluid must satisfy the initial and boundary conditions,

$$t = 0\colon T = T_1 \tag{7a}$$

$$t > 0\colon T = T_1,\ y = -h \tag{7b}$$

$$t > 0\colon T = T_2,\ y = h \tag{7c}$$

The viscosity of the fluid is assumed to vary with temperature and is defined as, $\mu = \mu_o f_1(T)$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form [7], $f_1(T) = \exp(-a_1(T - T_1))$. In some cases a_1 may be negative, i.e. the coefficient of viscosity increases with temperature [7, 15].

Also the thermal conductivity of the fluid is varying with temperature as $k = k_o f_2(T)$. We assume linear dependence for the thermal conductivity upon the temperature in the form $k = k_o(1 + b_1(T - T_1))$ [19], where the parameter b_1 may be positive or negative [19].

The problem is simplified by writing the equations in the non-dimensional form. To achieve this define the following non-dimensional quantities,

$$\hat{y} = \frac{y}{h},\quad \hat{t} = \frac{tU_o}{h},\quad \hat{G} = \frac{hG}{\rho U_o^2},\quad (\hat{u}, \hat{w}) = \frac{(u, w)}{U_o},\quad \hat{T} = \frac{T - T_1}{T_2 - T_1},$$

$\hat{f}_1(\hat{T}) = e^{-a_1(T_2 - T_1)\hat{T}} = e^{-a\hat{T}}$, a is the viscosity parameter,
$\hat{f}_2(\hat{T}) = 1 + b_1(T_2 - T_1)\hat{T} = 1 + b\hat{T}$, b is the thermal conductivity parameter,
$\mathrm{Re} = \rho U_o h/\mu_o$ is the Reynolds number,

$S = \rho v_o h/\mu_o$ is the suction parameter,
$Ha^2 = \sigma B_o^2 h^2/\mu_o$, Ha is the Hartmann number,
$\Pr = \mu_o c_p/k_o$ is the Prandtl number,
$Ec = U_o^2/c_p(T_2 - T_1)$ is the Eckert number.

In terms of the above non-dimensional quantities the velocity and energy equations (3) to (7) read (the hats are dropped for convenience)

$$\frac{\partial u}{\partial t} + \frac{S}{\mathrm{Re}}\frac{\partial u}{\partial y} = Ge^{-\alpha t} + \frac{1}{\mathrm{Re}} f_1(T)\frac{\partial^2 u}{\partial y^2} + \frac{1}{\mathrm{Re}}\frac{\partial f_1(T)}{\partial y}\frac{\partial u}{\partial y} - \frac{Ha^2}{\mathrm{Re}(1+m^2)}(u + mw) \tag{8}$$

$$\frac{\partial w}{\partial t} + \frac{S}{\mathrm{Re}}\frac{\partial w}{\partial y} = \frac{1}{\mathrm{Re}} f_1(T)\frac{\partial^2 w}{\partial y^2} + \frac{1}{\mathrm{Re}}\frac{\partial f_1(T)}{\partial y}\frac{\partial w}{\partial y} - \frac{Ha^2}{\mathrm{Re}(1+m^2)}(w - mu) \tag{9}$$

$$t = 0:\ u = w = 0 \tag{10a}$$

$$t > 0:\ y = -1,\ u = w = 0 \tag{10b}$$

$$t > 0:\ y = 1,\ u = w = 0 \tag{10c}$$

$$\frac{\partial T}{\partial t} + \frac{S}{\mathrm{Re}}\frac{\partial T}{\partial y} = \frac{1}{\Pr} f_2(T)\frac{\partial^2 T}{\partial y^2} + \frac{1}{\Pr}\frac{\partial f_2(T)}{\partial y}\frac{\partial T}{\partial y} + Ecf_1(T)\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{EcHa^2}{\mathrm{Re}(1+m^2)}(u^2 + w^2) \tag{11}$$

$$t = 0:\ T = 0 \tag{12a}$$

$$t > 0:\ T = 0,\ y = -1 \tag{12b}$$

$$t > 0:\ T = 1,\ y = 1 \tag{12c}$$

Equations (8), (9), and (11) represent a system of coupled non-linear partial differential equations which can be solved numerically under the initial and boundary conditions (10) and (12) using finite difference approximations. The Crank-Nicolson implicit method is used [20]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time levels. The non-linear terms are first linearized and then an iterative scheme is used at every time step to solve the linearized system of difference equations. All calculations have been carried out for $G = 5$, $\alpha = 1$, $\Pr = 1$, $\mathrm{Re} = 1$, and $Ec = 0.2$. Step sides $\Delta t = 0.001$ and $\Delta y = 0.05$ for time and space respectively, are chosen and the scheme converges in at most 7 iterations at every time step. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when any one of u, w, T, $\partial u/\partial y$, $\partial w/\partial y$ and $\partial T/\partial y$ for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1 < y < 1$ at every time step. Less than 7 approximations are required to satisfy this convergence criteria for all ranges of the parameters studied here. The transient results obtained here converges to the steady state solutions given in [17] in the case $m = 0$.

3 Results and Discussion

Figure 1 presents the velocity and temperature distributions as functions of y for various values of time t starting from $t = 0$ up to steady state. The figure is evaluated for $Ha = 1$, $m = 3$, $S = 0$, $a = 0.5$ and $b = 0.5$. It is clear that the velocity and temperature distributions do not reaches the steady state monotonically. They increase with time up till a maximum value and decrease up to the steady state due to the influence of the decaying pressure gradient. The velocity component u reaches the steady state faster than w which, in turn, reaches the steady state faster than T. This is expected as u is the source of w, while both u and w are sources of T.

Figure 2 depicts the variation of the velocity component u at the centre of the channel ($y = 0$) with time for various values of the Hall parameter m and the viscosity parameter a and for $b = 0$ and $Ha = 3$. The figure shows that u increases with m for all values of a. This is due to the fact that an increase in

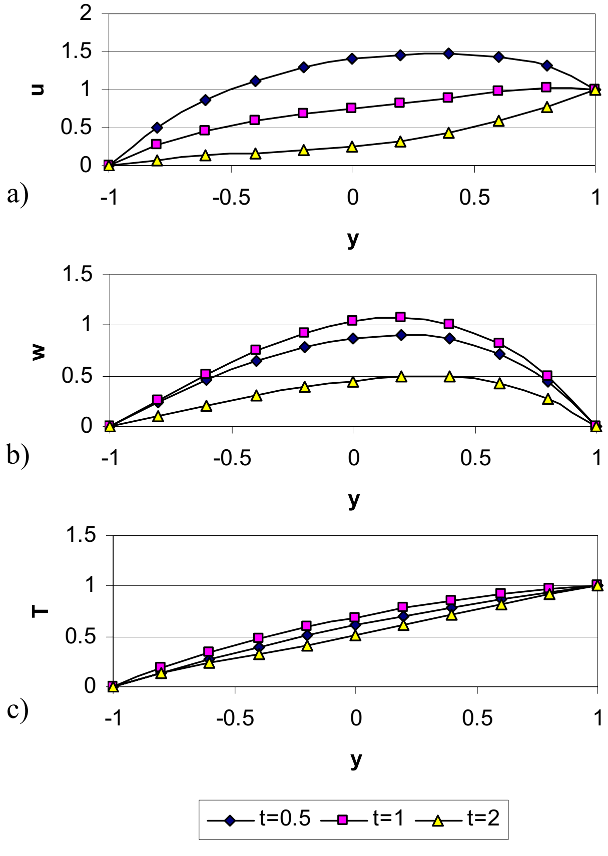

Figure 1 – The evolution of the profile of: (a) u; (b) w and (c) T. ($Ha = 3$, $m = 3$, $a = 0.5$, $b = 0.5$).

m decreases the effective conductivity ($\sigma/(1 + m^2)$) and hence the magnetic damping. The figure shows also how the effect of a on u depends on the parameter m. For small and moderate values of a, increasing a decreases u, however, for higher values of m, increasing a increases u. It is observed also from the figure that the time at which u reaches its steady state value increases with increasing m while it is not greatly affected by changing a.

Figure 3 presents the variation of the velocity component w at the centre of the channel ($y = 0$) with time for various values of m and a and for $b = 0$, $S = 0$ and $Ha = 3$. The figure shows that w increases with increasing m for all values of a. This is because w, which is the z-component of the velocity is a result of the Hall effect.

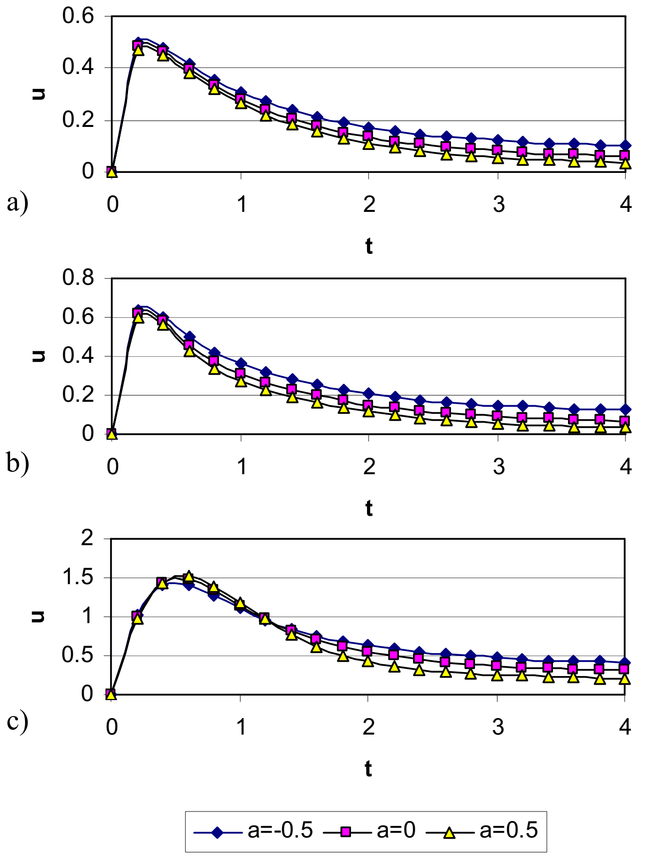

Figure 2 – The evolution of u at $y = 0$ for various values of a and m: (a) $m = 0$; (b) $m = 1$ and (c) $m = 5$. ($Ha = 3$, $b = 0$).

Although the Hall effect is the source for w, a careful study of Figure 3 shows that, at small times, an increase in m produces a decrease in w. This can be understood by studying the term $(-(w - mu)/(1 + m^2))$ in Eq. (10), which is the source term of w. At small times w is very small and this term may be approximated to $(mu/(1 + m^2))$, which decreases with increasing m if $m > 1$. Figure 3 shows also that for small values of m the effect of a on w depends on t. For small t, increasing a increases w, but with time progress, increasing a decreases w.

Figure 4 presents the evolution of the temperature T at the centre of the channel for various values of m and a when $b = 0$ and $Ha = 3$. The effect of m on T

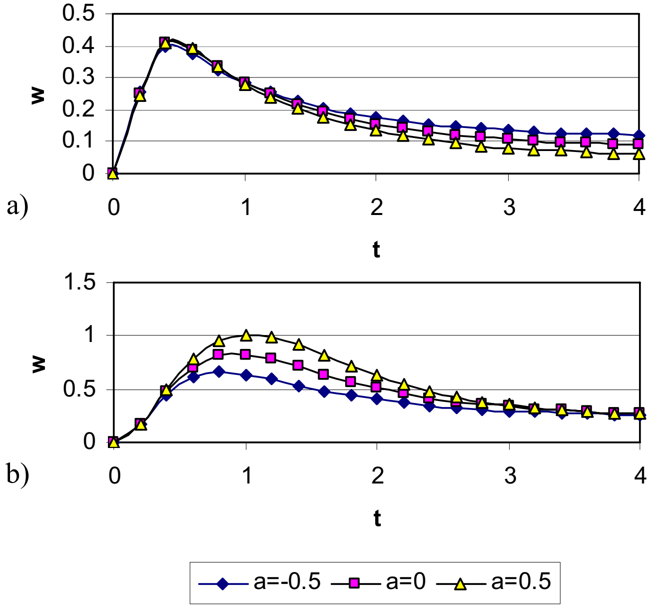

Figure 3 – The evolution of w at $y = 0$ for various values of a and m: (a) $m = 1$ and (b) $m = 5$. ($Ha = 3$, $b = 0$).

depends on t. When $m > 1$, increasing m decreases T slightly at small times but increases T at large times. This is because when t is small, u and w are small and an increase in m results in an increase in u but a decrease in w, so the Joule dissipation which is proportional also to $(1/(1 + m^2))$ decreases. When t is large, u and w increase with increasing m and so do the Joule and viscous dissipations. Increasing a increases T due to its effect on decreasing the velocities and the velocity gradients and the function f_1.

Figure 5 presents the evolution of T at the centre of the channel for various values of m and b when $a = 0$, $S = 0$ and $Ha = 3$. The figure indicates that increasing b increases T and the time at which it reaches its steady state for all m. This occurs because the centre of the channel acquires heat by conduction from the hot plate. The parameter b has no significant effect on u or w in spite of the coupling between the momentum and energy equations.

Table 1 shows the dependence of the steady state temperature at the centre of the channel on a and m for $b = 0$ and $S = 0$. It is observed that T decreases

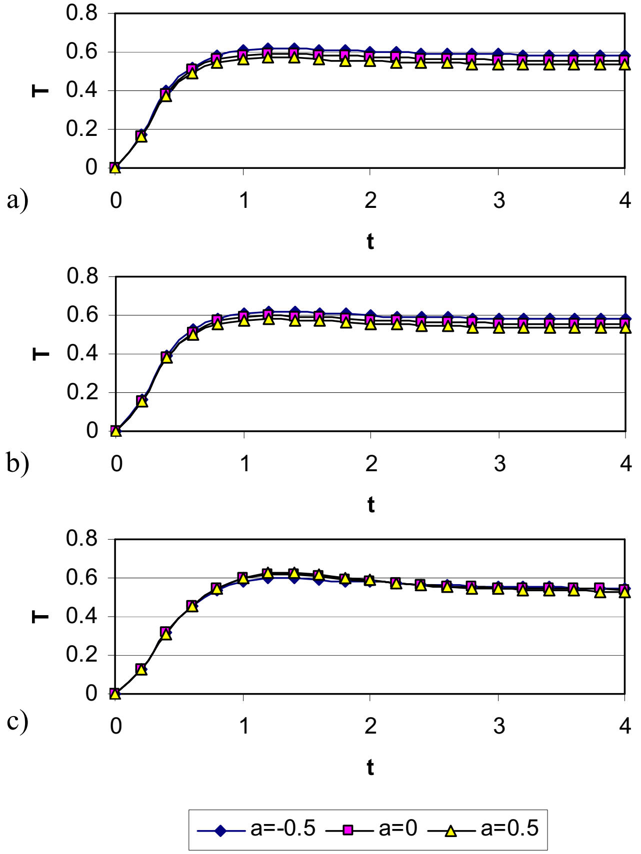

Figure 4 – The evolution of T at $y = 0$ for various values of a and m: (a) $m = 0$; (b) $m = 1$ and (c) $m = 5$. ($Ha = 3$, $b = 0$).

with increasing m as a result of decreasing the dissipations. On the other hand, increasing a decreases T as a consequence of decreasing the dissipations. Table 2 shows the dependence of T at the centre of the channel on m and b for $a = 0$, $S = 0$ and $Ha = 1$. The dependence of T on m is explained by the same argument used for Table 1. Table 2 shows that increasing b increases T since the centre gains temperature by conduction from hot plate. Table 3 shows the dependence of T on a and b for $Ha = 1$ and $m = 3$. Increasing a decreases T and increasing b decreases T.

Figures 6, 7, and 8 present the time development of the velocity components u and w and the temperature T, respectively, at the centre of the channel ($y = 0$) for

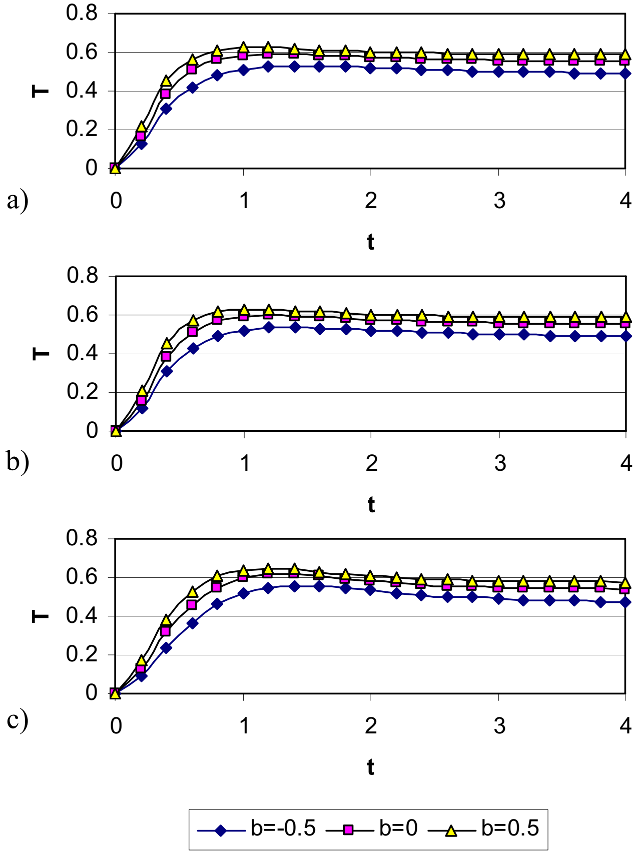

Figure 5 – The evolution of T at $y = 0$ for various values of b and m: (a) $m = 0$; (b) $m = 1$ and (c) $m = 5$. ($Ha = 3$, $a = 0$).

T	$m = 0.0$	$m = 0.5$	$m = 1.0$	$m = 3.0$	$m = 5.0$
$a = -0.5$	0.5541	0.5511	0.5452	0.5338	0.5314
$a = -0.1$	0.5459	0.5439	0.5398	0.5310	0.5291
$a = 0.0$	0.5435	0.5417	0.5382	0.5300	0.5281
$a = 0.1$	0.5412	0.5396	0.5365	0.5289	0.5272
$a = 0.5$	0.5325	0.5318	0.5301	0.5253	0.5240

Table 1 – Variation of the steady state temperature T at $y = 0$ for various values of m and a ($Ha = 1$, $b = 0$).

T	$m = 0.0$	$m = 0.5$	$m = 1.0$	$m = 3.0$	$m = 5.0$
$a = -0.5$	0.5541	0.5511	0.5452	0.5338	0.5314
$a = -0.1$	0.5459	0.5439	0.5398	0.5310	0.5291
$a = 0.0$	0.5435	0.5417	0.5382	0.5300	0.5281
$a = 0.1$	0.5412	0.5396	0.5365	0.5289	0.5272
$a = 0.5$	0.5325	0.5318	0.5301	0.5253	0.5240

Table 2 – Variation of the steady state temperature T at $y = 0$ for various values of m and b $(Ha = 1, a = 0)$.

T	$m = 0.0$	$m = 0.5$	$m = 1.0$	$m = 3.0$	$m = 5.0$
$b = -0.5$	0.4805	0.4781	0.4732	0.4621	0.4596
$b = -0.1$	0.5329	0.5311	0.5273	0.5187	0.5167
$b = 0.0$	0.5435	0.5417	0.5382	0.5300	0.5281
$b = 0.1$	0.5528	0.5512	0.5478	0.5401	0.5383
$b = 0.5$	0.5795	0.5781	0.5382	0.5690	0.5677

Table 3 – Variation of the steady state temperature T at $y = 0$ for various values of a and b $(Ha = 1, m = 3)$.

various values of the suction parameter S and the viscosity variation parameter a when $Ha = 3$, $m = 3$, and $b = 0$. Figures 6 and 7 indicate that increasing S decreases both u and w for all a due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. It is also clear that the influence of the parameter a on u and w becomes more pronounced for lower values of the parameter S. Figure 8 shows that increasing the suction parameter decreases the temperature T for all a as a result of the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel.

Figure 9 presents the time development of the temperature T at the centre of the channel ($y = 0$) for various values of the suction parameter S and the thermal conductivity variation parameter b when $Ha = 3$, $m = 3$, and $a = 0$. The figure shows that increasing S decreases T for all b. Figure 9a indicates that, for $S = 0$, the variation of T with the parameter b depends on time as shown before in Figure 5c for higher values of the Hall parameter m. Figures 9b and 9c present an interesting effect for the suction parameter in the suppression of the crossover points occurred in the $T - t$ graph due to changing of the parameter b.

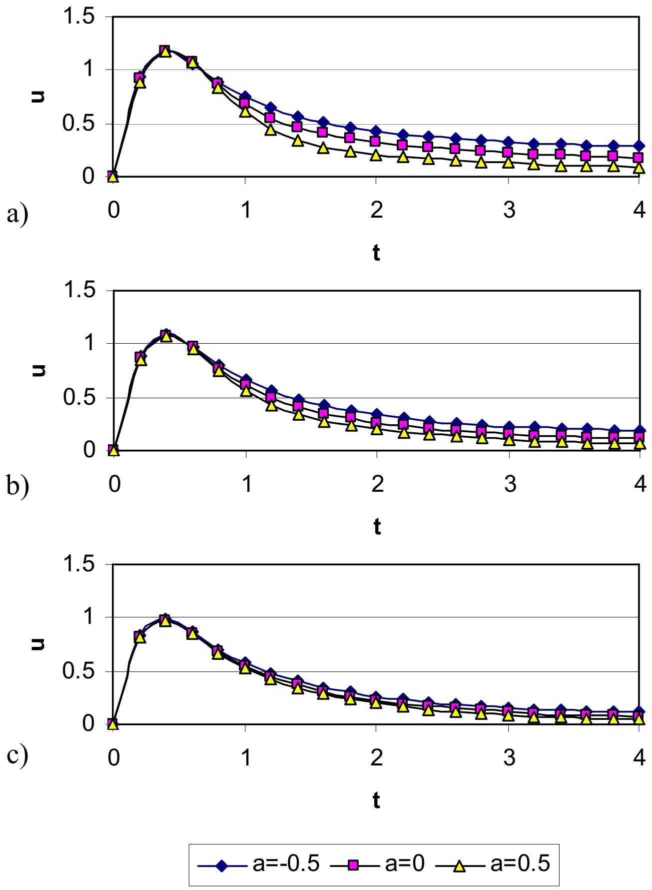

Figure 6 – The evolution of u at $y = 0$ for various values of a and S: (a) $S = 0$; (b) $S = 1$; (c) $S = 2$. ($Ha = 3$, $m = 3$, $b = 0$).

Note that the influence of increasing the parameter b on T is more apparent for higher values of suction velocity.

4 Conclusions

The transient MHD Couette flow between two parallel plates is studied with the inclusion of the Hall effect. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. The effects of the Hartmann number Ha, the Hall parameter m, the viscosity parameter a and the thermal conductivity parameter b on the velocity and temperature fields at the centre of the channel

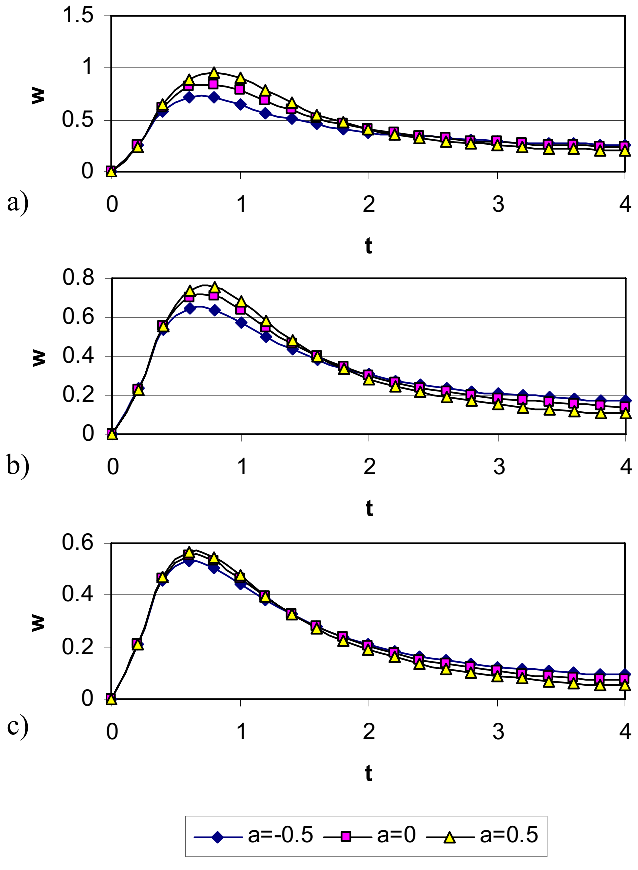

Figure 7 – The evolution of w at $y = 0$ for various values of a and S: (a) $S = 0$; (b) $S = 1$; (c) $S = 2$. ($Ha = 3$, $m = 3$, $b = 0$).

are discussed. Introducing the Hall term gives rise to a velocity component w in the z-direction and affects the main velocity u in the x-direction. It was found that the parameter a has a marked effect on the velocity components u and w for all values of m. On the other hand, the parameter b has no significant effect on u or w.

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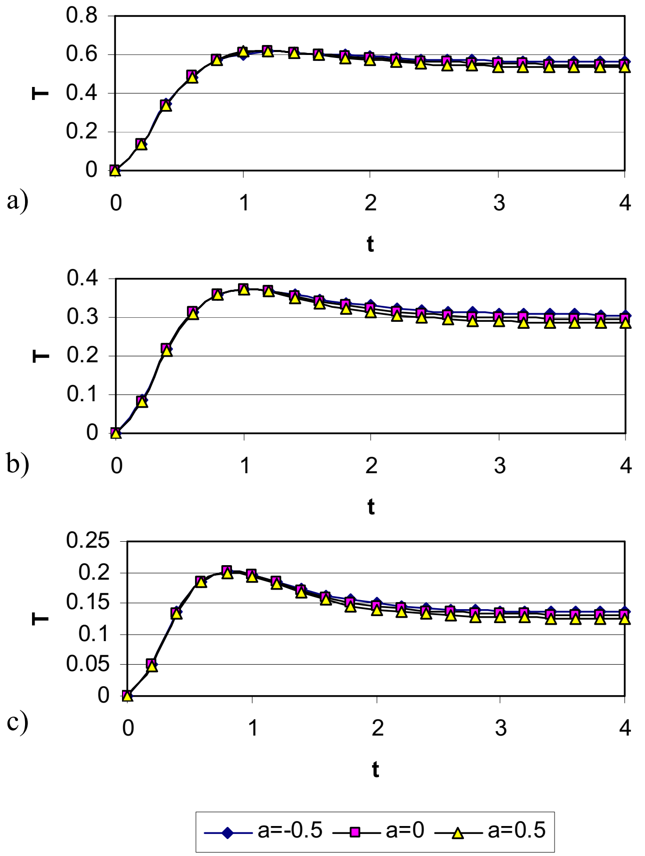

Figure 8 – The evolution of θ at $y = 0$ for various values of a and S: (a) $S = 0$; (b) $S = 1$; (c) $S = 2$. ($Ha = 3$, $m = 3$, $b = 0$).

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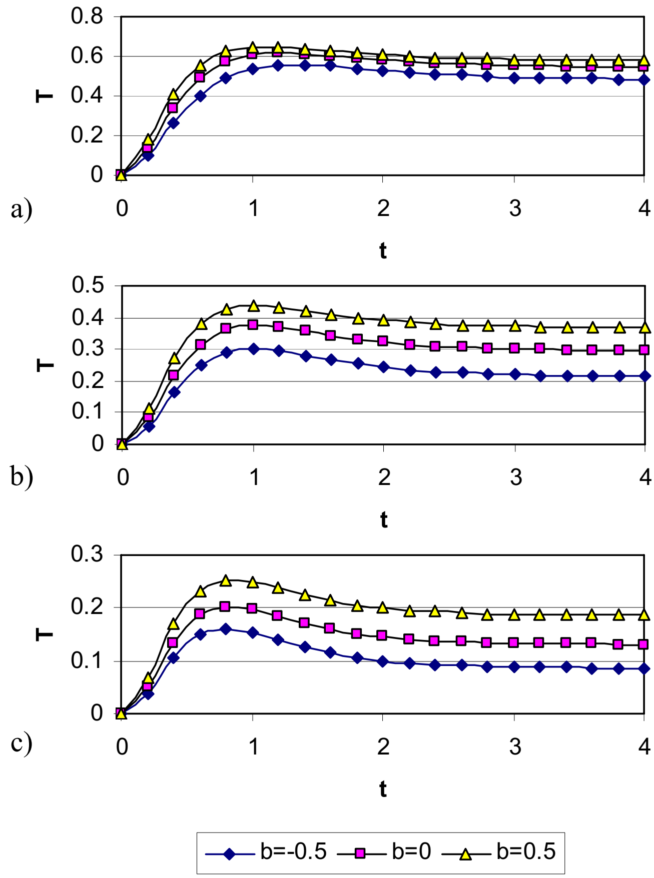

Figure 9 – The evolution of θ at $y = 0$ for various values of b and S: (a) $S = 0$; (b) $S = 1$; (c) $S = 2$. ($Ha = 3$, $m = 3$, $a = 0$).

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