ALTERNATIVE METHODOLOGY FOR CALCULATING THE MODULUS OF ELASTICITY OF WOODEN BEAMS OF STRUCTURAL DIMENSIONS

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ABSTRACT: This study aims to present an alternative calculation methodology based on the Least Squares Method for determining the modulus of elasticity in bending wooden beams of structural dimensions. The equations developed require knowledge of three or five points measured in displacements along the piece, allowing greater reliability on the response variable, using the statistical bending test at three points and non-destructively, resulting from imposition of measures from small displacements L/300 and L/200, the largest being stipulated by the Brazilian norm NBR 7190:1997. The woods tested were Angico, Cumaru, Garapa and Jatoba. Besides obtaining the modulus of elasticity through the alternative methodology proposed, these were also obtained employing the Brazilian norm NBR 7190:1997, adapted to the condition of non-destructive testing (small displacements) and for pieces of structural dimensions. The results of the modulus of elasticity of the four species of wood according to both calculation approaches used proved to be equivalent, implying the good approximation provided by the methodology of calculation adapted from the Brazilian norm.

KEYWORDS: wood, beam theory, modulus of elasticity, Least Squares Method.

METODOLOGIA ALTERNATIVA PARA O CÁLCULO DO MÓDULO DE ELASTICIDADE EM VIGAS DE MADEIRA DE DIMENSÕES ESTRUTURAIS

RESUMO: Este trabalho objetiva apresentar uma metodologia alternativa de cálculo fundamentada no Método dos Mínimos Quadrados para a determinação do módulo de elasticidade na flexão, em vigas de madeira de dimensões estruturais. As equações desenvolvidas requerem o conhecimento de três ou cinco pontos medidos em deslocamentos ao longo da peça, permitindo maior confiabilidade sobre a variável resposta, utilizando-se do ensaio de flexão estática a três pontos e de forma não destrutiva, decorrente da imposição das medidas de pequenos deslocamentos *L/300* e *L/200*, sendo a maior delas estipulada pela norma Brasileira NBR 7190:1997. As madeiras testadas foram Angico, Cumaru, Garapa e Jatobá. Além da obtenção dos módulos de elasticidade pela metodologia alternativa proposta, estes foram também obtidos do emprego da norma Brasileira NBR 7190:1997, adaptada para a condição de ensaio não destrutivo (pequenos deslocamentos) e para peças de dimensões estruturais. Os resultados dos módulos de elasticidade das quatro espécies de madeira, segundo ambas as abordagens de cálculo utilizadas, mostraram-se equivalentes, implicando na boa aproximação fornecida pela metodologia de cálculo adaptada da norma Brasileira.

PALAVRAS-CHAVE: madeira, teoria de vigas, módulo de elasticidade, Método dos Mínimos Quadrados.

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INTRODUCTION

The use of wood has grown over the last few years, because it is a material of renewable source, low density and good mechanical performance, usually used in civil and rural buildings, performing the structural role as beam elements, columns and others (CHRISTOFORO, 2011).

The design of wooden structures, as well as of other materials, requires the knowledge of some variables, including the modulus of elasticity, obtained through experimental tests advocated by normative documents, which may be destructive or not.

Because the wood is an orthotropic and heterogeneous material, aiming to increase reliability, its characterization in bending is more appropriate if performed in parts with structural dimensions. In this context, only international normative documents can be cited (EN 789:1995, ASTM D4761:1996, ASTM D198:1997), since the Brazilian norm ABNT NBR 7190:1997 (Wood Structures Design), which deals with the wood characterization, contemplates only the condition of destructive testing on sample parts with small dimensions and free from defects (Pigozzo et al, 2000; FIORELLI, 2005; MIOTTO & DIAS, 2009).

Structural models contained in the normative documents mentioned above consist of the static bending tests at three and four-points, obtaining the modulus of elasticity from the knowledge of two measurements of force and successive displacements, defined for the stretch of elastic and linear behavior of the material, based on the value of the maximum force applied to the part. The mathematical models of calculations contained in these codes do not include criteria for optimality (idealized formulations in the search for optimal solutions –neighboring), with displacements in the trial obtained from one or two different points along the elements.

As mentioned before, the characterization of wooden pieces of structural dimensions can also be accomplished through non-destructive testing, aimed at determining the physical and mechanical properties of a structural element without changing its use capabilities (ROSS et al., 1998, WANG et al, 2008; LIANG & FU, 2007; DONG and HAI, 2011; SALES et al, 2011). The advantage of employing non-destructive testing constitutes waiving the extraction of sample parts, further enabling the study of structural integrity (OLIVEIRA & SALES, 2002; MINÁ et al, 2004; BURDZIK & NKWERA, 2002), commonly performed by means of tests with transverse and ultrasound vibration.

From the above, the mathematical models contained in normative documents for the calculation of the modulus of elasticity in bending do not include optimality criteria, and with respect to the usual non-destructive testing (transverse and ultrasound vibration), the need to acquire specialized equipment for determining the modulus of elasticity is emphasized.

This paper proposes an alternative calculation methodology, based on the Least Squares Method and on the three-point bending test, non-destructively, for determining the modulus of elasticity for bending in pieces of lumber in structural dimensions, having evaluated the Angico , Cumaru , Garapa and Jatoba woods.

MATERIAL AND METHODS

For validity of the use of Euler Bernoulli beam theory used in the calculation of the modulus of elasticity by this method, the wooden beams must comply with the $L/h \geq 21$ relation (ROCCO LAHR, 1983), disregarding the effect of the shearing stress in the calculation of displacements, where L is the effective length of the piece (distance between supports - span) and h is the height of the cross section.

The experimental test used to determine the modulus of elasticity is considered non-destructive, because the highest values are in displacement in the experiments (midpoint) limited to the reasons $L/200$ and $L/300$ (L in cm), the largest of them being defined by the Brazilian norm ABNT NBR 7190:1997 as a measure of small displacements. Besides obtaining the modulus of elasticity with displacements restricted to these limits, they were also obtained with the use of the methodology prescribed in the norm ABNT NBR 7190:1997, adapted for nondestructive testing and for structural dimension parts, aiming to verify the differences between them.

Figure 1 illustrates the structural test schemes used to calculate the modulus of elasticity, where L is the span of the piece, F is the force applied at the midpoint of the span and b and e are the dimensions of the base and height of the rectangular cross section,

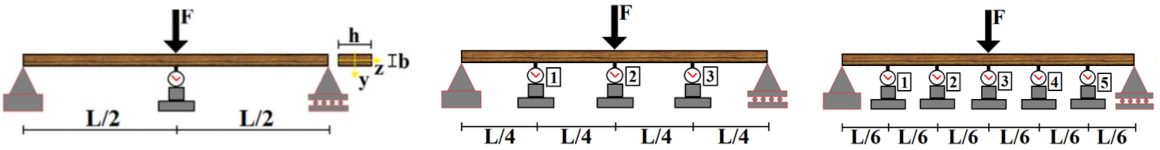


FIGURE 1. Testing setup.

Figure 1a shows the test scheme used to calculate the elasticity modulus according to NBR 7190:1997 (simplified approach), using an only dial indicator positioned at the midpoint of the piece. Figures 1b and 1c respectively illustrate the structural designs used to calculate the modulus of elasticity through the alternative methodology, using three and five dial indicators equally spaced along the elements.

From the structural test design illustrated in Figure 1b, equidistant experimental displacements of the restraints (δ1and δ3) are measured when the displacement at mid-span (δ2) is equal either to *L/300* or *L/200*. Similarly, the displacements (δ1, δ2, δ4 and δ5) of the dial indicators positioned in sixths of the span (Figure 1c) are obtained.

The alternative methodology used to calculate the effective modulus of elasticity is presented for the condition of three dial indicators (Figure 1b), being analogous to the condition of five (Figure 1c).

From the Euler Bernoulli beam theory, analytical displacements in the positions of dial indicators 1, 2 and 3 in Figure 1b are expressed by Equations 1 and 2, rewritten as a function of the longitudinal elastic modulus ($\delta_1(E)$, $\delta_2(E)$ e $\delta_3(E)$).

$$\delta_2(E)=\delta_{máx}=\frac{1}{E}\cdot\frac{F\cdot L^3}{4\cdot b\cdot h^3} \quad (1)$$

$$\delta_1(E)=\delta_3(E)=\frac{1}{E}\cdot\frac{11\cdot F\cdot L^3}{64\cdot b\cdot h^3} \quad (2)$$

Equation 1 is equivalent to the proposal by the Brazilian norm NBR 7190:1997 for the calculation of the elastic modulus (E_m), also consisting of a (simplified) calculation methodology to be evaluated in this work.

The elastic modulus to be calculated with the information derived from the test model of Figure 1b starts out from the idea of least squares (Equation 3), aiming to determine the value of the modulus of elasticity so that the residue generated between the analytical ($\delta_i(E)$) and experimental (δ_i) values of displacements is the least possible.

$$f(E)=\frac{1}{2}\cdot\sum_{i=1}^{n}\left(\delta_i(E)-\delta_i\right)^2 \quad (3)$$

Substituting Equations 1 and 2 in Equation 3 and deriving and equating the latter to zero one can get to the effective modulus of elasticity ($E_{m,3}$) for the structural design with three dial indicators, expressed by Equation 4, proving this the minimum and global point by the criteria of the second derivative .

$$E_{m,3} = \frac{249 \cdot F \cdot L^3}{32 \cdot \gamma \cdot b \cdot h^3},\ \gamma = 11 \cdot (\delta_1 + \delta_3) + 16 \cdot \delta_2 \tag{4}$$

By using the same methodology for the calculation of Equation 4 we arrive at the expression for the calculation of the longitudinal modulus of elasticity ($E_{m,5}$) in parts of lumber in structural dimensions using five dial indicators (Figure 1c), expressed by Equation 5.

$$E_{m,5} = \frac{2125 \cdot F \cdot L^3}{108 \cdot \beta \cdot b \cdot h^3},\ \begin{cases} \beta = 13 \cdot \gamma_1 + 23 \cdot \gamma_2 + 27 \cdot \delta_3 \\ \gamma_1 = \delta_1 + \delta_5 \\ \gamma_2 = \delta_2 + \delta_4 \end{cases} \tag{5}$$

Equations 1, 4 and 5 along with the displacement restrictions L/300 and L/200 were used to calculate the modulus of elasticity for Angico, Cumaru, Garapa and Jatoba woods. Woods had 35cm × 50cm × 130cm medium size, and 12 pieces of each species were evaluated.

RESULTS AND DISCUSSION

Tables 1, 2, 3 and 4 respectively show the results obtained for the modulus of elasticity (E_m; $E_{m,3}$; $E_{m,5}$) of the wood parts of the species Angico, Cumaru, Garapa and Jatoba for the displacement averages *L/300* and *L/200*.

TABLE 1. Angico timber modulus of elasticity (MPa).

L/200	***E_m* (MPa)**	***$E_{m,3}$* (MPa)**	***$E_{m,5}$* (MPa)**	**L/300**	***E_m* (MPa)**	***$E_{m,3}$* (MPa)**	***$E_{m,5}$* (MPa)**
A1	9727	8520	9498	A1	8951	8764	9118
A2	9972	8822	11285	A2	9964	10050	10811
A3	10875	11488	11971	A3	10609	10623	11184
A4	10099	8636	11804	A4	9887	8079	11062
A5	10492	10000	10126	A5	9979	9343	9982
A6	10914	9920	21065	A6	10551	10501	10405
A7	9892,6	13842	13829	A7	9599	5964,2	6189,1
A8	9422	10383,7	9686,4	A8	9501	9584	9547
A9	10201,2	10883	10817	A9	10193	10001	10356
A10	9976	10352	10581	A10	10180,5	10566	10161
A11	10403	10022	10774	A11	10196	10124	10304
A12	9186	8322	9358	A12	9043	9052	9217

TABLE 2. Cumaru timber modulus of elasticity (MPa)

L/200	***E_m* (MPa)**	***$E_{m,3}$* (MPa)**	***$E_{m,5}$* (MPa)**	**L/300**	***E_m* (MPa)**	***$E_{m,3}$* (MPa)**	***$E_{m,5}$* (MPa)**
C1	8987	8731	9498	C1	8743	8653	9118
C2	13292	10715	11285	C2	13190	12212	10811
C3	11854	11207	11971	C3	11714	10984	11183
C4	7731	7977	11804	C4	7636	7776	11061
C5	9968	9849	10126	C5	9890	9493	9981
C6	15949	18398	21065	C6	15323	15974	10404
C7	9014	8604	13829	C7	8872	5967	6189
C8	9190	8712	9686	C8	8966	8948	9546
C9	18585	18797	10817	C9	18589	17139	10356
C10	10403	10317	10580	C10	10370	10057	10161
C11	7900	7223	10774	C11	7710	7091	10303
C12	8329	7554	9357	C12	8336	7540	9216

TABLE 3. Garapa timber modulus of elasticity (MPa)

L/200	E_m (MPa)	$E_{m,3}$ (MPa)	$E_{m,5}$ (MPa)	L/300	E_m (MPa)	$E_{m,3}$ (MPa)	$E_{m,5}$ (MPa)
G1	8670	8178	8812	G1	8602	8030	8569
G2	22819	22009	21941	G2	22718	21894	21828
G3	10460	10514	7776	G3	10238	10160	10168
G4	9585	9429	9736	G4	9545	9321	9580
G5	11803	11534	11759	G5	11885	11389	11537
G6	13091	13354	13100	G6	12772	13302	13022
G7	7803	7480	7940	G7	7926	7359	7814
G8	8155	7375	6949	G8	7423	7218	7108
G9	21076	20670	21357	G9	20988	20357	21049
G10	9688	9913	10120	G10	9617	9600	10161
G11	7128	7155	7429	G11	7138	6993	7283
G12	8544	8195	8750	G12	8478	8119	8640

TABLE 4. Jatoba timber modulus of elasticity (MPa)

L/200	E_m (MPa)	$E_{m,3}$ (MPa)	$E_{m,5}$ (MPa)	L/300	E_m (MPa)	$E_{m,3}$ (MPa)	$E_{m,5}$ (MPa)
J1	9302	8901	9182	J1	9232	8739	8847
J2	22154	21224	19967	J2	21983	21172	20883
J3	10041	10385	10843	J3	10147	10165	10523
J4	7422	7450	6852	J4	7500	7273	6937
J5	13772	12893	13596	J5	13707	12913	13428
J6	22664	21688	22788	J6	22433	21963	22540
J7	8793	8583	8948	J7	8579	8427	8685
J8	10339	10123	10724	J8	10183	7304	10312
J9	20851	18851	21322	J9	20538	18697	20540
J10	11687	10909	10596	J10	11596	10714	11741
J11	4622	4130	4813	J11	4460	4104	7829
J12	8117	7601	8152	J12	8126	7524	7891

Figure 2 shows the residual graphs of the modulus of elasticity obtained from the use of the Brazilian NBR 7190:1997 (E_m) for the wood species Jatoba and Angico, in order to verify that the samples are in accordance with the assumptions of hypothesis testing. At the employment of the hypothesis testing procedure we start from the premise that both samples are drawn from independent populations, described by a normal distribution, and that the standard deviations or variations of the populations are equal (MONTGOMERY, 2005). It is observed that the points distributed uniformly along the line for Angico wood meet the conditions of normality and homogeneity required for validation of this test, which does not occur with the other species (Cumaru, Garapa and Jatoba).

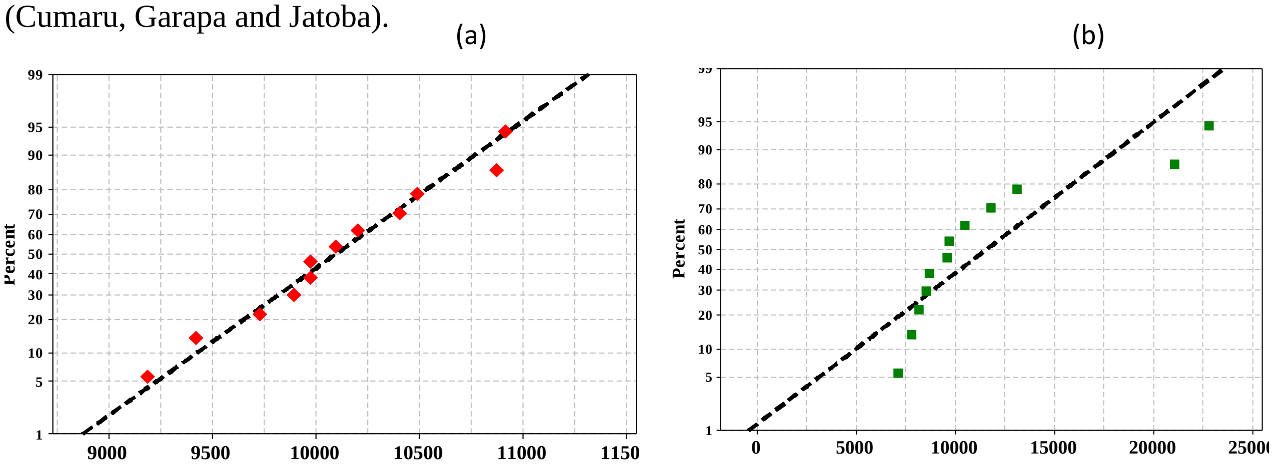
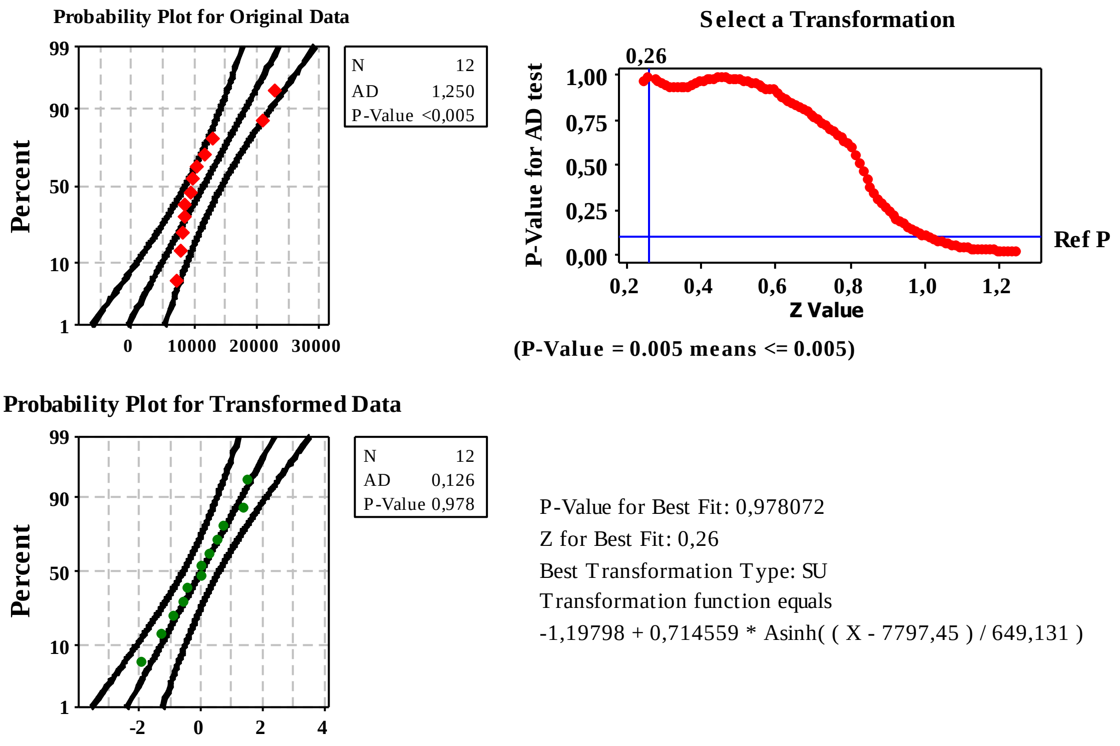


FIGURE 2. Normality test of the E_m modulus of elasticity for Angico (a) and Jatoba (b) timber.

Alternatively, the Johnson transformation for achieving standardization and homogeneity of data was applied to wood species Cumaru, Garapa and Jatoba. The result of the confidence interval (Probability Plot for Transformed Data) for Jatoba wood (Figure 3), as well as for Garapa and Cumaru woods, proves the normality of the transformed data by presenting P-value of 0.978, greater than 0, 05 (CHOU et al. 1998).



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P-Value for Best Fit: 0,978072
Z for Best Fit: 0,26
Best Transformation Type: SU
Transformation function equals
-1,19798 + 0,714559 * Asinh( ( X - 7797,45 ) / 649,131 )
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FIGURE 3. Johnson transformation for data standardization of Jatoba wood.

After normalization of the modulus of elasticity of Cumaru, Garapa and Jatoba woods, in order to verify the statistical equivalence between the values of the effective modules ($E_{m,3}$ a $E_{m,5}$) with the ones coming from NBR 7190:1997 (E_m) for the four wood species, the hypothesis test of the means of two independent populations was used, with the results for the Angico timber shown in Tables 5. The P-values greater than 0.05 (5% significance) or the appropriateness of the zero in the confidence interval proves the statistical equivalence between the modulus of elasticity (MONTGOMERY, 2005), which are also equivalent for the other three wood species.

TABLE 5. Hypothesis testing of the Angico timber.

Hypothesis test	Standard deviation	Mean	P-value	Confidence Interval
Test t: E_m, $E_{m,3}$, *L/200*	525	10096	0.995	$-1018.76 \le \mu \le 1013.26$
	1542	10099		
Test t: E_m, $E_{m,5}$, *L/200*	525	10096	0.108	$-3693.40 \le \mu \le 420.31$
	3194	11733		
Test t: $E_{m,3}$, $E_{m,5}$, *L/200*	1542	10099	0.131	$-3816.45 \le \mu \le 548.80$
	3194	11733		
Test t: E_m, $E_{m,3}$, *L/300*	529	9888	0.246	$-386.413 \le \mu \le 1386.91$
	1331	9388		
Test t: E_m, $E_{m,5}$, *L/300*	529	9888	0.950	$-859.99 \le \mu \le 913.24$
	1331	9861		
Test t: $E_{m,3}$, $E_{m,5}$, *L/300*	1331	9388	0.393	$-1603.576 \le \mu \le 656.32$
	1331	9861		

CONCLUSIONS

The present methodology enables obtaining the modulus of elasticity in bending wood parts of structural dimensions with higher reliability for being based on optimality concepts, allowing the use of three or five values in experimentally measured displacements.

The restriction of displacement in the bending test (*L/300* and *L/200*) confers the present methodology nondestructive character, which is interesting for the possibility of being able to use the part after tested.

The results between the modulus of elasticity from the use of the Brazilian ABNT norm NBR 7190:1997 (E_m) equation, adapted for pieces of structural dimensions and methodology of non-destructive testing, with what derives from the approach proposed herein ($E_{m,3}$ e $E_{m,5}$) were both equivalent. However, the results obtained cannot be extrapolated to other woods of the same or different species, justifying the use of the present calculation approach proposed.

Thus, the simplified model, adapted from the Brazilian NBR 7190:1997 norm, was able to provide results close to those obtained with the use of the least squares-based methodology (alternative), presenting itself as an effective alternative calculation methodology.

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