TECHNICAL PAPER

INFLUENCE OF STIFFNESS IN BOLTED CONNECTIONS IN WOODEN PLANE STRUCTURE OF TRUSS TYPE

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ABSTRACT: Trusses are structural systems commonly used in projects, being employed mainly in roof structures, present in most rural buildings. The design of trusses, as well as other structural systems, requires the determination of displacements, strains and stresses. However, the project is developed from an ideal model of calculation, considering free rotation between the elements of a connection. This paper presents a computer program for the analysis of bidimensional wooden trusses with connections formed with two screws per node. The formulation is based on the flexibility method, taking into account the influence of the effect of semi-rigid connections formed by two screws. An example of a structure is presented and analyzed by the program developed here, highlighting the importance of behavior analysis on semi-rigid connections.

KEYWORDS: trusses, semi-rigid connections, flexibility method, rural buildings.

INFLUÊNCIA DA RIGIDEZ DE LIGAÇÕES PARAFUSADAS EM ESTRUTURAS PLANAS DE MADEIRA DO TIPO TRELIÇA

RESUMO: Treliças são sistemas estruturais comumente utilizados em projetos, empregadas principalmente em estruturas de cobertura, presentes na maioria das construções rurais. O dimensionamento de treliças, assim como o de outros sistemas estruturais, requer a determinação dos deslocamentos, esforços, tensões e deformações atuantes em seus elementos constituintes. O cálculo é desenvolvido com base em um modelo ideal, considerando-se o giro livre entre os elementos componentes de uma ligação. Este trabalho objetiva apresentar um programa computacional destinado à análise de treliças planas de madeira com ligações formadas com dois parafusos por nó. A formulação é fundamentada no Método da Flexibilidade, levando-se em consideração a influência do efeito semirrígido das ligações formadas por dois parafusos. Um exemplo de estrutura auxiliar de cobertura é apresentado e analisado pelo programa desenvolvido, evidenciando-se a importância da análise do comportamento semirrígido sobre as ligações.

PALAVRAS-CHAVE: estruturas de cobertura, ligações semirrígidas, método da flexibilidade, construções rurais.

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INTRODUCTION

The use of wood as a structural element in Brazil has grown over the past few years because of the research carried out in order to make it more competitive than other construction materials.

In the case of structures, wood can be used in temporary works, such as anchors and forms, or as a structural element, such as beams, columns, poles, trusses and more. Wood is widely used in structures of truss-type roofs, as those found in sheds and diverse rural buildings.

The flat design of roof structures is usually done through ideal calculation models, in which the links are considered as perfectly flexible in rotation (trusses), i.e., transmitting only normal forces among their structural components.

In practice, flat connections in wooden structures are designed in different ways, and especially among them, the use of bolted connections. However, the existence of two or more screws forms a semi-rigid effect of these connections.

The semi-rigid effect of connections lies between two ideal calculating models: the free rotating (truss) and the perfectly rigid (porch) models.

According to RIBEIRO (1997), the study of connections began in England in the early nineteenth century, with the study of the riveted beam-column type, which the moment-rotation relationship was evaluated.

A work of great importance is that by JOHNSTON & MOUNT (1942), which analyzed frames with semi-rigid connections. Later, SHOROCHNIKOFF (1950) reported the influence of forces by the wind in semi-rigid connections for the same type of structure.

For metal structures, LOTHERS (1951) proposed equations to represent the elastic constraint of semi-rigid connections. KRISHNAMURTHY et al. (1979) applied the Finite Element Method (FEM) to obtain moment-rotation curves of connections with steel sheets, and JONES et al. (1983) studied the influence of semi-rigid connections in steel columns.

In the case of studies of connections in wooden structures, the work by OLIVEIRA & DIAS (2001) evaluated through the analysis of experimental results, the criterion for the design of the metal pin connections, proposed by the technical standard NBR 7190/97 (Design of wooden structures). Other relevant works involving the study of links and numerical methods applied to wooden structures are the following: SANTANA & MASCIA (2009), SOUZA JUNIOR & GESUALDO (2007), SOUZA JUNIOR & GESUALDO (2006), STAMATO & CALIL (2002), CARVALHO (2002), GESUALDO (2001), SERAPHIM & FRANCO (2001), EMERSON & FRIDLEY (1996), GROOM (1996) among others.

The present work aims at developing a computer program PS-R (Semi-Rigid Porch), developed from the foundations of the flexibility method, to evaluate the effect of semi-rigid connections formed by two screws in wooden plane structures of truss type. A roof structure is evaluated by the PS-R program in order to verify the importance of this analysis.

METHODS

The modified stiffness matrix $[S_M^0]$ that accounts for the semi-rigid effect of the rotational component for each element of the structure is developed according to the method of flexibility, as expressed by Equation 1. Further details on obtaining the modified stiffness matrix can be found in the work by WEAVER & GERE (1986).

$$[S_M^0]=\begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & 6Ce_{Rij} & 3CLe_{Rj2} & 0 & -6Ce_{Rij} & 3CLe_{Ri2} \\ 0 & 3CLe_{Rj2} & 2CL^2e_{Rj3} & 0 & -3CLe_{Rj2} & CL^2 \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -6Ce_{Rij} & -3CLe_{Rj2} & 0 & 6Ce_{Rij} & -3CLe_{Ri2} \\ 0 & 3CLe_{Ri2} & CL^2 & 0 & -3CLe_{Ri2} & 2CL^2e_{Ri3} \end{bmatrix} \qquad ((1)$$

where,

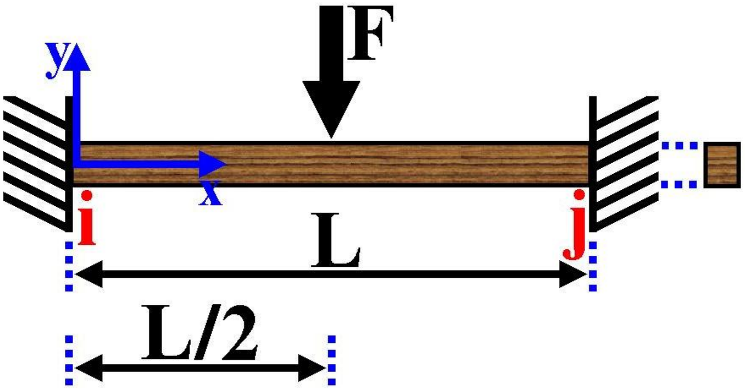
A - cross-sectional area of the structural element;
E - longitudinal modulus of elasticity or Young's modulus;
L - length of the structural element;
I - moment of inertia of cross section;

C - coefficient of flexibility, obtained by: $C=\dfrac{2EI}{L^3\left[4e_{Rij}+3\left(4e_{Ri}e_{Rj}-1\right)\right]}$, and

e_{Rij}; e_{Ri}; e_{Rj} - dimensionless parameters of flexibility.

The coefficients of flexibility contained in the modified stiffness matrix element of the beam, allow assessing the structural behavior through three different forms of analysis: the first form accounts for the effect of semi-rigid connection, the second one considers the structure as a porch and the third one considers it as a truss. To analyze the behavior of semi-rigid connections via the software developed in FORTRAN language, it is necessary to know the coefficients of flexibility that make the modified stiffness matrix (equation 1) in a stiffness matrix for porch beam and in a stiffness matrix for truss beam, respectively.

The determination (calibration) of the coefficient of flexibility responsible for transforming the stiffness matrix in the modified stiffness matrix for a porch beam is performed according to the structural model of a beam embedded in their edges with a concentrated force of intensity F, applied in mid-span, as illustrated in Figure 1.



For the structural variables were assigned the following numeric values: F = -5 kN ; A = 50 cm^2 ; I = 467.67 cm^4 ; E = 2000 kN/cm^2 ; L = 200 cm

FIGURE 1. Bi-clamped beam.

Several attempts were made to find the rotational coefficient of semi-stiffness, which aims to transform the stiffness matrix into the modified stiffness matrix of a beam porch element. It was found that for a coefficient of 1.0×10^9, the values of the transverse displacements at the point of application of force as well as the relative rotations at the ends of the beam converged to the results provided by commercial software SAP 2000. For values above 1.0×10^9, the results for the displacements with the use of these programs, PS-R and SAP 2000, remained constant.

To determine the coefficient of rotational semi-stiffness that represents the structure as bolted, the same tests (variation of rotational stiffness) were carried out; however, the structure considered in SAP 2000 was bi-clamped (hinged). The coefficient of semi-rigid rotation found for this case was 1.0×10^{-9}.

It is important to note that the rotational coefficients of semi-rigidity herein determined are independent of the geometrical and physical parameters used in numerical simulations and that the axial stiffness was kept constant over the whole data analysis, varying only the rotational stiffness. In addition to analyzing the effect of semi-rigid connections, the PS-R program also allows evaluating the structure as a porch or a truss.

In the case of the connection formed by two screws, the standards adopted for the calculation and the respective provisions were established by the normative NBR 8800/1986 (Project and execution of steel structures in buildings), replaced by the current version NBR 8800/2008, to quantify the forces acting on the screws of the connection. The removal of the screws (Figure 2) confers the presence of a moment of resistance in the connection.

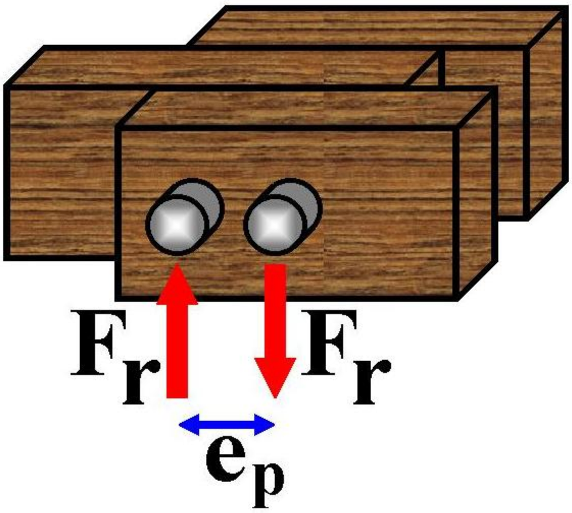


FIGURE 2. Detail of bolted connection.

The shear resistance force of the screws F_R and the resistance moment M_R of the connection are expressed by Equations 2 and 3, respectively:

$$F_R = \Phi_V R_{NV}, \; R_{NV} = 0{,}42 A_P f_u \quad (2)$$

$$M_R = F_R e_p \quad (3)$$

where,

e_p - spacing between screws;

A_P - gross area, based on the nominal diameter “d_p” of the screw;

f_u - tensile strength of the screw material;

R_{NV} - nominal shear strength, and

Φ_V - weighting coefficient of the shear resistant strength.

According to the version of the normative document NBR 8800/1986, the weighting values for bolts ASTM A325 and ASTM A490 equals to 0.65 and the minimum distance between their centers should not be less than $3d_p$ (constructive disposition).

Table 1 indicates, according to the specifications of the material, the shear resistance of the screw according to their diameter.

TABLE 1. Materials used on screws.

Specification	Resistance (kN cm^{-2})	Nominal Diameter "d_p" (mm)
ASTM A325	82.50	$12,7 \leq dp < 25,4$
	72.50	$25,40 \leq dp < 38,10$
ASTM A490	103.50	$12,7 \leq dp < 38,10$

The analysis of the behavior of semi-rigid connections formed by two screws through the program PS-R is carried out according to the correction of rigid connections, which is due to an iterative process. This correction is made only when the resistance moment of the connection becomes lower than that of the applicant. To determine in advance the strains on the structure, the connections are considered to be perfectly rigid, and soon after this consideration, the bending stresses acting on the connection are compared in module with the resistance that it presents.

The calculation of the resistance moment of the connection by the PS-R program is conducted as a function of the spacing and diameter of the screws, being equal for all nodes of the structure and constant throughout the analysis (equation 3). Attention should be paid for the minimum dimension of the screws provided by the normative, to not impair the reliability of the results.

After calculating the bolted moment and the resistance moment of the connections, the program compares these two values. If the bolted moment is less than or equal to the resistance moment, the connections are considered rigid and the value of the moment acting on the connection is the bolted moment. If the bolted moment in a connection is greater than the resistance moment, the program recalculates the whole structure so that it finds a new equilibrium configuration. This procedure is incremental and iterative and, for the bolted connections beyond their resistance, the value of the coefficient of semi-rigid rotational connection was successively decreased in 1% and all strains in the structure (redistribution of the surplus moment) were recalculated.

EXAMPLE OF APPLICATION AND RESULTS

Figure 3 illustrates the structure analyzed by the PS-R program.

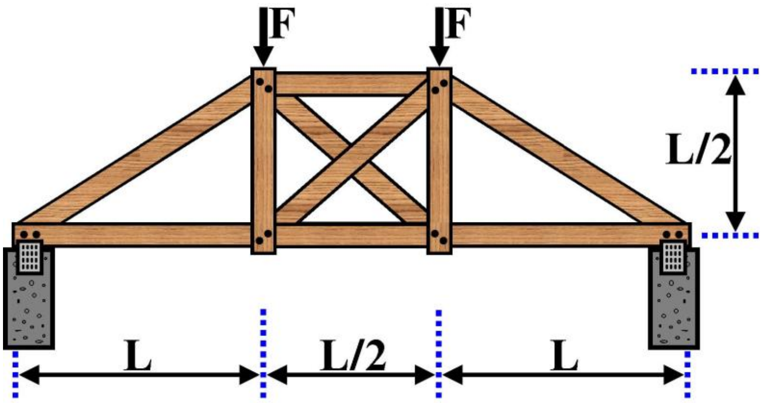
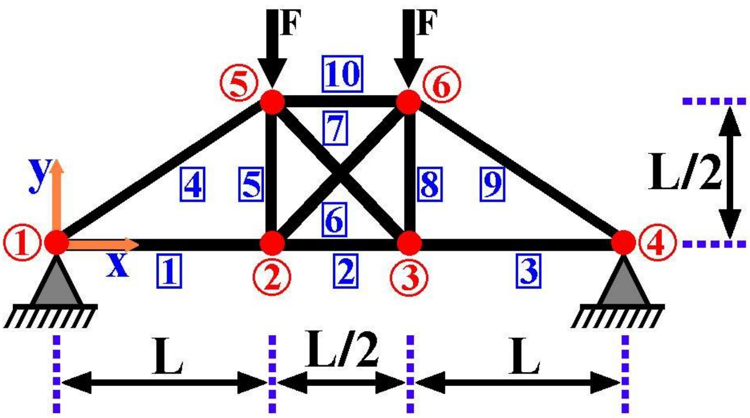


FIGURE 3. Type-A structure.

The identification of nodes and elements of the components of type A structure are illustrated in Figure 4.



In the analysis of the type-A structure, the following values of structural variables are used: F = 50 kN ; A = 90 cm^2 ; I = 1,687.50 cm^4 ; E = 2000 kN/cm^2 ; L = 200 cm; specification of screw material = ASTM A325; d_p= 1.3 cm ; e_p= 4 cm.

FIGURE 4. Discretization of the type-A structure.

The type-A structure is evaluated by three different forms of analysis: first, the displacements of nodes 2 and 3 (Figure 4) are limited to values less than L/200, according to the NBR 7190/1997 specifications, so to ensure that the structure to be designed is within the linear elastic regime (as required in the project), in order to verify the influence of the semi-rigid effect under small displacements; in the second analysis, all structural variables are maintained, with the exception of force, which is gradually increased in order to verify the intensity responsible for applying two connections beyond their resistance; the third form of analysis aims to determine the amount of force necessary to apply four connections of the structure beyond its resistance, allowing to compare the values of bending moments for both, rigid and semi-rigid connections.

It is important to make clear that the whole structure is designed to withstand the strain and suffer small displacements, ensuring linear elastic behavior of materials, the designer's task is to find the best dimensions and arrangements of structural elements as well as choose the most suitable material. Thus, the second and third forms of analysis aforementioned are presented merely to verify the stiffness loss of the connections calculated by the program.

Table 2 shows the values of nodal displacements of the type-A structure, expressed in centimeters, obtained under the three forms of analysis that the PS-R program performs, with a intensity force F = 50 KN.

TABLE 2. Values of nodal displacements for the type-A structure.

Node	Flexible (truss)		Semi- Rigid		Rigid (porch)	
	Displ. (x) (cm)	Displ. (y) (cm)	Displ. (x) (cm)	Displ. (y) (cm)	Displ. (x) (cm)	Displ. (y) (cm)
1	0	0	0	0	0	0
2	0.00235	-0.36612	0.00248	-0.36150	0.00248	-0.36150
3	-0.00235	-0.36612	-0.00248	-0.36150	-0.00248	-0.36150
4	0	0	0	0	0	0
5	-0.02484	-0.36025	-0.02456	-0.35602	-0.02456	-0.35602
6	0.0248	-0.36025	0.02456	-0.35602	0.02456	-0.35602

where: Displ. (x) - nodal displacement of the element in direction of x-axis; Displ. (y) – nodal displacement of the element in direction of y-axis.

According to the PS-R program, no connection was required beyond their capacity limits, i.e., the calculated acting moments were all less active to the resistance moment, of intensity

105.60 kN cm, noting that this value was calculated by the program and was the same for three analysis presented. This result is observed in Table 2, where displacement values for the structure with rigid and semi-rigid connections are exactly the same.

For the second form of analysis, the intensity of the force responsible for applying the first two connections beyond their resistance limits equals to 75 kN. Table 3 shows values of bending moments acting on the structure nodes, expressed in kN cm, considering semi-rigid and rigid connections.

TABLE 3. Values of bending moments acting on type-A structure by a force of 75 kN.

			Rigid (porch)		Semi-Rigid	
	Connectivity		Bending moment (kN cm)		Bending moment (kN.cm)	
Element	Node (i)	Node (j)	Node (i)	Node (j)	Node (i)	Node (j)
1	1	2	-24.01698	-121.53668	-23.6591	-120.17243
2	2	3	36.96272	-36.962762	36.64768	-36.64786
3	3	4	121.53668	24.01690	120.17243	23.65691
4	1	5	24.01690	-63.12227	23.65691	-63.27271
5	2	5	28.97094	29.16207	28.39852	28.96128
6	2	6	55.60302	3.19366	55.12605	2.89919
7	3	5	-55.60302	-3.19366	-55.12605	-2.89919
8	3	6	-28.97094	-29.16277	-28.39852	-28.96128
9	6	4	63.12227	-24.01690	63.27271	-23.65691
10	5	6	37.15385	-37.15385	37.21062	-37.21062

According to the program, nodes 2 and 3 (Figure 4) were applied beyond their resistance capacities and, for the redistribution of the residual bending moment to the entire structure, 465 iterations were necessary.

For the third form of analysis, the intensity of the force responsible for applying the first four connections beyond their limits of resistance is equal to 372 kN. Table 4 shows the values of bending moments acting on the nodes of the structure, expressed in kN cm, considering the semi-rigid and rigid connections.

TABLE 4. Values of bending moments acting on the type-A structure by a force of 372 kN.

			Rigid (porch)		Semi-Rigid	
	Connectivity		Bending moment (kN.cm)		Bending moment (kN.cm)	
Element	Node (i)	Node (j)	Node (i)	Element	Node (i)	Node (j)
1	1	2	-119.12384	-602.82193	-56.83838	-120.08168
2	2	3	183.33508	-183.33508	68.54579	-68.54579
3	3	4	602.82193	119.12384	120.08168	56.83838
4	1	5	119.12384	-313.08645	56.83838	-120.13901
5	2	5	143.69586	144.64384	-31.55450	-7.31987
6	2	6	275.79100	15.84055	83.09039	-32.70217
7	3	5	-275.79100	-15.84055	-83.09039	32.70217
8	3	6	-143.69586	-144.64389	31.55450	7.31987
9	6	4	313.08645	-119.12384	120.13901	-56.83838
10	5	6	184.28311	-184.28311	94.75671	-94.75671

According to the program, nodes 2, 3, 5 and 6 (Figure 4) were applied beyond their resistance capabilities and, for the redistribution of residual bending moments to the entire structure 1,135 iterations were necessary.

CONCLUSIONS

The force value of 50 kN responsible for causing small displacements (L/200) in the structure originated applying moments on the connections for both, rigid and semi-rigid calculation models, lower than the resistance moment (105.60 kN cm). The displacements values shown in Table 2 indicate great similarity in terms of displacement between the flexible (truss), the rigid and the semi-rigid models. For design conditions in which the applicant moment is lower than the resistance moment, the truss model represents as a good calculation alternative, since it does not require the use of iterative processes, contrary to what happens with the PS-R program, which does consider the effect of semi-rigid connections.

The use of the PS-R program allowed finding values of the forces responsible for applying some connections beyond their respective resistances, which represents an alternative calculation tool for the analysis of truss displacements and stresses. However, it should be noted that the rigidity of the connection is usually greater than that of the beam elements, resulting in failure of the wood before the occurrence of maximum application to the connection. Studies involving the consideration of non-linear physics to the wood allow a more precise analysis of this effect, being the subject of interest for the development of future work.

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