***History, philosophy, and sociology of mathematical education in teachers training**: a research program*

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Abstract

We present and discuss in this article some features of a research program whose central object of investigation is the way in which the recent fields of history, philosophy, and sociology of mathematical education could take part in a critical and qualified manner in the initial and continuing training of teachers in this area. For that, we endorse the viewpoint that the courses for mathematics teacher education should be based on a conception of specificity through which a new pedagogical project could be established. In such project those new fields of investigation would participate, in an organic and clarifying way, in the constitution of multidimensional problematizations of school practices, in which mathematics would be involved, and that would be guided by academic investigations about the issues that currently challenge teachers in the critical work of incorporation, resignification, production, and transmission of mathematical culture in the context of the school institution.

Keywords

Mathematical school education - Teacher education - Basic education.

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Introduction

In the last years much has been said, both in Brazil and in other countries, about the critical and formative potentialities of an organic participation of the history of education in the school mathematics education, and consequently also in the training of mathematics teachers (Fauvel; van Maanen, 2000; Miguel; Miorim, 2002, 2004; Miguel, 2001; Miorim; Miguel, 2001, 2002; Miguel; Brito, 1996; Katz; Michalowicz, 2004). In a less emphatic tone, there has also been talk about the critical and formative potentialities of the philosophy of mathematics for such formation (Bkouche, 1997; Bicudo, 1999, 2003; Ernest, 1991, 1994, 1995; Garnica, 1999; Jesus, 2002; Miguel, 2003, 2004b; Steiner, 1987; Vianna, 2004). Now, although there is a fair amount of discussion these days concerning the sociocultural and political dimensions both of mathematics and of mathematics education – discussions that have been taking place above all inside movements born in the 1980s, such as those of *ethnomathematics*, of *critical mathematics education,* and of *mathematics education and society* (D'Ambrosio, 1990, 2001; Knijnik, 2002; Frankenstein; Powell, 1997; Barton, 1999; Ferreira, 1990; Gerdes, 1991; Skovsmose, 2001) – we cannot identify any written production about the equally critical and formative role that could be played by the so-called sociology of mathematics in the education of mathematics teachers.

Although discussions about the relationships between thought and society have surfaced at least since the late Renascence, sociology has only constituted itself as an autonomous and scientific area of knowledge in the 19th century with the appearance of the Comtean positivism. The sociology of knowledge produced its first developments in the 1930s, the moment when sociologists such as Schütz, Merton, Berger, Luckmann, and Searle recognized and began to consider the circularity of the relationship between society and knowledge, leading them to conceive society itself, in its totality, as the result of a social construction (Crespi & Fornari, 2000, p. 9-16). The independent constitution of sociology of science as an academic field, in its turn, seems to have occurred in the 1930s in the U.S.A., the work of Robert Merton having exerted a decisive influence for such autonomous constitution of the sociology of science, seeing that around 1945 "*Merton had already created an approach in which he identified science as a social institution with its own ethos*" (Barnes, 1980, p. 12). Finally, the expression *sociology of mathematics* seems to have been used for the first time in the 1940s, more precisely in a paper by mathematics historian Dirk J. Struik published in 1942 in the *Science and Society* journal under the title *On the sociology of mathematics*. Since then, social, cultural, or sociocultural approaches to mathematics began to be more common. Among them, we can highlight: *The cultural basis of mathematics* by Raymond Wilder in 1950; the works by David Bloor – such as *A naturalistic approach to mathematics*, *Negotiation in logical and mathematical thought*, and *Can there be an alternative mathematics?* – which began to be published in the 1970s; and the more recent work by Sal Restivo, which started to appear in the 1990s, from which we can mention the article *The social roots of pure mathematics*[1] . It is also from the 1970s the suggestive *Sociology of Mathematics and Mathematicians* by J. Fang and K. P. Takayama (Fang; Takayama, 1975).

The emergent research fields in the history, philosophy, and sociology of mathematics education

To talk about the history, philosophy, and sociology of mathematics is, in our view, quite

1. All these works have been translated to the Portuguese by the members of the TEM (Theory of Mathematics education) Research Group of the Universidade Nova of Lisbon, Portugal. The reader can find these works in the reference (TEM, 1998).

different from talking about the history, philosophy, and sociology of mathematics education. Although mathematics, since Antiquity, has been the object of historical studies[2], and although we have knowledge that histories of mathematics began to be written since that time, it was only at the end of the 20th century that the first studies related to the history of mathematics education began to appear[3].

We know that since Antiquity mathematics has been the object of reflections and isolated philosophical studies. According to Fang & Takayama (1975, p. 33), *Philosophia mathematica* was the title given by Erhard Weigel to a book he had published in 1693. To those authors, the contents of this book, of a certain theological slant, said very little about "*philosophized mathematics or about mathematized philosophy*". Still according to them, the dubious title of Weigel's work could be translated in two different ways: as 'philosophical mathematics' or as 'mathematical philosophy', expressions that would nowadays be representative of two different disciplines or fields of investigation. The first of them – philosophical mathematics – would be a proper domain of mathematicians, since such field is generally seen by that community as being created by taking as its object of study theories potentially 'mathematizable' or 'formalizable' in the sense of being possibly capable of integrating the field of mathematics proper. It is in this sense that many logicians to this day employ the expression *philosophy of mathematics*, that is, as philosophical mathematics. A representative of this conception is the work *The Philosophy of Mathematics* published in 1969, in which its editor J. Hintikka compiles several articles about symbolic logics. For Fang & Takayama (1975, p. 33), on the other hand, the *mathematical philosophy* could be seen *manifestly as a region for philosophers*, and would have as its object of investigation certain mathematical theories of a speculative nature that would have been introduced by mathematicians, but that remained or still remain little developed.

The philosophy of mathematics education is, however, a much more recent subject, and it seems that only in the 1980s it began to be constituted as an autonomous field of study. According to Bicudo (1999, p. 22), the first work with the title of *philosophy of mathematics education* was Eric Blaire's doctoral thesis, presented to the Institute of Education of the University of London in December 1981. But it has been mainly thanks to the work developed by Paul Ernest, a professor at the School of Education and Lifelong Learning, University of Exeter, U.K., that this field has been kept in good development. In fact, Ernest is the editor of the international electronic journal *Philosophy of Mathematics Education Journal*, which since 1990 has been publishing and publicizing studies and investigations in this field[4].

As far as we could ascertain, it was only in 1998 that a work appeared bringing in its title the expression *Sociology of Mathematics education*. It is the book entitled *The Sociology of Mathematics Education: Mathematical Myths/Pedagogic Texts* by Paul Dowling, a sociologist and professor at the Institute of Education of the University of London.

In the preface to Dowling's book, its editor, Paul Ernest, explains in the following manner the emergence of this new field of study:

> With its roots in mathematics, psycology and everyday classroom practice, it might be said that mathematics education is permeated

2. Mathematics historians are unanimous in pointing out the fact that histories of arithmetic, geometry, and astronomy have been written at around 335 B.C. by Eudemo of Rhodes, a member of the Aristotelian school. Because these works were lost, the little information we have about them are due to Proclus, Simplicius, and to Eutocius of Ascalon, commentators of Greek mathematics who lived, respectively, in the 5th, 6th, and 6th centuries A.C. (Miorim; Miguel, 2001, p. 35)

3. In our country, two references must be noted in this respect: (Miorim, 1998) and (Valente, 1999). We also note here that from a total of 169 works published, partially or in full, in Annals of the National and Luso-Brazilian Meetings in the History of Mathematics up to 2002, around 20 percent were inscribed in the field of the investigation on the history of mathematics education (cf. Miorim; Miguel, 2002, p. 10). At the international level, deserve to be mentioned, amongst others, the works that are being developed in this field by Gert Schubring from the Institute for the Didactics of Mathematics of Bielefeld University (Germany), and Bruno Belhoste from the Institut National de Recherche Pédagogique (INRP) of Paris (France).

4. The reader can access the full text of the articles published in this journal at the following site: http://www.ex.ac.uk/~PErnest/.

> with the ideology of individualism. (...) Mathematics education came of age in the era of the Cold War when individualism ruled supreme in the West and communitarianism and social perspectives were backgrounded. In the past decade counterpoising the individualistic voice of developmental psychology a new voice has been heard in mathematics education. This is the voice of sociology and associated social theories. Although a social strand has long been present in mathematics education in such seminal works as Griffiths and Howson (1974), deep applications of sociological theory are as yet rare. Sociology concerns not only individuals and groups and their patterns of inter-relationships. Modern sociology also weaves knowledge and social practice into a complex whole. Until the last decade, studies which recognized this complex character were virtually non-existent in mathematics education. The feminist movement offered a social critique of mathematics, but until works such as Walkerdine (1988) [the author refers here to the work 'The Mastery of Reason'], these were under-theorized. Likewise, the multiculturalist and ethnomathematical movements offered valuable social insights for mathematics teaching, and have become widely endorsed vehicles for the reform of mathematics education. But all too often they have been offered uncritically or as under-theorized perspectives. Up to the present day there remains a dearth of fully worked out sociological approaches to mathematics education able to supply the missing theoretical perspectives and critique. (Ernest, P. in: Dowling, 1998, p. xiii-xiv).

In view of this rich and complex current picture of proliferation of new fields of research related to two specific sectors of culture – the mathematical culture proper, and the educational culture in mathematics – let us present and discuss in what follows, albeit preliminarily, some guiding principles of a research program that proposes to take as its central object of investigation the way in which the fields of history, philosophy, and sociology of mathematics education could participate, in a critical and qualifying manner, in the initial and continuing education of mathematics teachers.

Guiding principles of the Research Program

A first guiding principle of this program is that the participation of those fields in teacher education should not take place through their simple transformation into new autonomous disciplines – such as, for example, history of mathematics or history of mathematics education; philosophy of mathematics or philosophy of mathematics education; sociology of mathematics or sociology of mathematics education – to be added to those already part of the curriculum of mathematics teacher education courses. Firstly, because such addition would practically make it impossible to include those fields in the curricula, due to the unbearable extra load they would represent. Besides that, we believe that the education would be much more qualitatively enriched if it were revised in the light of a new idea of specificity, anchored in a pedagogical project in which those research fields, amongst others, could participate in an organic and clarifying way in the constitution of multidimensional problematizations[5] of school practices involving mathematics, problematizations oriented by academic investigations about the issues that challenge today the teachers in the critical and productive work of incorporation, resignification, production, and transmission of

5. By multidimensional problematization of the school practices related to mathematics education we understand all critical and clarifying debate that puts in evidence and focuses upon the various dimensions constitutive of the social practices involving mathematics, and that take place under the conditions of the school institution, namely: the mathematical dimension, the epistemological, the logical, the sociological, the methodological, the anthropological, the axiological, the historical, the political, the ethical, the didactic, the linguistic etc (Miguel; Miorim, 2004, p. 154).

mathematical culture under the conditions imposed by the school institution.

A second guiding principle of this research program relates to the fact that in it we are not conceiving the philosophy, history, and sociology of mathematics education as simply a mechanical juxtaposition of themes or problems extracted from the fields of philosophy, history, and sociology of education with other themes produced inside the fields of philosophy, history, and sociology of mathematics.

That is because we believe that the objects and problems upon which both the pedagogical investigation and the pedagogical action in the domain of mathematics education focus are not restricted to those customarily belonging to the fields of *history of mathematics*, *philosophy of mathematics*, and *sociology of mathematics*, neither are they of the same nature, or possess the same pedagogical relevance.

In other words, what this second principle suggests is that the objects upon which the investigations carried out in the fields of history, philosophy, and sociology of education should focus do not result from the sum of knowledges produced by the philosophy, history, and sociology of mathematics constituted by the problems faced by the professional mathematician in her research activities in the domain of pure or applied mathematics with knowledges produced by general histories, philosophies, and sociologies of education constituted under the pedagogue's perspective.

That is because it is unlikely that a conception of such nature would produce more than a compartmentalized, disarticulated and little effective composition of philosophical choices extracted from those two fields. Besides, such composition would not be clarifying, and it would be of little use to face the problems that challenge the mathematics teacher.

On one hand, the shortcomings we see in taking the problems that have been constituted in the fields of philosophy, history, and sociology of mathematics as the research backbone of the fields of philosophy, history, and sociology of mathematics education are of at least two orders. First, the movements around the fields of philosophy, history, and sociology of education, in their recent histories, have chosen as their almost exclusive objects of analysis and reflection the mathematical activities and culture of professional mathematicians, thereby ignoring, because of lack of interest, knowledge, or for prejudice, other forms of mathematical activity and culture that have been produced in various social practices carried out in institutional contexts other than the academic-scientific. Second, in the context of that manner of conceiving and doing the history, philosophy, and sociology of mathematics, the knowledges produced and the different conceptions of mathematics that were revealed were not constituted based on the problems and concerns attending the mathematical activity that is carried out in the exercise of different social practices and, above all, in those carried out in the school institution.

On the other hand, the shortcomings we see in taking the problems that were constituted inside the domains of the history, philosophy, and sociology of education as the research backbones of the fields of philosophy, history, and sociology of mathematics education are also of at least two different orders, analogous to the previous ones. First, the so-called general philosophies of education have chosen as their almost exclusive object of analysis and reflection a general, universal, uniform, and abstract educative activity, thereby ignoring by ignorance, lack of interest and/or prejudice not just the specific form taken by education as a discipline carried out in the school institution context of all countries, but also all other forms of educative activity that have been carried out in social practices other than those related to schools, in different geopolitical contexts, and at different epochs.

The third guiding principle of our program proposes that the object upon which investigations should focus in these fields should be the mathematics education that has

been carried out at schools, that is, under the particular conditions of the school institution. This principle, to be fully understood, as we want it to be, deserves a few clarifications.

It suggests, first of all, that the object upon which the investigations in these fields should focus should be neither the generic and abstract concepts of *mathematical knowledge* or of *mathematical culture*, nor the more delimited concepts of *school mathematics* or of *school mathematical culture*, but that of *school mathematics education*.

Even if talking about *school mathematics education* instead of about *school mathematics* is an option, such option is not, in our view, a mere terminological choice without greater consequences. It takes us, first of all, to the controversial problem of knowing to what extent and in which ways the social practices of an educative character – school-related or not – would take part, in an active and creative way, in the production of mathematical culture or of culture in general. Would social practices producing mathematical culture be incommensurate with social practices producing an educative culture related to mathematical culture? We already had opportunity to consider the wider issue of the project of disciplinarization of educative practices related to mathematical culture in (Miguel et al., 2004, p. 80-89). Let us recover here some arguments already put forward in that reference, so that we can justify our choice of the expression 'school mathematics education'.

Schubring (2001, p. 297) espouses the viewpoint that the researches in the field of the history of mathematics education should avoid any separation between production and reproduction of culture. That means, in other words, that the researcher should avoid in his/her investigations working implicitly or explicitly with the Manichean assumption that associates production with invention, and teaching with socialization, propagation, or passive reception of culture. It is important to avoid such assumption because it takes us ineluctably to the inadmissible establishment of a hierarchy between invention and transmission, and then causes us to see research as a noble, original, and indispensable activity, and teaching as a secondary activity, whose exercise does not require the same degree of talent, imagination, and education.

But, on the other hand, we must admit that the mathematical activity does not take place or manifest itself solely in one social practice, namely, that in which its promoters put consciously before themselves the task of producing mathematical culture. This implies that the so-called professional mathematicians – because they are also teachers, but not just because of that – carry out an educational activity, as well as producing educational culture – even if that is not the intentional, conscious, and predominant dimension of their activity. But that implies further that other practice communities – including, of course, the community of mathematical educators – also conduct mathematical activity, and also produce mathematical culture – even if that is not the intentional, conscious, and predominant dimension of their activity.

We can then say that, apart from a specific and particular culture intentionally produced and absolutely necessary for a social practice to be conducted and survive, the communities that conduct it also incorporate, in a re-signifying and institutionally conditioned way, cultures produced in the exercise of other social practices, and end up also producing an *educational culture of survival*, cultures that, although not perceived as just as important as those intentionally produced when carrying out the reference social practice, are also absolutely necessary for the reference social practice to take place, survive, and fulfill its social purposes. And so, each in his own way, a professional mathematician is not a non-'mathematical educator', in the same way that a mathematical educator is not a non-'professional mathematician'.

Our viewpoint is endorsed by Belhoste when he says that even if mathematicians, in

their vast majority, are teachers nowadays, given that their activities take place within a university or school context, and even if the public opinion sees mathematics essentially as a school discipline, mathematicians do not see themselves that way. For them, the research activity is the defining element of their professional identity, and teaching mathematics is not seen as an activity sufficient to make a mathematician; for that, one would still, and above all, have to produce mathematical results (Belhoste, 1998, p. 291).

Yet, proceeds Belhoste, such representation mathematicians make of their own identities is quite recent, harking back to the late 19th century. And when we consider the status of mathematician, not as an ahistorical category, but as a social construction, nothing authorizes us to say that Descartes would have been a mathematician instead of a philosopher, whereas the Lycée mathematics teacher Galois would not have been a mathematician, since it was mainly through his teaching activity that the mathematical activity itself became professionalized in Europe, giving birth to the professional mathematician of our days. So, even if today mathematicians and mathematics educators increasingly constitute two practice communities with different aims, both in the research domain, and in pedagogical action, these communities should not be seen as radically distinct, considering that they not only share at least some objectives, but also carry out activities that are mutually influencing. Still, such influence may not be immediate, and its nature is not of a passive subordination of one of them to the other; besides, both activities are also conditioned by other activities in the same way that they influence other social practices.

In this sense, in the controversy he established with Chevallard (1991), Chervel defended, rightly in our opinion, the epistemological and methodological viewpoint partly contrary to the one that guided the former author that school disciplines are not reflex, vulgarization or pure and simple adaptation of *knowledges* produced by the sciences of reference. Alternatively, Chervel then stated that the concept that in his opinion should be put in the center of a reflection about school culture should not be the concept of *knowledge*, but that of *discipline*, or even better, that of *school discipline* or of *teaching discipline* (Chervel, 1992, p.195-198). The nature of this subtle, but fundamental and reorienting, distinction is better characterized in Chervel's own words:

> My studies do not reveal at all the existence of a social group independent from the school, whose function would be that of transforming the scholarly knowledge into knowledge that can be taught. On the contrary, they lead me to see in the school (in a wide sense) a place of production of culture, of a school culture, of teaching contents, of 'disciplines'. It is therefore necessary to present a different theoretical framework, in which the school can be conceived as creator of 'cultural contents'. But it is necessary, first of all, to draw the boundaries of the domain: that in which the teachings are 'disciplines', that is, contents directed to children or teenagers in a process that is not just of instruction, but also of education.(Chervel, 1992, p.197)

On the one hand, Chervel's point of view draws our attention to the specific, discipline-related, and compartmentalized form taken by school culture by virtue of the historical conditions to which it is subjected, school mathematical culture being one of those compartments. On the other hand, this point of view warns us against the danger of identifying the concept of *knowledge* with that of *teaching contents*. Notwithstanding that, by giving that warning, Chervel ends up suggesting a dichotomy that seems to contradict his own viewpoint, and that brings him closer than he would like to Chevallard's opinion. Indeed, according to Chervel, if the teaching contents, even when understood as cultural contents, cannot be seen as proper knowledges, then school, even if it should be seen as an

autonomous locus of cultural production, would not be, strictly speaking, a locus for the production of knowledge. As we can see, the concept of *culture* that informs Chervel's point of view – and that dispels the contradiction that seemed to surround him – says that every form of knowledge is a cultural production, but not all cultural production – here included the school cultural production – is a form of knowledge.

Although we do not share these notions of *culture* and of *school culture*[6], we believe that Chervel's viewpoint – but not that of Chevallard – allows us to state that the culture produced by the *teaching discipline* called *mathematics* should not be confused with the culture produced by the mathematical activity carried out by different practice communities and, above all, by the community of professional mathematicians.

This same type of warning is also suggested by the viewpoint defended by João Filipe Matos – a professor at the University of Lisbon, Portugal – of substituting the discipline *mathematics* in the school curriculum by one called *mathematics education* (Matos, 2003).

But the third guiding principle of our research program requires still a second type of clarification. When we propose to replace the concept of *school mathematics* with that of *school mathematics education*, the adjective *school* that qualifies the expression *mathematics education* is not a simple detail. It not just contextualizes the mathematics education that we wish to consider as object of historical, philosophical, and sociological investigation; more than that, it *institutionalizes* it.

To better understand this point, we must say a few words about the way in which we understand here the sociological concept of *institution*. An *institution* for us is any dynamic and mutable collection of norms socially instituted with the purpose of organizing in a given way the social relations of the members of practice communities which, under the influence of those norms, carry out actions in various places or environments. Thus, for instance, when we refer to IBM, or to science, or still to catholic religion as institutions we are, strictly speaking, referring to the collection, explicit or implicit, of norms that at each given moment organizes, controls, and conditions the interpersonal relations of any nature, as well as the personal modes of thinking and acting of the members of practice communities that have submitted to those norms, independent of the locus or physical space where they are acting or thinking[7]. Thus, the activity of producing this article – which was limited by several factors, such as the topic about which I proposed to write about, the maximum number of pages it should have, the nature of the literary genre etc – can be seen as an institutional activity, not because I wrote it in a specific physical place, but because this activity and all the others I did in my home or at UNICAMP to write this paper were conditioned by the way I have agreed and represented to myself the collection of rules of various institutions: of the institution journal of the Faculty of Education of USP, of the institution scientific text, of the institution school, of the institution history etc.

This manner of conceiving the concept

6. Alternatively, historians of education have proposed other conceptions of school culture. To Jean Claude Forquin, school culture is the "collection of cognitive and symbolic contents which, selected, organized, 'normalized', 'routinized' under the effect of the imperatives of didactization, constitute habitually the object of a deliberate transmission in the context of schools". To Antonio Viñao Frago, school culture is the collection of institutionalized aspects that characterize the school as an organization, including practices and conducts, ways of life, habits and rituals — the everyday history of the school doing —, material objects — function, use, distribution in space, physical materiality, symbolism, introduction, transformation, disappearing (…) —, and modes of thinking, as well as meanings and shared ideas". Dominique Julia states that school culture presents itself as "a collection of norms that define knowledges to be taught and conducts to inculcate, and a collection of practices that allow the transmission of those knowledges and the incorporation of those behaviors, norms, and practices ordained according to purposes that may vary from epoch to epoch (religious, sociopolitical, or simply socialization purposes)". (cf. Valdemarin; Souza, 2000, p. 5-6).

7. We are here using the word *institution* in a way very close to that employed by (Thompson, 1995), that is, "as a structure – not necessarily embodied in a material property of public or private character – definite, specific, and relatively stable of social relations established and organized by rules and financial resources, and socially constituted with the purpose of carrying out actions of social or collective interest" (Miguel; Miorim, 2004, p. 157-158).

of institution makes us see all culture – and mathematical culture in particular – as a social institution. But it also stops us from imagining a historical moment in which mathematical culture would have existed under a state of institutional emptiness. So, the adjective *school*, which qualifies the expression *mathematics education*, more than institutionalizing it, re-institutionalizes it, which means always seeing the school educative practices that surround mathematical culture – in each historical time, in each geopolitical context, and in each concrete situation – as dynamic, creative, productive, original, and singular processes, even if conditioned by not always identifiable collections of norms originated in different social institutions, apart from those originated in the school institution itself.

According to this viewpoint, mathematical culture is then seen as each and every normative and public system of signals produced through the mathematical activity conducted by different practice communities, and not just by the community of professional mathematicians. However, the mathematical activity producer of mathematical culture is not conceived as a type of activity carried out and conditioned just by a given type or by a unique collection of institutional norms, nor is it seen, alternatively, as an activity that would not be subjected to any kind of institutional conditionings. Thus, mathematical culture is no longer seen in a uniform manner, that is, as carrying characteristics, properties, and purposes always universal, fixed, good, and noble. Indeed, we are becoming more and more aware of the fact that the nature of mathematical activity, as well as the nature of the culture produced by that activity, varies not only according to time and different geopolitical contexts, but also – within each time and context – according to the nature, purposes, and forms of organization of the social institutions that condition that activity. This means that the mathematical activity ends up, almost always uncritically, incorporating and retransmitting the guiding interests and values of the political purposes of the social groups that finance the constitution and functioning of the social institutions in which such activity takes place.

Obviously, this sociological point of view on the mathematical activity has immediate repercussions in the sphere of school mathematics education. In Skovsmose's words, this means that

> Mathematics education cannot simply serve as Mathematics 'ambassador', aiming at bringing it to the students or facilitating its construction by them. The Mathematics education must also deal with a form of knowledge that, as part of a technological enterprise, creates wonders and horrors. (Skovsmose, 2004, p. 53)

We are, therefore, faced with the surprising need of questioning the traditional relations that the culture produced in school mathematics education establishes with the very mathematical cultures produced under the conditionings from other institutions and practices, and consequently of challenging the traditional postulate of thinking school mathematics education as a mere uncritical conveyer belt of a mathematical culture regarded as pure, universal, formal, autonomous, absolutist, uncontroversial, certain, and neutral. We are, therefore, faced with a problem simultaneously historical, philosophical, and sociological that has seldom been formulated or duly appreciated by the fields of history, philosophy, and sociology of mathematics. Notwithstanding, this problem, turning out to be fundamental for the exercise of the pedagogical action in the context of school mathematical culture, is, by extension, equally essential to the education of mathematics teachers.

We therefore think that Chervel's viewpoint – which found, and still finds, strong resonance among some historians of mathematics education, and of education in general in our country – had not only the merit of defending and highlighting

the specificity and singularity of school culture vis-à-vis the other forms of cultural manifestation, but also of seeing the school as a space of cultural production.

However, having in mind the critique to the conception of culture underlying Chervel's point of view, and also the nature of our own research program, we believe that the more circumscribed notion of *teaching contents*, although representing a conceptual progress when compared to the generic notion of *knowledge*, should go through a refinement in order to show its true potential. For that, we propose to replace the notion of *school discipline* or of *teaching contents* by the equally complex and controversial notion of *social practice*. We can then state the fourth guiding principle of our research program in the following terms: the object upon which should focus the investigations in the fields of history, philosophy, and sociology of mathematics education is the collection of *social practices surrounding mathematics*, and that have been or still are been conducted at schools, that is, under the conditions of the school institutions.

To understand what this latter principle means we should emphasize here the fact that, when we speak of *social practices*, we do not conceive of practice as a locus, that is, as a place or physical space institutionally conditioned in which we develop a professional activity. For us, a social practice – and school practices are examples of social practices – is a collection of activities or physical-affective-intellectual actions characterized by being: 1. consciously oriented by certain purposes; 2. spatial-temporally configured; 3. conducted on the natural and/or cultural world by practice communities whose members establish among themselves institutionalized interpersonal relations; 4. producers of knowledges, actions, technologies, discourses, artifacts, works of art etc or, in a word, producers of culture, that is, of a collection of symbolic forms[8] (Miguel; Miorim, 2004, p. 165).

It should be noted that not all practices carried out in the school, that is, under the conditions of the school institution, are carried out solely in the school, and that some practices that are carried out in the school happen only in the school, and still, that not all practices carried out outside school also happen in the school. For example, the social practice of carrying out written calculations according to the rules of the Hindu Arabic system is a practice that takes place in the school, but not only in it. But the social practice of orienting oneself spatially with the help of a technological artifact operating a GPS[9] system is not a practice that takes place in the school institution, although it may come to be one day. It may still happen that certain school practices that have taken place in the school for a given period of time may have become obsolete, and may have stopped being carried out in the school. For example, the school practice of verifying the result of an arithmetic operation using the so-called "proof of the nines"[10] has become obsolete, and does not seem to take place nowadays.

Although the fourth guiding principle of our research program suggests that we should choose the school educative practices involving mathematics as the central object of historical, philosophical, and sociological investigation, this does not mean that such investigations should limit themselves to look exclusively at the school institution and at the educative practices involving mathematics that happen in it, as if those practices could be analyzed, understood,

8. We use here the expression *symbolic forms* in the sense given by Thompson, that is, as "a wide variety of signifying phenomena, from actions, gestures, and rituals, to verbal manifestations, texts, television programs, and works of art" (Thompson, 1995, p. 182-183). In that reference, Thompson distinguishes five features of symbolic forms, namely their intentional, conventional, structural, referential, and contextual aspects. Accordingly, we here use the word *culture* in quite a wide semantic conception, as the collection of symbolic forms produced by humankind.

9. The GPS, acronym for Global Positioning System, (…) is a radio navigation system developed by the U.S.A. Department of Defense (…) with the purpose of being the main navigation system for the U.S.A. Armed Forces. (…) It has become an extremely useful and innovative technology for a whole series of positioning activities. One may mention those related to Cartography, Environment, Control of Fleets of Vehicles, Air and Maritime Navigation, Geodynamics, Agriculture etc (Monico, 2000, p. 15 e p. 21).

10. This is a procedure to check the results of a simple arithmetic operation, which used to be taught to schoolchildren. (Note of the Translator)

explained, re-signified, and transformed exclusively based on the analysis of that which can be observed immediately in the school.

Instead, we believe that, to be useful to the mathematics teacher, the investigation of a school educative practice in which mathematics is involved should, whenever fit, be guided by the comparative method of the manners in which this social practice constituted and transformed itself in different geopolitical and institutional contexts, one of them being the school institutional context. That is the fifth guiding principle of our research program.

As an example of this principle, it is not enough to investigate unilaterally the nature of the current school educative practices involving trigonometry; it is necessary to investigate also the manners and reasons through which those school practices constituted and transformed themselves in our country, as well as the nature of the influences that the knowledge about trigonometry historically constituted and/or conveyed in other social practices conducted in other geopolitical and institutional contexts may have had, such as social practices of topography, navigation, astrology, astronomy, cartography, finances and commerce, music, war, construction of measuring instruments etc.

We believe that comparative investigations of this nature, developed in the fields of history, philosophy, and sociology of mathematics education, could reveal the institutional mechanisms of a political, economic, legal, sociological, axiological, psychological, and ideological order that influence the process of reception, transmission, incorporation, re-signification, and transformation of school educative practices involving mathematics. Such mechanisms would tend, therefore, to reveal the dynamic interplay of asymmetrical power relations at the root of the explanations to specific and concrete qualitative changes that took place in the context of school mathematics education.

The constitution of knowledges associated to that interplay of asymmetrical power relations with which school mathematics education found and/or still finds itself is somehow involved establishes the sixth guiding principle of our research program. Those knowledges could offer elements to a deeper qualitative evaluation of what currently goes on in the classrooms, namely: the students' resistance to the process of incorporation of mathematical culture; the teachers' difficulties in the process of reception, re-signification, and transmission of mathematical culture; the artificiality of the school practices involving mathematics; the algorithmic and little meaningful nature of school mathematics education etc. They could still offer concrete elements for the decision making, and for the reorientation of pedagogical actions at school, with the purpose of making them more meaningful to students, and more suited to the nature of the challenges that were place and are still placed before humankind in the sphere of the relations between mathematical culture, mathematics education, society, democracy, and citizenship.

A work recently carried out by Souza (2004) along these lines could illustrate our point of view. Working in interactive sessions with a group of teachers from the initial series of Basic Education, her investigation problem consisted of identifying the values that would give support to the naturalization of the process of transmission of the social practice of written calculations in the school institutions. Such identification was done through a problematization of the dialogue occurred during the sessions between the researcher and the participating teachers. That problematization took as its reference some aspects of the history of the processes of incorporation of the social practice of the written calculation in the Hindu Arabic manner, particularly in the Portuguese geopolitical context of the 15th and 16th centuries. Inspired in the work of Michel Foucault, Souza's work is an excellent example of drawing from history as a possibility to conduct a historical-philosophical problematization of a social practice of a mathematical character, amply valued and promoted in the schools nowadays.

Finally, a seventh guiding principle of our research program is effectively trying to make the boundaries between the research fields of history, philosophy, and sociology of mathematics education progressively indistinct. And that because if all of them take as their common object of investigation the school educative practices involving mathematics, a possible distinction between them could only be sustained by an argument that would defend a supposedly distinct nature of the analysis that each of those fields could produce by virtue of the diversity of purposes and methods behind those analyses. We could then ask ourselves how different could those purposes and methods be, and if these supposed differences would not be much more related to different forms of conceiving those purposes and methods than to incommensurable distinct features inherent to each of those fields. Nevertheless, given that when we talk here about history and philosophy of school mathematics education we, in fact, mean the socio-institutional history and socio-institutional philosophy of school mathematics education, we tend to think that the meta-field of sociology of mathematics education could function as a meta-field articulating the three, building between them a common dialogical territory inside which the discussion about the interchange, sharing, and constitution of new conceptual, methodological, and hermeneutic resources that would ground the investigations could flow in an effective and productive way.

As regards this last principle, we would like to add to the reflection the timely and suggestive warning made by Struik in 1942:

> (...) We must be always conscious of the fact that a mathematical discovery, a state of mind regarding mathematics, or a teaching system, are never explained by a single cause. Life is complex, and even the most modest or subtle act reflects, in one way or another, an infinity of aspects of the real. We cannot say that a particular factor was responsible for a particular occurrence or mental state. We have to discover how all factors – sociological, logical, , artistic, and personal – played a role in the subject under investigation, never forgetting, however, that man is a social being, even when he is concerned with straight lines and hyper cones in a space of seven dimensions. (Struik, In: Grupo TEM, 1998, p. 29)

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