ARTICLES

# Intervention in quantitative reasoning as a possibility to develop arithmetic knowledge[1]

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## Abstract

The low performance in mathematical abilities at the beginning of school life can lead to learning problems in the long term. Therefore, we highlight the importance of knowing the basic mathematical abilities that influence specifically the learning of arithmetic, so that it would be possible to indicate interventions focused on performance predicting abilities in this area and help the development of students' mathematical knowledge, as well as teachers' practice. In this sense, this study aimed to investigate the effects of an intervention in quantitative reasoning on the arithmetic performance of students from the 4th and 5th Grades of Elementary School. We separated 40 children into experimental and control groups. The experimental group participated in an intervention program focused on quantitative reasoning, organized in 7 sessions, twice a week, lasting 45 minutes each session. The control groups participated in sessions with mathematical games and mindfulness practices. The results indicated that the intervention showed no significant effect when comparing the performance of both groups. However, we found a significant improvement in the performance of 5th Grade students who participated in the experimental groups. The findings of this study help understand efficient instructional principles and highlight the importance of educational practices based on evidence.

## Keywords

Intervention in quantitative reasoning – Arithmetic performance – Elementary School.

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## Introduction

Mathematical interventions grounded on cognitive abilities underlying learning are key to promoting students' mathematical performance and, consequently, reducing learning difficulties in this knowledge area. Studies have been researching mathematical learning in the first school years and pointing out interest to identify cognitive abilities that predict students' mathematical performance (ARAGÓN *et al.*, 2019; CHING; NUNES, 2017; GEARY, 2011; MALONE; BURGOYNE; HULME, 2019). Knowing these predictors is essential to developing intervention programs for specific abilities that intend to prevent difficulties in mathematical learning (PASSOLUNGHI; COSTA, 2016).

In this sense, some interventions focused on promoting cognitive abilities of general and/or specific domains indicate positive effects on the mathematical performance of students in the first school years (FUCHS *et al.*, 2013; NUNES *et al.*, 2007; PASSOLUNGHI; COSTA, 2016; SPERAFICO *et al.*, 2019). More specifically, evidence shows that interventions in early numerical abilities – counting, representation on a number line, one-to-one correspondence, and quantity comparison – are efficient to promote early mathematical knowledge in Childhood Education (PASSOLUNGHI; COSTA, 2016). Added to that, the training in a numerical sense has shown significant results to improve performance in numerical abilities and problem-solving, also among students of Childhood Education (STERNER; WOLFF; HELENIUS, 2020). Fuchs and collaborators (2013) conducted another study with an important result for the theme. They held training to promote early numerical knowledge, mainly arithmetical calculations, with students in the 1st Grade. The main results indicated significant benefits in students' performance in arithmetic, numerical knowledge, and reasoning (FUCHS *et al.*, 2013).

These results indicate that intervention in initial numerical abilities, in the early years of schooling, can benefit students' learning and improve their mathematical performance. However, regarding children in later school years, other abilities may also be considered important to develop mathematical understanding, such as quantitative reasoning. This ability requires the development of early numerical abilities and plays a role in students' mathematical performance (NUNES *et al.*, 2007, 2012). Quantitative reasoning is an ability that involves the understanding of relations between the quantities involved in a calculation, it is essential to develop arithmetic knowledge and the numeral system (NUNES *et al.*, 2007, 2016). In this sense, children need to understand the relationships established between quantities to understand how to represent the numbers and the quantities before solving an arithmetic calculation (NUNES *et al.*, 2007; NUNES; BRYANT, 2015). Besides this, studies have also shown the predictive value of quantitative reasoning for mathematical performance, i.e., indicating it as an explicative ability of the performance in arithmetic and problem-solving (NUNES *et al.*, 2007).

Nunes and collaborators (2007), in a study held in England, combined longitudinal method and intervention. In the longitudinal study, they evaluated 59 children at 6 years old. The results indicated that quantitative reasoning and working memory are predictors of mathematical performance, even when evaluated 16 months after the initial evaluation of their cognitive abilities. In the intervention study, they held a program focused on quantitative reasoning for 12 weeks with weekly sessions of 40 minutes. Twenty-seven

6 -year-old students participated in the activities in groups of five children maximum separated into experimental and control groups. The intervention group faced problems involving additive composition, an inverse relationship between addition and subtraction, and one-to-one and one-to-many correspondences. From the intervention, they found significant results in the learning of children who were at risk to develop mathematical difficulties, indicating positive effects on students' mathematical learning. Thus, we can see that understanding the relationships between quantities establishes a base for learning how to represent and operate with these quantities, extending itself to school teaching and classroom work.

This intervention program was adapted and applied in a Brazilian study with students with Attention Deficit Hyperactivity Disorder (ADHD) (SPERAFICO *et al.*, 2019). In this study, the researchers separated 46 ADHD students of the 3rd and 4th Grade into two groups who received different interventions. The study aimed to compare the effects on students' arithmetic performance of an intervention combining work memory and quantitative reasoning with another focused only on work memory. The interventions were organized in 22 one-hour sessions, twice a week, for 11 weeks, in groups of 14 students maximum. The results indicated a significant improvement in students' performance, showing a higher effect of the combined intervention. Hence, we can see that this model of intervention can benefit ADHD students or those with difficulties in arithmetic, and, more than that, highlights the positive effects of an intervention collectively applied and in a school context.

From this, we can see that there is still a need for studies that indicate intervention programs focused on predictor abilities of mathematical performance to reduce the number of children at risk of developing mathematical difficulties. We also need studies that show options for students at higher educational levels. Considering that learning difficulties can arise at different ages and mathematical content, an intervention might be needed in different moments of school life and related to varied mathematical abilities (KROESBERGEN; VAN LUIT, 2003). Otherwise, studies vary considerably on the duration of the interventions, and studies of literature review and meta-analysis indicate that the time dedicated to this specific work with students might be influenced by the approached content. That is, the broader the domain to be studied the longer the time needed for the intervention (KROESBERGEN; VAN LUIT, 2003; MONONEN *et al.*, 2014). Such studies indicate that short interventions, i.e. less than 12 weeks, are more efficient to work on specific content or only one domain. Longer interventions, more than 12 weeks, are needed to encompass more content or a broader domain (KROESBERGEN; VAN LUIT, 2003; MONONEN *et al.*, 2014).

Another important aspect to be considered is the form to apply the intervention: individual, in small groups, or with the whole class. In general, children benefit more from individual instructions or in small groups (FUCHS; FUCHS; COMPTON, 2012; KROESBERGEN; VAN LUIT, 2003; MONONEN *et al.*, 2014). However, this configuration demands more attention from the teacher, more available time, and resources for implementation (FUCHS; FUCHS; COMPTON, 2012; KROESBERGEN; VAN LUIT, 2003), therefore less feasible in the traditional school context. Thus, it is also essential to think of teaching strategies that can be applied to the whole class. Hence, the teacher will be able to intervene more effectively and keep the classroom routine, allowing general instructions and adaptations that can reach all students (FUCHS; FUCHS; COMPTON, 2012). Besides this, abilities considered in the post-test to evaluate intervention effects can also make a difference in the results, that is, if the efficiency of the intervention is measured based on the same abilities trained or if the evaluation adds other abilities (RUIZ; BALBI, 2019), which can be even evaluated through standardized tests of general performance. All these factors create difficulties to generalize and compare intervention studies.

From these pieces of evidence, this research aims to see the effect of an intervention of quantitative reasoning on the arithmetic performance of students in the 4th and 5th Grade, considering the possibility of collective implementation in the classroom. As they are students in the last years of Elementary School, we have opted to implement an intervention program seeking to develop essential abilities for mathematic learning, predominating the study of numbers and arithmetic operations (BRASIL, 2018). Thus, we chose quantitative reasoning because it implicitly involves initial numerical abilities and is indicated in the literature as one of the cognitive abilities that predict arithmetic performance.

We should also point out other types of interventions not based on the explicit instruction of mathematical content, which have interesting results. Among these approaches is game-based instruction, useful to develop quantitative and numerical competencies (RAMANI; SIEGLER, 2008; STEBLER *et al.*, 2013; VOGT *et al.*, 2018). Board and card games only require an initial explanation for students to play independently. Through games it is possible to contextualize mathematical contents, allowing more student involvement, besides deepening numerical competencies previously learned (STEBLER *et al.*, 2013). Therefore, board and card games can benefit students' learning in different moments. Other interesting aspects of game-based instruction are the promotion of interaction among players and the possibility of conversation involving mathematics (STEBLER *et al.*, 2013; VOGT *et al.*, 2018). During the games children also monitor their learning and help each other in the development of a better understanding of the mathematical competencies involves, besides repetitively practicing the same abilities (STEBLER *et al.*, 2013; VOGT *et al.*, 2018).

Another interesting approach to promoting mathematical learning is *mindfulness*, which consists of practices and methods that contribute to a state of full attention (YOUNG, 2016). From a cognitive point, literature shows that *mindfulness* interventions lead to improvements in the performance of work memory, especially the central executive component and the inhibitory system (CHIESA; CALATI; SERRETTI, 2011). Added to that, studies also indicate that these interventions reduce stress and anxiety connected to mathematical tasks (LAGUE; EAKIN; DYKEMAN, 2019; ZENNER; HERRNLEBEN-KURZ; WALACH, 2014). In this sense, decreasing anxiety and controlling emotions better when doing mathematical tasks can help improve performance in this knowledge area.

Hence, aiming to intervene in the predictive abilities of mathematical performance in students in the final years of Elementary Education, we checked the effect of an intervention focused on quantitative reasoning in arithmetic performance from the comparison of the performance of two groups. The experimental group received a specific intervention in quantitative reasoning. The control group received combined sessions of mindfulness and mathematical games. For both groups to have some benefit, the control group also received mathematical instruction through playful teaching activities combined with mindfulness practices. This choice considered alternative methods that did not use explicit instruction of mathematical concepts.

# Method

## Participants

A total of 40 children, between 9 and 12 years old (M=10.58, SD=0.70), developed all tasks proposed in this study. The participants are students from the 4th and 5th Grade of Elementary Education in a public school in the city of Porto Alegre/RS, Brazil. Out of 112 students enrolled in both school years evaluated, 70 handed the authorization of their guardians to participate in the study, and 40 of those filled the criteria to compose the sample. These criteria included: (a) intellectual quotient over the percentile 25 in the Raven's Colored Progressive Matrices-Special Scale (ANGELINI et al., 1999). Above this percentile, the intellectual level is considered 'average', and below is percentile is classified as 'below average' or cognitive deficit; and (b) having participated in all the activities proposed, thus with complete data for analysis. The intellectual evaluation took place to eliminate possible intellectual deficit cases which would demand specific forms of teaching, adequate to the needs. The characterization of the study participants can be seen in Table 1.

**Table 1 –** Characterization of sample

| | Total (N=40) | Experimental (N=22) | Control (N=18) |
|---|---|---|---|
| | N (%) | N (%) | N (%) |
| School-level | | | |
| 4th Grade | 19 (47.5) | 9 (40.9) | 10 (55.6) |
| 5th Grade | 21 (52.5) | 13 (59.1) | 8 (44.4) |
| Sex | | | |
| Female | 25 (62.5) | 13 (59.1) | 12 (66.7) |
| Male | 15 (37.5) | 9 (40.9) | 6 (33.3) |
| Age[1] | | | |
| 9 | 10 (25.0) | 4 (18.2) | 6 (33.3) |
| 10 | 17 (42.5) | 12 (54.5) | 5 (27.8) |
| 11 | 12 (30.0) | 5 (22.7) | 7 (38.9) |
| 12 | 1 (2.5) | 1 (4.6) | 0 (0.0) |

1Mean (M) and standard deviation (SD) of ages: Total (M=10.58, SD=0.70); Experimental Group (M=10.58, SD=0.67); Control Group (M=10.57, SD=0.76).
Source: Research data.

As agreed between the researchers and the school, the intervention took place during school time in the classroom. Therefore, the activities were enacted by all students in the class, however, only those who handed in the authorization were considered in the sample. From the total of classes available in the school, two of the 4th Grade and 3 of the 5th Grade, and considering the number of authorizations received, each class participated in one of the activity groups. The control group had 18 students, 10 from the 4th Grade and 8 from the 5th; and the experimental group had 22 students, 9 from one class in the 4th Grade, and the other 13 from 2 classes in the 5th Grade.

## Procedures

In the first moment, we evaluated the arithmetic performance with a pre-test, previous to the intervention. After, two researchers conducted the intervention organized in 7 sessions, twice a week, lasting approximately 45 minutes. Finally, we evaluated once more the arithmetic performance with a post-test to check the effects of the intervention on students' performance. The participants were divided into two groups: intervention and control. The intervention lasted the time allotted by the school for the research activities. Therefore, the original proposal was to carry out 11 sessions, which were condensed into 7 sessions plus 2 days for the pre-and post-test evaluations, a total of 9 meetings.

## Evaluation of arithmetic performance

Mathematical performance was evaluated through the TDE II- Arithmetic Subtest (STEIN; GIACOMONI; FONSECA, 2019), a subtest regulated for the Brazilian population. It evaluates the ability of arithmetic calculations that, in Elementary Education, involve the operations of addition, subtraction, multiplication, and division, as well as basic notions of fractions. We applied the test collectively during school hours.

### Activities in the experimental group

The intervention was a training in quantitative reasoning, adapted from the material of Nunes (2009) grounded on the Program *Numeracy Corner*, developed by a group of researchers from the Department of Education of the University of Oxford. As they were students from 4th and 5th Grades, i.e., from more advanced educational levels than those to whom the original intervention was created, we have opted to add more questions of multiplicative reasoning, besides the additive reasoning proposed. This way, the adaptation of the intervention program encompassed the same separation of situations of quantitative reasoning indicated by Nunes and collaborators (2016). The adaptation was organized following an increasing order of difficulty of the activities proposed, which were grouped considering the time allotted by the school. Thus, we started with two sessions with only additive reasoning situations, followed by three sessions combining situations of additive and multiplicative reasoning, ending with two sessions involving only situations of multiplicative reasoning. The topics approached in each session can be seen in detail in Table 2. We chose this intervention program because it agrees with the theoretical and practical principles indicated in recent literature, the possibility of adaption to the Brazilian population and because it could be collectively applied in school.

**Table 2 –** Organization of experimental group sessions

| Session | Description |
|---|---|
| 1 and 2 | **Additive reasoning**<br>**Types of situation**: Composition; Comparison; Inverse relation between addition and subtraction; Transformation<br>**Learning objectives:**<br>a) Understand that any number can be composed of other two numbers<br>b) Be able to use logical reasoning to know how to count;<br>c) Understand verbal problems involving addition and subtraction;<br>d) Understand the inverse relationship between addition and subtraction;<br>e) Understand that when adding and subtracting the same number of blocks from a role, the original number of blocks does not change;<br>f) Understand that when removing more blocks than adding, the answer will be "less"; and by removing fewer blocks than adding, the answer will be "more". |
| 3 | **Additive and Multiplicative reasoning**<br>**Types of situation**: Composition; Comparison; Inverse relation between addition and subtraction; Transformation; Direct relationship<br>**Learning objectives:**<br>a) Understand the composition of quantities;<br>b) Understand the inverse relationship between addition and subtraction;<br>c) Understand how to use different forms of counting to solve verbal problems of addition and subtraction;<br>d) Understand the reasoning of one-to-many correspondence, using drawings to understand problem-situation. |
| 4 | **Additive and Multiplicative reasoning**<br>**Types of situation**: Inverse relation between addition and subtraction; Transformation; Direct relationship<br>**Learning objectives:**<br>a) Understand the inverse relationship between addition and subtraction;<br>b) Understand verbal problems involving addition and subtraction;<br>c) ) Understand how to use different forms of counting to solve problems;<br>d) Understand the reasoning of one-to-many correspondence. |
| 5 | **Additive and Multiplicative reasoning**<br>**Types of situation**: Transformation; Direct relationship; Inverse relation; Proportion; Product measures<br>**Learning objectives:**<br>a) Understand verbal problems involving addition and subtraction<br>b) Understand the reasoning of one-to-many correspondence;<br>c) Understand the inverse relationship between two quantities, that is, that as one amount increases the other decreases;<br>d) Identify the reason among the parts that form a whole from a proportional relation;<br>e) Use counting to solve problem situations by combining possibilities. |
| 6 and 7 | **Multiplicative reasoning**<br>**Types of situation**: Direct relationship; Inverse relation; Proportion; Product measures<br>**Learning objectives:**<br>a) Understand the reasoning of one-to-many correspondence;<br>b) Understand the inverse relationship between two quantities, that is, that as one amount increases the other decreases;<br>c) Identify the reason among the parts that form a whole from a proportional relation;<br>d) Use counting to solve problem situations by combining possibilities. |

Source: Created by authors.

Regarding the procedures, the session started by handing notebooks with only drawing of the problem situation to be worked in the day. The researchers gave the instructions orally and the students were asked to think individually or collectively about a possible solution. After the solutions given by students were discussed by the whole class and the researchers systematized and explained one or more strategies to solve each problem situation.

## Activities in the control group

The control group participated in mindfulness activities, organized from the central principles of the practice, using intervention methods systematics and adapted to children and teenagers (BRODERICK, 2013; LYONS; DELANGE, 2016), and in mathematical games, adapted from Rechsteiner and collaborators (2018), involving basic abilities related to arithmetic performance, such as comparison of quantities, correspondence number and quantity, and numerical sequence. The sessions of the group were organized with 15 to 20 minutes of *mindfulness* first, followed by 25 to 30 minutes of games. In Table 3 below, we present the organization and objectives.

**Table 3 –** Organization of group control sessions

| Session | Part 1 – Mindfulness | Part 2 – Mathematical games |
|---|---|---|
| 1 | **Do not swallow the candy** – Each student received a candy. They were challenged to keep it in their mouths, without swallowing it, for 5 minutes.<br>**Objective:** develop focus and self-control | **To the middle** – the aim is to use cards to move all your pieces to the center of the board.<br>**Abilities involved**: correspondence of number and quantity; identification of quantities. |
| 2 | **Attention to abdominal breathing** – While sitting the students were invited to close their eyes and focus on their abdominal movements while breathing, for 7 minutes.<br>**Objective:** develop focus and attention, reduce stress and anxiety; increase the ability to block external stimuli | To the middle (same game session 1). |
| 3 | **Listen to the environment sounds** – We invited students to list on paper all the environmental sounds they could perceive at that moment, for 5 minutes.<br>**Objective**: develop focus and selective attention | **Snakes and ladders** – the board has snakes and ladders spread following the numerical sequence from 1 to 100. The aim is to be the first one to reach 100.<br>**Abilities involved**: identification of quantities; addition. |
| 4 | **Statue** – Students had to walk around the class. When the researchers gave them a sign, they had to form groups according to a determined number of students and make a pose also indicated by the researchers. For example, researchers would say "feet with feet, five". Students had to organize themselves into groups of 5 and touch their feet. This pose had to be kept until a second sign, also given by the researchers, then students could move around the room again.<br>**Objective:** develop self-control, develop body consciousness, and tolerance to adversities | **More is more** – the aim is to get rid of your pile of cards. Each card can be discarded if there are more points of the same color than the reference card on top of the center pile.<br>**Abilities involved**: comparison of quantities; identification of quantities. |
| 5 | **Body Scan** – With their eyes closed, the students should focus on specific body parts, guided by the researchers.<br>**Objective:** develop body consciousness, reduce stress and anxiety, develop focus and selective attention | Neighbor numbers – the aim is to place different previous and following numbers, correctly creating a numerical sequence and thus discarding their cards as fast as possible.<br>Abilities involved: correspondence of number and quantity; numerical sequence |
| 6 | **Copy the animal** – Each student received a card with a drawing of an animal. Researchers instructed them to look at the picture for two minutes, trying to memorize all the details. After this time, the figure was covered and they were asked to write all they could remember about the picture. After that, they compared their register with the card.<br>**Objective:** develop focus and attention, work short-time memory, develop self-perception of own abilities | High five – create a numerical sequence from 1 to 10 starting with the number 5. To do so, they have to add one card in the sequence that corresponds to the previous or following number of the one on the table.<br>Abilities involved: correspondence of number and quantity; numerical sequence. |
| 7 | Do not swallow the candy (same activity as session 1). | Splashing monster – the aim is to establish the maximum number of pairs with the cards and not finish the game with the monster card. Each pair is formed by a number and its representation in quantity.<br>Abilities involved: comparison of quantities; correspondence of number and quantity; identification of quantities. |

Source: Created by authors.

The *mindfulness* sessions were designed and adapted according to participants' age and their possibilities. All sessions followed this order: introduction gathering students in a circle, inviting them to close their eyes and focus on the sound of a bell raising their hands when they could no longer hear the instrument sound. After, researchers would start the programmed activity. During the initial conversation, students could talk about their mental state that day, aiming to approximate them with researchers, the activities were also explained at this moment. This was followed by the main activity when researchers led the mindfulness activity planned. After that, there was a final conversation when students could report their perceptions of the experience and researchers systematized the objectives of the activity.

The mindfulness practice was followed by the instructions of the mathematical game proposed for the day. Researchers divided students into groups of 4 (maximum) and the material was distributed. The mathematical games were used to broaden initial mathematical knowledge previously developed by the children at this school level, which is also needed to develop arithmetic knowledge.

## Data analysis[3]

The analysis was quantitatively made, using statistical tests adequate to check the effects of the intervention on students' arithmetic performance and compare the performance of experimental and control groups. To do so, we used the *software* R v.3.6.3 to produce descriptive analysis considering the mean and the standard deviation of the performance of each group in the pre-and post-tests, analysis of internal consistency of the arithmetic evaluation in two moments (pre-and post-tests), through Cronbach's alpha test, and the Pearson Correlation test to check if the pre-and post-tests were related. Besides this, we also conducted the Students' t-test to compare the groups' performance and verify if the intervention was meaningful. The sample was also separated, into each group, according to the school year, and by students with and without difficulties in Mathematics. Added to that, we did the Cohen's D effect test to get the effect of the intervention in the cases where there was a significant improvement in students' performance. Finally, we did a mixed model analysis of variance (mixed model ANOVA) aiming to identify the influence of the variables 'school year' and 'having difficulties' in the intervention results.

## Results

The test of arithmetic performance presented a normal distribution and a good level of internal consistency in the pre-test ($\alpha=0.652$) and the post-test ($\alpha=0.654$). Thus, the measures used are reliable and normally distributed. Besides this, the Pearson Correlation test between pre-and post-test indicated a strong association between these two moments of evaluation ($r=0.73$, $p<0.001$).

**3-** The set of data that supports our findings is not publically available, as they are still in the analytical phase for other derived studies. Researchers can demand access to data directly from the author, by email camilapnogues@gmail.com

The main hypothesis of the study was that the intervention in quantitative reasoning would improve students' performance in arithmetic. For that, the performance of the experimental and control group was compared. The result of this analysis and the descriptive analysis of each group can be seen in Table 4.

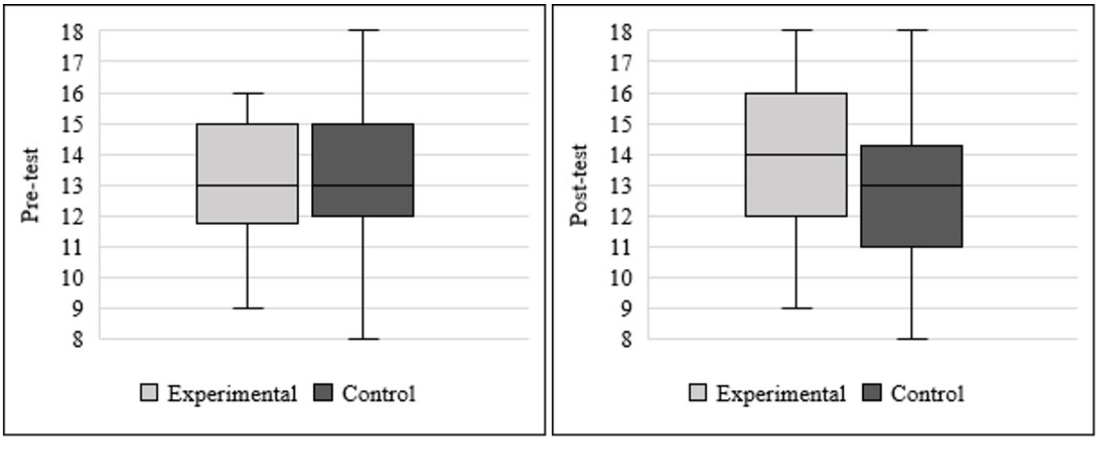
**Table 4 –** Descriptive analysis and comparison between experimental and control groups

| | Experimental | | Control | | Comparison | |
|---|---|---|---|---|---|---|
| | **Mean (SD)** | **Min. – Max.** | **Mean (SD)** | **Min. – Max.** | **T (df)** | **p-value** |
| Pre-test | 13.1 (2.2) | 9 – 16 | 13.0 (2.5) | 8 – 18 | 0.12 (33.7) | 0.90 |
| Post-test | 13.8 (2.9) | 9 – 18 | 12.9 (2.3) | 8 – 18 | 1.12 (38) | 0.27 |

Source: Research data.

From these results, we can see that there was no statistical difference between the means of both groups in the post-test (t(38)=1.12, p=0.27). Therefore, we can affirm that the intervention had no effect comparing the groups (Graphic 1).

**Graphic 1 –** Performance in the pre and post-test by groups




Source: Research data.

However, when comparing the results of the pre-and post-tests separately within each group, we can see that there was a statistically significant difference only in the experimental group (t(21)=2.20, p<0.05), indicating that the students in this group had an increase in their performance by the end of the intervention, though the effect size was small (Cohen's D= 0.24).

Aiming to explain the data in more detail, we separated the participants of the experimental group were separated by school year and if they had difficulties in mathematics. The classification of students with or without difficulties was done through the score in the arithmetic subtest. For that, we have used as a criterion the percentile 25, which was calculated based on participants' scores, i.e., from the distribution of data in the sample. Thus, students under the 25 percentile were considered 'with difficulties' (WD) and those above this percentile with 'no difficulties' in Mathematics (ND). This classification of students can be seen in Table 5 for each group, also considering their separation by school year.

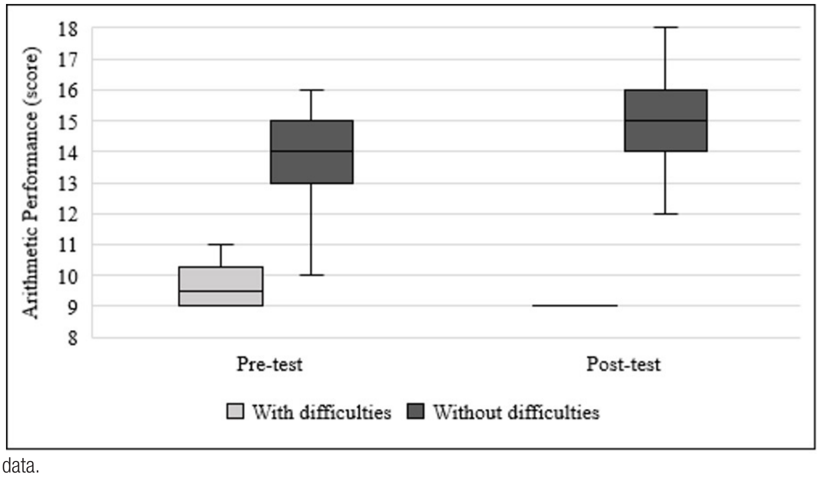
**Table 5 –** Classification of participants in each group

| | Experimental Group | | | | Control Group | | | |
|---|---|---|---|---|---|---|---|---|
| | WD | ND | 4th Grade | 5th grade | WD | ND | 4th Grade | 5th grade |
| N | 4 | 18 | 9 | 13 | 5 | 13 | 10 | 8 |
| Pre-test Mean (SD) | 9.7 (1.0) | 13.8 (1.6) | 12.2 (2.0) | 13.7 (2.1) | 11.6 (3.4) | 13.5 (2.1) | 12.3 (2.6) | 13.9 (2.4) |
| Post-test Mean (SD) | 9.0 (0.0) | 14.9 (1.9) | 11.9 (2.5) | 15.2 (2.4) | 10.2 (1.3) | 13.9 (1.7) | 12.7 (2.4) | 13.1 (2.5) |

Source: Research data.

After this classification, students' performance was again compared, but considering only the experimental group because it presented more significant results within the group. Hence, we found a significant difference regarding the presence or not of difficulties in Mathematics ($t(17)=-12.9$, $p<0.001$), that is, the students with no difficulties have higher performances than the students with difficulties in the post-test, as expected. Besides this, when comparing the performance between the tests of students without difficulties, we identified a meaningful increase in the number of correct answers in the pre-and post-tests ($t(17)=3.04$, $p<0.05$), with a mean effect of the intervention (Cohen's D = 0.56). Therefore, students with difficulties in Mathematics from the experimental group improved their performances in arithmetic at the end of the intervention. The same cannot be said for students with difficulties, as there was no significant difference between the performances pre-and post-tests ($t(3)=-1.57$, $p=0.21$). These results can be seen in Graphic 2.

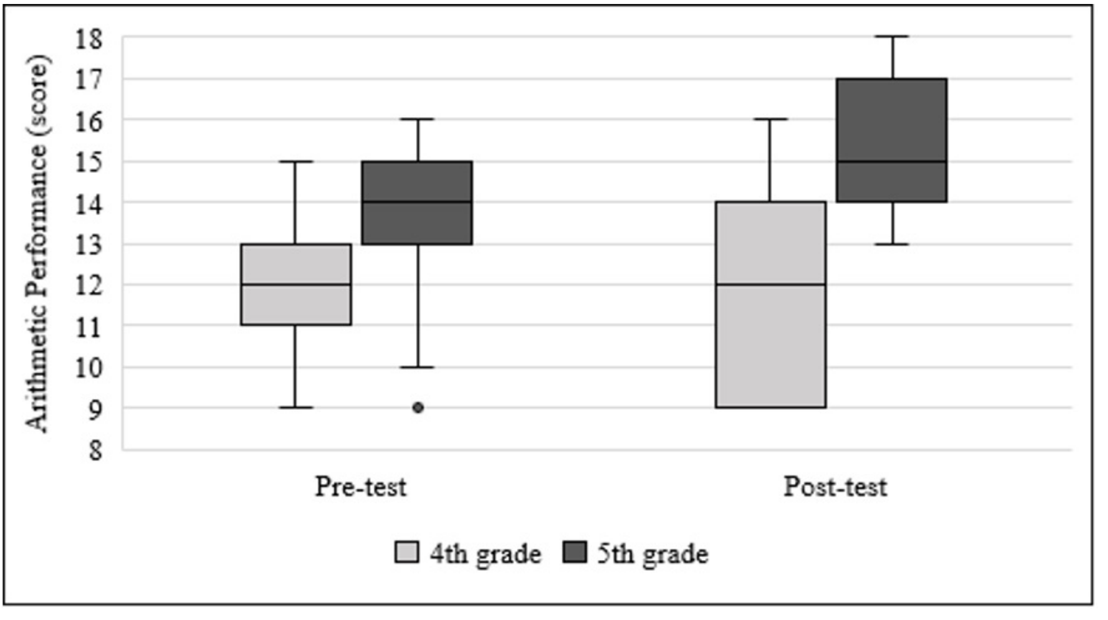
**Graphic 2 –** Arithmetic performance in the Experimental Group by the level of difficulty




Source: Research data.

The same analysis was conducted considering the classification per school year. First, we identified a significant difference between the performance of students in the 4th and 5th Grades ($t(16.8)=-3.04$, $p<0.01$), indicating that 5th Grade students had a higher performance than the 4th Grade ones in the post-test. More than that, only 5th Grade students who participated in the experimental group presented significant improvements in the performance of pre-and post-tests ($t(12)=-3.63$, $p<0.01$), with a strong intervention effect (Cohen's D = 1.008). For 4th Grade students, this result was not seen ($t(8)=1$, $p=0.35$), i.e., these students did not have significant improvement between the pre-test and the post-test. These results can also be seen in Graphic 3.

**Graphic 3 –** Arithmetic performance of experimental group by grade




Source: Research data.

Therefore, we have raised the hypothesis that the 'school year' was a variable of confusion, which could play some bias in the effect of the test, that is, that the improvement in the performance presented in the experimental groups, in general, was because the 5th Grade presented better results. Hence, to test the hypotheses, we conducted an analysis of mixed variance (Mixed ANOVA), considering the variables of arithmetic performance, in the pre-and post-tests, and school year. From that, the result points out that the variable 'school year' was significant ($F(20,1)=6.2$, $p<0.05$), i.e., the school year influenced the result of the group in general. So, the performance of 5th Grade students might have been better in the pre-test and in the post-test was enough to lead to a significant result for participants in the experimental group.

Regarding the development of the intervention program and the strategies used by students, it was possible to perceive that some types of problems were more challenging to them, mainly situations involving the inverse relationship between addition and subtraction in additive reasoning, and the product of measures in multiplicative reasoning. Furthermore, we also observed in their resolutions that most students used the algorithm even in situations they did not need it, possibly reflecting the influence of an education focused on procedures.

## Discussion

The aim of the study described was to check the effect of an intervention of quantitative reasoning on arithmetic performance. Thus, students' performances were evaluated in pre-and post-tests and compared between experimental and control groups. The experimental group participated in 7 sessions focused on quantitative reasoning, while the control group participated in 7 sessions involving mindfulness activities and mathematical games. As a hypothesis, we expected that the experimental group would have a better performance than the control one at the end of the intervention, considering it was an intervention adapted and based on the literature, starting from a predictive ability of arithmetic performance. However, this hypothesis was not confirmed.

An important result to highlight was that students from the experimental group significantly improved their arithmetic performances from the pre- to the post-test. More than that, this performance increase was seen only among 5th Grade students. This suggests that it is an effective intervention to improve arithmetic performance, mainly among older students, corroborating previous studies, which indicated that intervention in quantitative reasoning is efficient to improve mathematical performance in standardized tests (NUNES *et al.*, 2007; SPERAFICO *et al.*, 2019). However, there were no statistically significant data between the experimental and control groups, that is, this increase in the performance of the experimental group in the post-test was not enough to have a significant difference from the control one, differently from the main findings of intervention studies on the same ability (NUNES *et al.*, 2007; SPERAFICO *et al.*, 2019). Therefore, we cannot say that intervention in quantitative reasoning is more efficient than mindfulness activities and mathematical games. Thus, we now present three possible reasons that might explain these results.

As a first reason, we highlight that there is the possibility that a similar but longer intervention could have more consistent results and significant effects, considering that the content approached in the intervention was very broad, involving several types of situations of additive and multiplicative reasoning. We can raise the hypothesis that the seven sessions grouping many different situations of quantitative reasoning might have been an ambitious decision. So, we consider the possibility of too much new information for the students to understand in a short time. Therefore, we suggest that this same intervention program, with more sessions, could have significant improvements in students' arithmetic performance. Considering that, as already pointed out by the literature, broader knowledge spheres demand longer interventions (KROESBERGEN; VAN LUIT, 2003; MONONEN *et al.*, 2014).

Secondly, the configuration of the intervention implementation can also have influenced students' performance, seeing that the intervention was collectively applied to all students of the class. Studies indicate smaller effects in interventions with all students in a class (FUCHS; FUCHS; COMPTON, 2012; KROESBERGEN; VAN LUIT, 2003; MONONEN *et al.*, 2014), as this limits the benefits of the intervention because the evaluator cannot pay the proper attention to everyone. Furthermore, it needs more time and resources to collectively apply the intervention in a large group (FUCHS; FUCHS; COMPTON, 2012; KROESBERGEN; VAN LUIT, 2003).

Finally, we highlight that the evaluation of the intervention effect, through standardized tests, has smaller effects than when evaluated by informal tests of research, including measures during the intervention itself (DE BOER; DONKER; VAN DER WERF, 2014; RUIZ; BALBI, 2019). However, the hypothesis of this study was to check the effect of the intervention specifically on arithmetic performance, and, because of that, we have used the standardized test that evaluates students' arithmetic ability.

Still, we should mention that the problems proposed might not have been familiar to the students, as they approach mathematical situations not common in school, and the instruction was given orally. Both elements might have hindered students' development of solutions. We also understand that social factors, such as students' socio-economic levels, and cultural and pedagogical characteristics related to school might have had some influence on students' performance, but such information was not considered because they were outside the objective of this study.

As limitations, we point out that the intervention duration was very short for the amount of content approached, less than what was originally planned due to school complications. The lack of an evaluation of the quantitative reasoning before and after the intervention is an important limitation because, if enacted, would allow us to see the effect of the intervention on the ability taught during the sessions.

Even so, the construction of solid evidence regarding the interventions for the development of students' arithmetic development can help schools to incorporate more efficient educational strategies. The ability of quantitative reasoning, essential for arithmetic proficiency, needs to be better understood by teachers so that they can teach it in school considering students' needs and previous knowledge. Through quantitative reasoning, one develops an understanding of the relations between the quantities involved in an arithmetic calculation, giving meaning to the traditionally taught algorithms. The findings of this study are not enough to provide consistent evidence to guide teaching practice in school, thus, we need more studies that consider interventions in quantitative reasoning to help teachers lead teaching strategies based on evidence.

Therefore, we suggest future studies about longer intervention programs, considering the amount of content approached and school years evaluated. Besides this, it is also important to consider the effects of interventions in quantitative reasoning in the long term. For so, we need longitudinal studies that evaluate the efficiency of teaching quantitative reasoning in different moments of development of mathematical knowledge. There is also the need for more studies based on the implementation of different intervention programs in quantitative reasoning to evaluate their implications for learning. More than that, the next studies must be aware of the applicability of intervention programs in real classroom contexts, which is one of the main objectives and the ultimate goal of Education research.

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