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Conceptual thinking of fractions: the Davydovian mode of teaching organization*

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Abstract

This article was produced in the context of teaching organization from a historical and dialectic perspective. More specifically, its main reference is the developmental teaching proposed by Davýdov and his collaborators. The specification can be expressed in the following research question: what are the manifestations of the movement of conceptual thinking of fractions in the particular tasks in textbooks and teacher's guides, when their reference is the Davydovian mode of teaching organization? In this bibliographical research, the analyzed data represents a sample of five particular tasks taken from the 5th Year textbook, based on Davýdov's proposal and accompanied by the teacher's guide. The first three tasks reveal the movement of reduction from the concrete to the abstract; the remaining tasks reveal the movement of thought as an ascension from the abstract to the concrete. This study reveals that the processes of reduction and ascension direct thinking towards the acquisition of the essential relations constituting the concept of fractions. Essence manifests in the measurement problem which creates the need for a division of the unit of measurement that will establish an intermediary unit. Both movements are characterized by mediated processes leading thinking into the abstraction and generalization of the inner connections constituting the law and cause of the concept.

Keywords

Fraction – Davydovian proposal – Thought movements.

* English version by Marcelo Martinez.

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https://doi.org/10.1590/S1678-4634202248236913eng

Introduction

This research studies thought movement, expressed in the teaching proposal by Davýdov2 and collaborators, towards the acquisition of the conceptual system of fractions. This research is linked to a broader project having at its center a mode of organization of teaching aimed at the formation and acquisition of mathematical concepts. Such a mode of organization has as its foundation the Cultural-historical theory. Among the scholars who have dedicated themselves to the organization of teaching, the principal reference is Davýdov, a Russian psychologist and educator concerned with the objectivization of the theoretical presuppositions of historical and dialectical materialism into a teaching proposal. Davýdov and a team of interdisciplinary collaborators dedicated over 40 years to research with the goal of formulating a "developmental teaching theory" in order to provide students with the development of theoretical thinking (LIBÂNEO; FREITAS, 2013). Their conclusions highlight that both the content and the method composing the curriculum of their country (Russia) did not fulfill the necessary conditions for the theoretical intellectual development of students. Teaching up to that moment had been based on the didactic principles of traditional schools3, which aim at cultivating empirical thinking among students. This acquisition follows formal logic and is based on visual character, whose processes of abstraction and generalization are inferred from the empirical perception of objects via immediate observation of the concrete material visually given and captured by the senses. (DAVÍDOV, 1988).

In his search for the overcoming of empirical thinking, Davýdov (1982) presents a teaching proposal oriented towards the development of study activities. Its contents are designed for the formation of theoretical thinking in students. This form of thinking demands, on the students' part, the elaboration of theoretical abstractions and generalizations shaped by the acquisition of scientific concepts.

According to Davýdov (1988), the process of acquisition focuses on the development of movements of reduction from the concrete to the abstract, as well as movements of ascension from the abstract to the concrete. The reduction movement acts as a point of departure since it brings about an elevation of thinking in the cognitive process via theoretical abstractions and generalizations. Such elevation of thinking goes from the sensorial/concrete to its universal (abstract) base. Having revealed the universal base, thinking flows from the abstract definition to the reproduction of the diversity of concrete phenomena (ROSENTAL, 1962).

It is within that conceptual scope that the object of the investigation in this study is defined: the movement of thinking—of reduction and ascension—for the acquisition of the conceptual system of fractions, based on the analysis of tasks proposed in the examined books. This study is justified by the absence of other studies—performed from the same

2- We will use the Davýdov spelling throughout the text. However, in the case of references, we will preserve the original spelling.

3- Davýdov (1987, p. 143) understands a traditional school as a "relatively unique system of European education which, in the first place, was formed during the period of rebirth and flourishing of capitalist production, which it serves; in the second place, has been based in the works of Ya Komenski, I. Pestalozzi, A. Diesterweg, K. Ushinski and other main pedagogues of the period; in the third place, has conserved up to the present its initial principles as a basis for the selection of contents and methods of the present-day school."

Cultural-historical theoretical perspective—addressing the movement of ascension from the abstract to the concrete in the teaching of the conceptual system of fractions.

Tangencial elements can be found in the research of Amorim (2007), who develops a sequence of tasks for the study of the way in which students acquire the conceptual system of fractions and its operations. Santos (2017) studies the conceptual movement of fraction, with a basis on dialectic logic, in learning situations. Romeiro (2017) addresses teachers' movement of theoretical thinking on the concept of fractions, referencing the meanings they attribute to didactic materials in teaching activities. Isidoro (2019) researches what is revealed by the testimonies of Pedagogy scholars about the mode of organization of Developmental Teaching, having as the study's object the acquisition of the concept of fractions following research by Santos (2017) and Freitas (2016). Therefore, none of these studies, despite having the same theoretical basis, addressed as a central question the following subject: the movement of thinking (reduction and ascension) in the analysis of particular tasks proposed by didactic books in the Davydovian teaching system for the acquisition of the concept of fractions.

It is within this context that the following research problem is defined: what are the manifestations of the movement of conceptual thinking of fractions in the particular tasks in textbooks and teacher's guides, when their reference is the Davydovian mode of teaching organization? To that end, we establish as a general objective: to investigate possible manifestations of the aforementioned movements of thinking for the acquisition of ideas which are introductory to the concept of fractions, having as reference particular tasks from didactic books in the Davydovian teaching system.

As for the up-to-date status of the Davydovian proposal, we will cite the discoveries of authors such as Schittau (2011) and Zuckerman et al. (2017).

Methodological considerations

This study's presupposition is that the process of conceptual acquisition considers the movement expressing the logic of its formation. In other words, we do not seek to simply reproduce the phenomenon as it is given. Instead, our intention is to interpret the laws of transition from one theoretical model to another in order to discover the general and universal laws promoting the movement of theoretical thinking (KOPNIN, 1978).

It must be emphasized that this investigation is aimed at the possibility of manifestation of movement of conceptual thinking in the Davydovian mode of teaching regarding the introduction of the concept of fractions. To that end, our analytical reference is the 5th Year textbook (ГОРБОВ *et al*., 2011), specifically Chapter VIII, titled "Ordinary Fraction." The chapter presents a set of "particular tasks"—as named by Davýdov (1988)—which make explicit the need for a new method of mediation for the introduction of the concept of fractions. Furthermore, the 5th Year teacher's guide (ГОРБОВ *et al*., 2006) contains detailed instructions on how to lead the process of resolution of the tasks in order that students may acquire essential contents.

We selected five tasks for analysis that present the substantial characteristics of the concept of fractions. They reveal the essentiality of the research object: the reduction and ascension movements of thinking in between the abstract and the concrete. They translate the essence of mathematical concepts which, according to Davýdov (1988), is the measurement relation between two quantities. Specifically, the concept of fractions applies to the measurement of a quantity when a unit of measurement will not be contained in it an integer number of times.

The two aforementioned books are written in Russian. Therefore, a translation into Portuguese became necessary and was performed by a team of official translators4. We studied the tasks after the translation work was done and identified the peculiar characteristics of conceptual movement.

The entire analytical process on the tasks for conceptual thinking movement established a dialogue with the theoretical basis, more specifically two kinds of foundations: philosophical (ROSENTAL, 1956; 1962; KOPNIN, 1958; 1978) and psychological and didactical (DAVÍDOV, 1987, 1988). Within this framework, the study of thinking assumes a materialist, historical and dialectical conception, with the understanding that the movement of thinking—reduction and ascension—is relevant to the apprehension of reality. It is that movement that is the focus of the analysis of particular tasks proposed by the Davydovian mode of teaching organization.

Analysis of movement of thinking in Davydovian particular tasks on the concept of fractions

Before we focus on the analysis of particular tasks, it is important to say that such analysis has as its foundation the materialist dialectic logic, which recognizes the principle of reflection, i.e., practical-sensorial activities, as the “immediate basis for the emergence of all intellectual faculties, including thinking itself” (KOPNIN, 1978, p. 50).

However, Rosental (1956, p. 60) points to the limitation of sensorial perceptions in the process of object apprehension since such perceptions only reflect the phenomenon in its isolated aspects. Knowledge, taken separately, may lead to error when determining “its principal, essential characteristic.” In sensorial knowledge, the apprehension of the aspect determining the existence of the object/phenomenon is not exhausted. It must be understood that, in the process of acquisition, thinking moves in search of the apprehension of inner properties through the processes of analysis and synthesis, essence and phenomenon, abstraction and generalization.

The concrete is reflected twice: as sensorial concrete and as thought-of concrete. As a consequence, there is a movement of thinking that promotes the overcoming of the former by the latter. Sensorial concrete consists of the “immediate sensorial perception” of knowledge. It guides thinking towards the acquisition of external properties and characteristics of a given object (KOPNIN, 1958, p. 316, our translation). Thought-of concrete manifests the ideal image of the object as a captured reflex in sensorial-material form. However, not as a mere copy of the object but as a synthesis of multiple relations established between internal and external aspects, which expresses its essence in the

4- Translation performed by a group of Russian teachers residing in São Paulo, who are part of the Clube Eslavo (Slavic Club). Available at <http://www.cursorusso.com.br/>

form of a concept (KOPNIN, 1958). The thought-of concrete brings about the revealing knowledge of the movement of development of the object which, through the process of acquisition, makes possible the apprehension of the internal relations that lead to the revelation of its essence.

According to Rosental (1962), the revelation of the essence occurs in the analytical procedure of investigation – the reduction process – as much to display the external characteristics and properties of the object as to express the internal contradictions that constitute it. In the reduction process, the essential relation is constituted of an initial/substantial abstraction and manifests both the essence of the phenomenon and the cause of its development. Additionally, it presents itself as a point of departure for the posterior movement in the process of apprehension—ascension from the abstract to the concrete. The ascension movement investigates, via the process of synthesis, the way in which a given basis manifests in the concrete diversity of its parts and properties. The synthetic procedure follows the initial movement of abstraction along the diversity of phenomena to reach a conceptual generalization (ROSENTAL, 1962).

The procedures of analysis and synthesis present distinct movements. Analysis departs from the investigation of phenomena and experience to reach the initial abstraction. Synthesis follows the opposite route, departing from the abstraction to explain the diversity of concrete phenomena. However, both are essential to cognition since they make it possible to reproduce the object and the system of its relations (ROSENTAL, 1962).

Such conceptual ideas regarding the movement of thinking, in relation to the acquisition of the internal nexus of the concept, support the process of introductory analysis in the particular tasks for the concept of fractions proposed by Davýdov's mode of teaching organization. His proposal for teaching has the objective of putting students to study activity. Davýdov (1987) understands that the study activity presupposes the acquisition of procedures which make possible the realization of object transformations in order to model and recreate internal properties that convert into concepts. His mode of teaching organization has the following structure: study tasks, developed by six study actions which, in turn, require a system of particular tasks. Study tasks, "linked to the substantial generalization, lead the school student to mastering the generalized relations of the studied area of knowledge and to master new action procedures." (DAVÍDOV; MÁRKOVA, 1987, p. 324, our translation).

The study task for the acquisition of the concept of fractions consists of the apprehension of a new measurement method5 based on analysis situations in which the unit of measurement cannot be contained in a quantity an integer number of times (DAVÍDOV, 1988). For that, a system of particular tasks is proposed for each of the six study actions[6]:

> 1) transformation of task data in order to reveal the universal (general) relation of the object of study; 2) modeling of the universal relation in the unity of object, graphical, or letter forms; 3)

5- The term method here refers to the procedure of measuring quantities. Along the text, the terms "*old method*" and "*new method*" are faithful to the translation. Old method refers to the measurement procedure for the acquisition of the concepts of number, multiplication and division. New method refers to the process of measurement yet to be acquired over the concept of fractions.
6- The fifth and sixth study actions are included in the development of the four initial actions (DAVÍDOV, 1982).

> transformation of the model of the [universal] relation to study its properties in "pure form;" 4) construction of the system of particular tasks to be solved with a general procedure; 5) control over the completion of the aforementioned actions; 6) assessment of the assimilation of the general procedure as a result of the solution of the given task. (DAVÍDOV, 1988, p. 181, our translation).

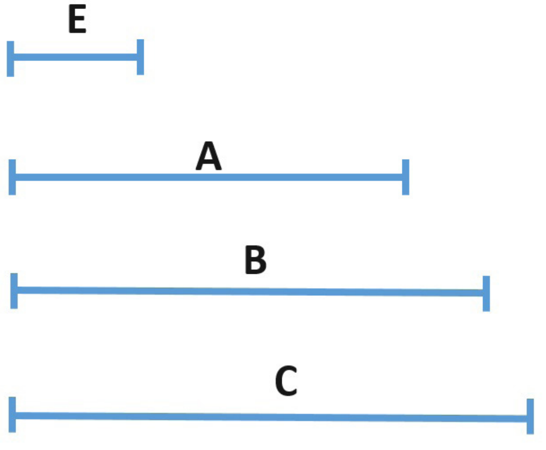
The particular tasks which are the data for analysis in this investigation are among those which make possible the elaboration of the universal model of the concept of fractions and reveal the manifestation of its essence. Five particular tasks are presented, of which the first three (1, 2, and 3) reveal the movement of reduction from the concrete to the abstract which corresponds to the second study action. In the development of those actions, students abstract the essential relation of the investigated object through the analysis of the transformation of the data given by the tasks. The abstraction of that relation revealed in the models – object, graphical and literal – fixates the internal properties of the concept and generalizes them in the form of a universal model (DAVÍDOV, 1988).

Particular tasks 4 and 5 manifest the movement of ascension from the abstract to the concrete for the acquisition of the conceptual system of fractions. They cover the third and fourth study actions. With the revelation of the universal model, the organization of tasks enables students to transform the model to study its properties.

The resolution of the first particular task requires the measurement of segments, which is the basis of theoretical concepts in mathematics (DAVÍDOV, 1988). The task is organized in such a way that students may perceive the impossibility of measuring a segment when they make use of procedures which had been used before, in similar situations.

Task 1 consist of measuring the length of A, B and C using the unit of measurement E, as shown in Figure 1 (ГОРБОВ *et al*., 2006).

Figure 1 – Data referring to Task 1



Source: elaborated according to instructions in Горбов *et al.* (2011).

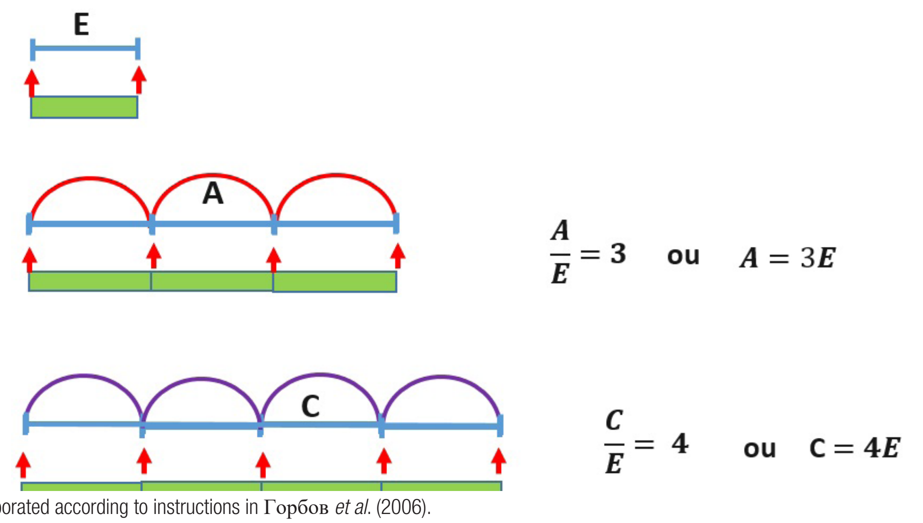
In order to determine the quantitative aspect of number which corresponds to the measurement of each segment, the adoption of a measurement instrument (a cardboard cutout) is proposed. Such a procedure becomes indispensable since the simple observation of the task data does not allow for the apprehension of essential nexus – multiplicity and divisibility – between quantities. The presupposition is that, to reach the knowledge of essence, a procedure capable of apprehending the internal contradictions of the phenomenon is required since these contradictions constitute the source of its development (ROSENTAL, 1962).

According to Rosa (2012), in the process of measurement, number emerges as the numeric property of quantity. In other words, number expresses the result of measurement obtained from the multiplicity and divisibility relation between the unit and the quantity being measured. This is, therefore, the essential – the *genetic* – relation of the concept of number which, according to didactic books, is introduced in the first year.

However, Task 1 brings a few contradictions that generate the necessity for a new measurement procedure which, according to Горбов *et al.* (2006), manifest to students as conceptual limitations. That is the case because some of the knowledge acquired that far is sufficient to solve some, but not all, of the measurement situations. Consequently, numbers need new meanings and representations.

The analogous situations to those developed in previous years consist of determining the amount of times the unit can be contained (divisibility) or repeat itself (multiplicity) in the quantity being measured, without leaving a remainder. That occurs in the measurement of segments A and C since the unit E is contained an integer number of times (3 and 4) in the respective segments (Figure 2).

Figure 2 – Process of measurement in relation to segments A and C

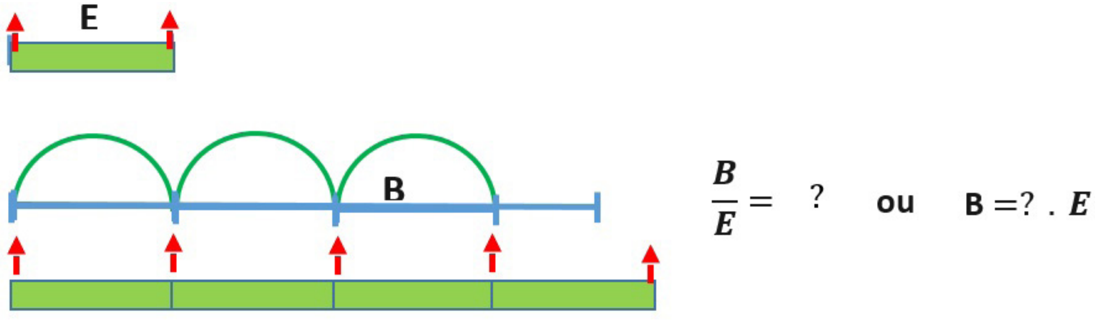


Source: elaborated according to instructions in Горбов *et al.* (2006).

When the reference is segment B, Горбов *et al.* (2006) predict students will find themselves in completely new situations since they will realize the unit of measurement

E cannot be contained an integer number of times in the segment being measured, as in Figure 3.

Figure 3 – Process of measurement in relation to segment B



Source: elaborated according to instructions in Горбов *et al.* (2006).

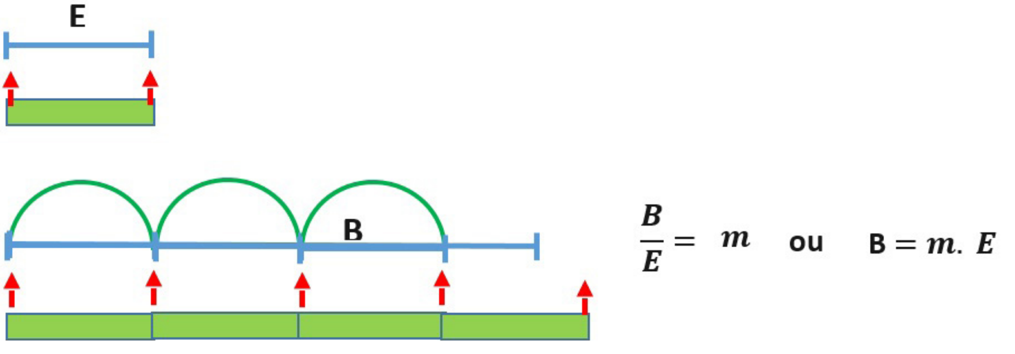
In the Davydovian proposal, unknown situations are presented with the purpose of placing students in constant investigative action, an indispensable condition in developmental teaching. According to Rosa (2012, p. 70), investigative action is imperative to the reduction movement of thinking, as it enables students to develop "the ability to autonomously structure, as well as transform creatively, their own study activity."

In the process of measurement of segment B, data analysis (in object form) makes it possible to ascertain that the adoption of unit of measurement E prevents students from obtaining the length of segment B since the ratio $\left(\frac{B}{E}\right)$ does not result in an integer. The abstraction – non-integer measurements – confronts students with two revelations. One is the expression of already acquired knowledge, in which the mastering of a measurement procedure includes all the necessary characteristics and relations to express a numeric singularity, the natural. In terms of thinking development, the abstractions $\frac{A}{E}=3$ and $\frac{C}{E}=4$, obtained from the measurement of segments A and C, translate a thought-of concrete since they result from a process of measurement acquired by students since their first school year (ГОРБОВ *et al.*, 2006). Such abstractions express the cognitive image of the concept of number in unity of its nexus and properties "as a whole composed of different aspects, qualities and relations" (KOPNIN, 1958, p. 298, our translation). However, they become a point of departure for the analysis of the concept of number since they express the fragility of previous knowledge before new measurement situations.

The other revelation is characterized by a lack of knowledge that will confront students with a new need: the search for another measurement procedure that may allow for the expansion of the numeric field (ГОРБОВ *et al.*, 2006). This specific situation (measurement of B with the unit E) aims at an expansion of the measurement process that extrapolates exact measurements. The different types of numbers (natural, rational, irrational, negative) emerge from a common universal basis expressing a generality: the relation between quantities. The new measurement process translates another possibility of representation of the unit of measurement beyond the natural number – the rational number (DAVÍDOV, 1988).

The interrelation between the known and the unknown generates the chaotic concrete in thinking, expressed as fuzzy knowledge because it does not translate the essential relation of the new measurement process and, as a consequence, of another numeric singularity. The question manifesting at this uncertain moment relates to the quantitative aspect present in the the process of measurement of segment B with unit E since it refers to an unknown number. Therefore, Горбов *et al.* (2006) suggest its representation by an arbitrary letter $\left(\frac{B}{E}\right)=m$ or B=m.E, as shown in Figure 4.

Figure 4 – Representation (literal) of the measurement of segment B



Source: elaborated according to instructions in Горбов *et al.* (2006).

The various representations (object, graphical and literal) are part of the learning process of students from the first year of school. They provide the revelation of the universal model expressed in the form of a law (DAVÍDOV, 1988). However, the law presented in the first task $\left(\frac{B}{E} = m\right)$ does not immediately translate the internal relations of the concept of fractions. It does, however, manifest the point of departure for the apprehension of that concept: relations of multiplicity and divisibility among quantities.

In summary, Task 1 is organized to manifest the contradiction – emergence of non-integer measurements – that is generated when the unit of measurement cannot be contained an integer number of times in the quantity, without remainder. In this contradiction, the necessity for a new measurement method appears. Such a method must be capable of overcoming the difficulties arising during the resolution process.

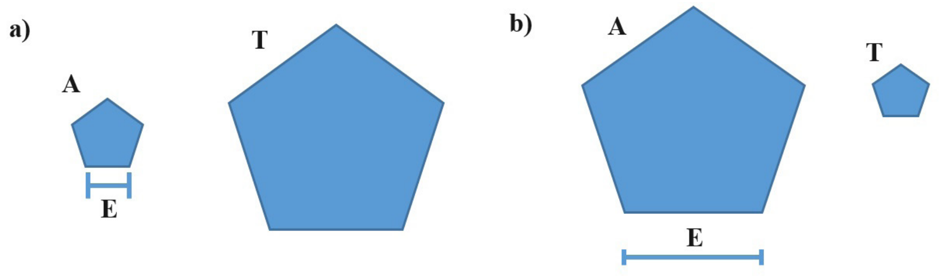
For Горбов *et al.*, the impossibility of measurement present in the resolution of Task 1 generates conceptual necessities emerging from the limitations

> [...] in our primary knowledge, associated with the limited capacity of measurement, the measuring of quantities. [...] Therefore, in order to know new numbers it is necessary to open (invent) new forms of measuring values in situations where the unit cannot be contained in the measured value without leaving a remainder. (ГОРБОВ *et al.*, 2006, p. 122).

Task 2 (Figure 5) fulfills the objective declared by the authors of providing students with the acquisition of a new method of measurement. Its goal is the introduction of graphical modeling (schematic diagram with arrows) through the relations of multiplicity and divisibility between quantities which requisitions two units: basic and intermediary.

The formulation of Task 2 includes the following instruction: “adopt measurement E, draw segments with length equal to the perimeter of the regular pentagons A and T” (ГОРБОВ *et al*., 2006, p. 31).

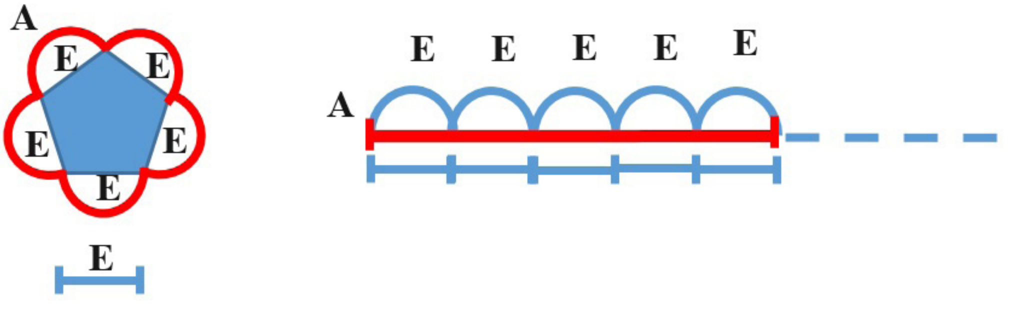
Figure 5 – Situations a and b in Task 2



Source: elaborated according to instructions in Горбов *et al.* (2011).

Situation a confronts students with two possibilities of measurement. One of them relates to the relation between A (pentagon) and E (unit). The abstraction that will move the students’ thinking in a process of reduction is the relation of equality between the measurement of pentagon A’s side and the unit of measurement E (l_A = E). In the construction of the necessary segment, corresponding to the perimeter of pentagon A, the unit of measurement E will be repeated 5 times (Figure 6).

Figure 6 – Construction of segment: perimeter of pentagon A



Source: elaborated according to instructions in Горбов *et al.* (2006).

In this first case, the method of measurement adopted is the one corresponding to the obtainment of the concept of number, in which can be verified that the measurement unit can be contained an integer number of times in the quantity without remainder, i.e.,

$\left(\frac{A}{E} = 5\right)$ or A=5E. Such established relations are represented by the schematic in Figure 7. Its appearance in this task fulfills one of its objectives: the fixation of procedures that had been thus far acquired (ГОРБОВ *et al.*, 2006).

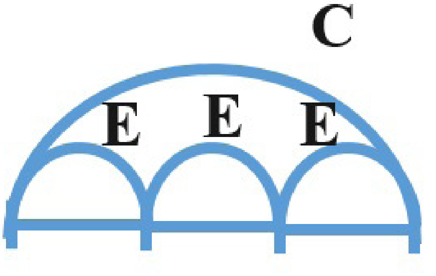
Figure 7 – Measurement schematic of segment A

Source: elaborated according to instructions in Горбов *et al.* (2006).

It can be observed in situation a that the length of measurement unit E is not equal to the side of pentagon T ($l_T \neq E$). The resolution of this situation will face students' thinking with another abstraction of visual content: the inequality of quantities (less than/greater than relations) substantiating the identification of limitations in the previous measurement process. In other words, for the construction of the segment corresponding to the perimeter of T, the adoption of unit E becomes infeasible since unit E is smaller than a side of T ($E < l_T$).

According to Горбов *et al.* (2006), students are instructed to adopt a new unit of measurement, the intermediary unit, obtained from the relation between the concepts of multiplication and division. The intermediary unit is constructed from the grouping of the basic unit to facilitate the process of measurement of quantities, and allows for the control of very extensive quantities (MADEIRA, 2012). In this case, the new unit (intermediary C) corresponds to the length of the side of pentagon T, ergo $l_T = C = 3E$ (Figure 8).

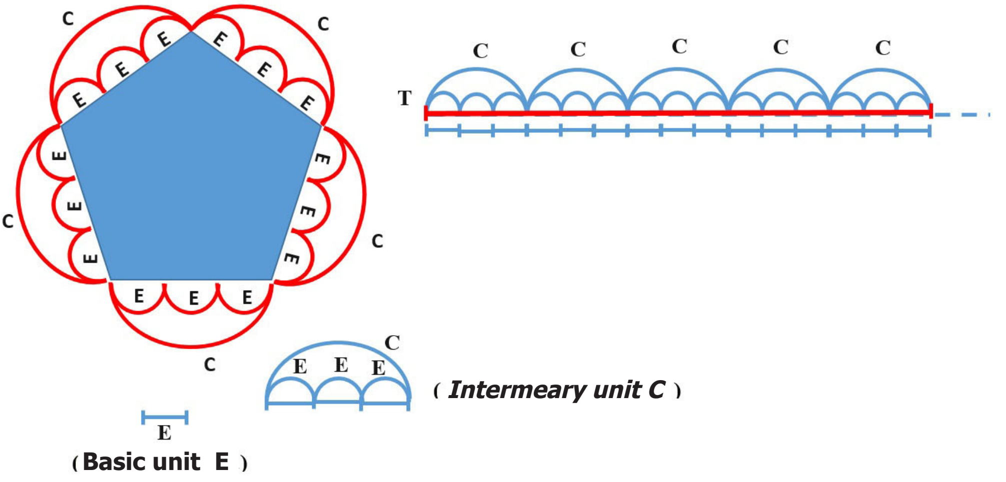
Figure 8 – Intermediary unit C



Source: elaborated according to instructions in Горбов *et al.* (2006).

In the construction of the segment (Figure 9) corresponding to the perimeter of pentagon T, the intermediary unit C will be repeated 5 times.

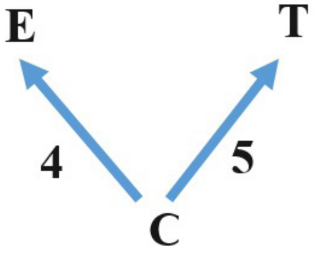
Figure 9 – Construction of segment: perimeter of pentagon T



Source: elaborated according to instructions in Горбов *et al.* (2006).

This method of measurement, according to Горбов *et al.* (2006), can also be presented in schematic form (Figure 10).

Figure 10 – Schematic of measurement of segment T



Source: elaborated according to instructions in Горбов et al. (2006).

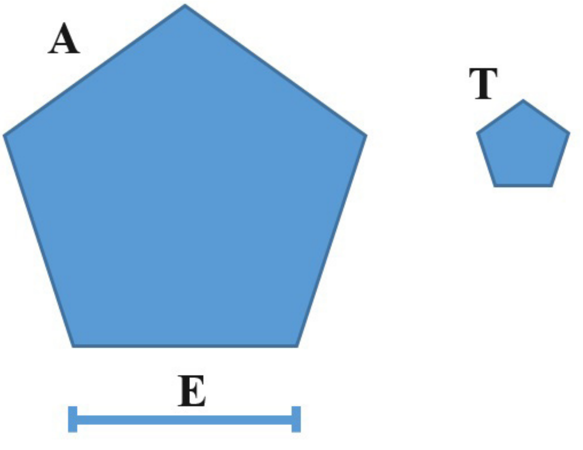
The schematic indicates that the basic unit (E) is repeated 3 times, resulting in the intermediary unit (C = 3E). C is in turn reproduced 5 times to obtain the perimeter of pentagon T, i.e., T=15E. The construction of the segment representative of the perimeter of pentagon T by basic unit E is only possible through intermediary unit C, which constitutes a mediating element in the process of measurement. It will enable students to develop theoretical multiplicative thinking, overcoming the one by one counting characteristic of empirical thinking (MADEIRA, 2012).

The analysis of measurement methods adopted in situation a shows something in common among them: the relation of multiplicity and divisibility between quantities. The difference is in the quantity of relations. In the first case, there is a direct relation E → A. In the second, when the intermediary unit is adopted as the basic unit, the E →

T relation comes to be mediated by two others: E → C and E → T (MADEIRA, 2012). The establishment of mediated relations – repetition of basic and intermediate units – allows for the emergence of theoretical abstractions. According to Davýdov (1982, p. 338, highlighted by the author), "these allow for the formulation of the requisites for the initial abstract definition." By way of mediated processes, abstract thinking aims at both the separation of essential and inessential evidence and the manifestation of internal connections which are intrinsic to the object and cannot be apprehended by the direct look of phenomena.

Another situation linked to the process of measurement, having pentagons A and T as a basis for the analysis, is presented with the same goal as the former, i.e., to construct a segment with the same length as the perimeter of the pentagons A and T (Figure 11).

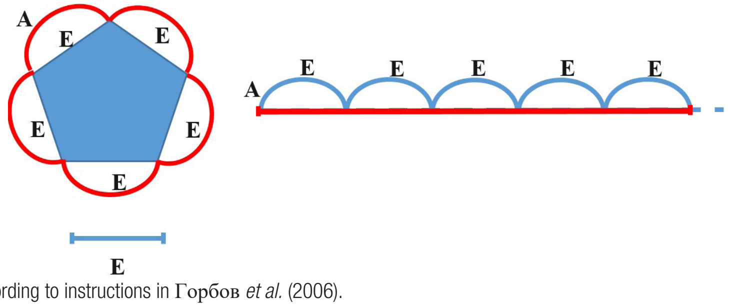
Figure 11 – Measurement based on pentagons A and T



Source: elaborated according to instructions in Горбов *et al.* (2011).

Situation b also involves two measurement processes. One is analogous to the one developed in situation a. Students verify that the unit of measurement E is equal to the length of the side of pentagon A. In this case, the segment being constructed will measure 5 times unit E (Figure 12).

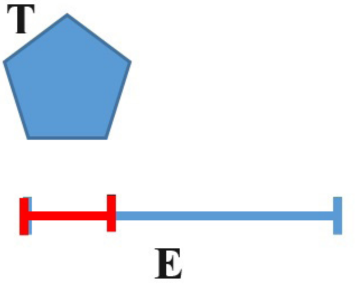
Figure 12 – Construction of segment: perimeter of pentagon A



Source: elaborated according to instructions in Горбов *et al.* (2006).

In the case described below, the unit of measurement E is not equal to the measurement of the side of pentagon T (E ≠ l_T). Again, the relation of inequality between quantities is presented to students. However, the unit of measurement E is greater than the length of the side of pentagon T (E > l_T). It is impossible to construct a new intermediary unit from the grouping of basic unit E. This is made evident when it is proposed that l_T be measured with unit E since it cannot be contained any integer number of times in l_T (Figure 13).

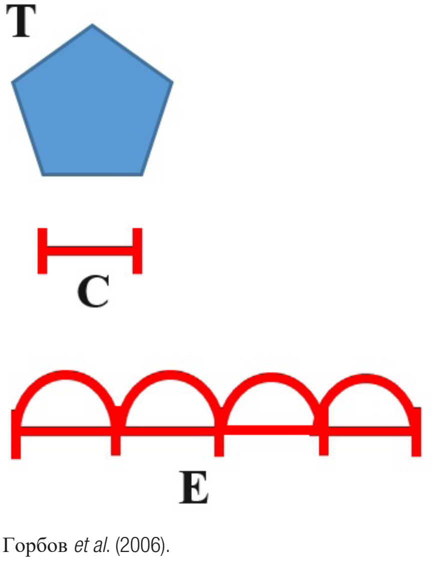
Figure 13 – Impossibility of measuring 1_T using unit E



Source: elaborated according to instructions in Горбов *et al.* (2006).

The task starts a process of searching for a procedure which will lead to the determination of a new unit of measurement of intermediary character, but of still unknown meaning. According to Горбов *et al.* (2006), the instruction turns to the inverse relation, i.e., l_T can be contained a number of integer times in unit of measurement E (Figure 14).

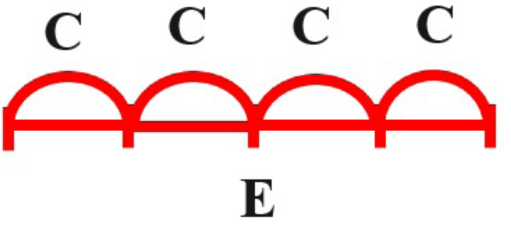
Figure 14 – Measurement process of basic unit E



Source: elaborated according to instructions in Горбов *et al.* (2006).

The measurement will lead to the finding that the length of the unit of measurement E is 4 times greater than the side of pentagon T, also expressed as E = 4C (Figure 15).

Figure 15 – Length of basic unit E



Source: elaborated according to instructions in Горбов *et al*. (2006).

The measurement promotes the emergence in thinking of a new abstraction – the subdivision of the unit – in a chaotic state. The process complexifies, since “the unit cannot be contained an integer number of times in the quantity, which leads to the need to divide it in equal parts and subsequently use one of the parts as a new unit” (ГОРБОВ *et al*., 2006, p. 121). It is valuable to reiterate, according to Aleksandrov, Kolmogorov and Lavrentyev (1973, p. 44), that the impossibility of measuring if that which gives rise to “the need to fraction [divide] the unit of measurement in order to express the quantity with greater exactitude in parts of the unit.”

The process of constituting the intermediary measurement is then represented by the schematic (Figure 16):

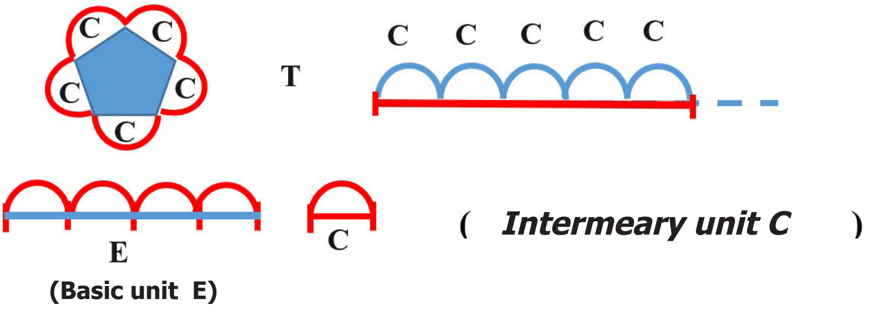
Figure 16 – Schematic of the process of measurement of E

Source: elaborated according to instructions in Горбов *et al*. (2006).

The schematic appears with the contrary indication of what is proposed by the abstract model for the acquisition of the concepts of multiplication and division. According to Madeira (2012), this happens because, in the schematic, the arrow represents the relation of multiplicity which goes from the smaller to the greater quantity.

In situation a, the intermediary unit for the measurement of Twas a multiple of the basic unit with the representation of the relation E → C. In the new situation, the intermediary unit C is a submultiple of the basic unit, being represented by the relation E ← C which indicates that C is smaller than the basic unit E. The relation expressed by the schematic of the construction of the segment (Figure 17) referring to the perimeter of pentagon T indicates that unit E must be divided in four equal parts. One of such parts is the intermediary measurement C, which will be repeated five times.

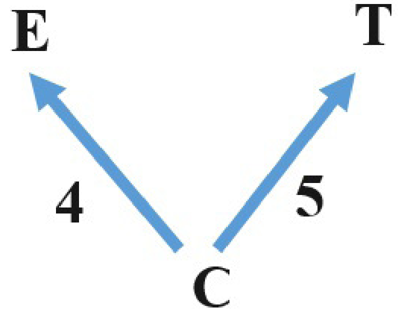
Figure 17 – Construction of segment: perimeter of pentagon T



Source: elaborated according to instructions in Горбов *et al.* (2006).

The whole process is then registered in the arrow schematic (Figure 18):

Figure 18 – Schematic of measurement of segment T



Source: elaborated according to instructions in Горбов *et al.* (2006).

With support from the schematic it is possible to elaborate the synthesis: "the value of a new [intermediary] unit can be obtained not only by the multiple of the basic unit [grouping] but also by its division in equal parts" (ГОРБОВ *et al.*, 2011, p. 124). According to Rosental (1962), the process of synthesis occurs through the development of the process of analysis, and vice versa.

The analysis and synthesis make possible, in the resolution of the task, the revelation of the essential relation – subdivision of the unit of measurement – which plays the role of initial abstraction in the ascending path from abstract to concrete for the reproduction of the conceptual system of fraction. The initial abstraction "reflects the essence, the law of phenomena, in an abstract way: in its pure aspect" (ROSENTAL, 1962, p. 493).

Such abstraction appears during the process of construction of the intermediary measurement, with the presentation of a new quality: submultiple of the basic unit, abstracted via the analysis of the transformation of task data. Unit subdivision constitutes a mediator element for the revelation of the new method which expresses the obtainment of rational/fractional numbers. It makes explicit internal relations – multiplicity and divisibility – between quantities: basic unit and intermediary unit.

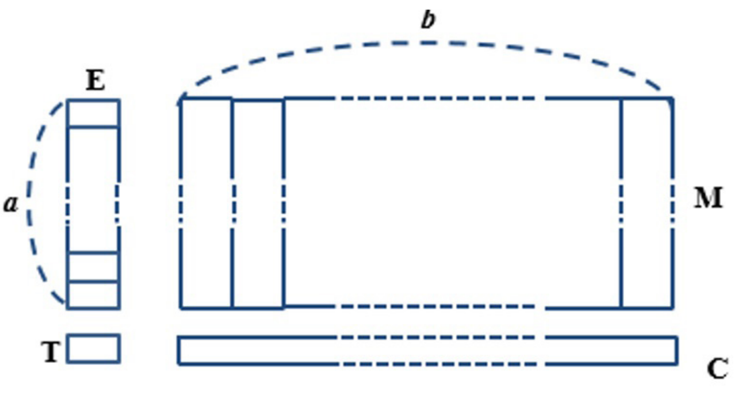
The movement of revelation of the essential relation takes place by the analysis of the process of measurement of the length quantity, represented by segments. These segments are presented, initially, in the object-sensorial form, which permits the abstraction of the first general relations, in both situations: $l_A = E$ (relation between the side of pentagon A with unit E); $l_T \neq E$, $l_T > E$ and $l_T < E$ (relation of the side of pentagon T with unit E). However, it is in the abstraction of relation lT < E that emerges the finding of the insufficiency of the adoption of unit E for the measurement of the perimeter of T. When the inverse relation between the two lengths (l_T and E) is proposed, the ascertainment of how many times l_T can be contained in unit E makes it possible to construct a new intermediary unit, to be obtained by the subdivision of the basic unit into equal parts.

Therefore, Task 2 makes explicit in its process of development the reduction movement mediated by abstractions and theoretical generalizations which direct thinking toward the acquisition of the universal relation of the concept of fractions. Such acquisition is achieved from the apprehension of a new method of measurement. For Rosental (1962), the universal relation manifests through the revelation of the essence, which is a complex procedure since the essence is not immediately given in the analyzed object. Furthermore, the simple observation and contemplation of external evidence does not allow for the acquisition of the internal nexus constituting the object, given that the internal consists in the contradictions generating its development. The promotion of the abstraction of the essential relation constitutes in this task the main focus, because it leads to the revelation of the universal basis (the law expressing the development of the concept of fraction).

Task 3 is aimed at the theoretical generalization of the new measurement method, a condition for the acquisition of the concept of fraction which is characterized by the reproduction of the internal relations manifested in the universal model. According to Davýdov (1988), the modeling of the universal relation (the law) is gradually revealed by the object, graphical and literal modeling.

In Task 3, the "design shows schematically how the area quantity C is measured with the unit E by the adoption of [Figure 19]: 1) the intermediary measurement T; 2) the intermediary measurement M" (ГОРБОВ *et al*., 2011, p. 37-38).

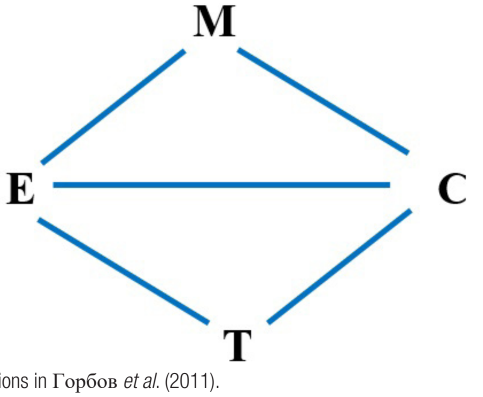
Figure 19 – Geometric representation: commutativity relation



Source: elaborated according to instructions in Горбов *et al.* (2011).

The task (Figure 20) also proposes: “describe the two ways of constructing a schematic” (ГОРБОВ *et al.*, 2011, p. 38).

Figure 20- Schematic to be completed in Task 3



Source: elaborated according to instructions in Горбов *et al.* (2011).

Горбов *et al.* (2006) propose a geometric (object) analysis, as presented in Figure 19, as a mediating relation for the abstraction of the commutation of factors a and b to be represented in the form of a schematic (graphically) (Figure 20) with the objective of introducing the conventional notation for the new method, which is expressed in literal form. Central to that goal is the movement of reduction in thinking, aimed at the abstraction and generalization of internal relations in order to model the universal basis of the concept.

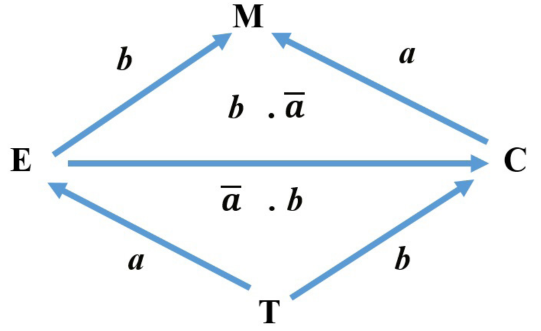
The first relation to be abstracted consists of measuring quantity C with unit E. The other relations, fixed in the schematic, are abstracted through the analysis of quantities, presented in sensory form, which makes it possible to verify that quantity T is the smaller among the areas, thus revealing the abstractions E > T and C > T. However, that is not sufficient since what is sought is the level of theoretical thinking, for which it is not enough to identify whether a quantity is greater than another. It is necessary to find a way to represent their quantities. Therefore, an analysis of relations E > T and C > T is called for, based on the essential idea in order to find that, respectively, T is repeated *a* times in E and *b* times in C. Additionally, other relations are established when the greatest quantity (M) is considered, i.e., M > E and M > C. In the first, the *b* factor corresponds to the amount of times E is repeated in M. In the second relation, M > C, factor *a* represents the amount of times C is repeated in M.

However, a representation of the direct relation between unit E and quantity C – mediated by intermediary units T and M – is still to be determined. In this case, the function and order of factors comes under analysis. When the reference is intermediary measurement T, factor a presents a subdivision function of unit E, while b represents the repetition of T in C. The direct relation (E → C) is represented by $\bar{a}.b$.[7] In turn, when the

7- Such representation is introduced by Горбов *et al.* (2006) in a particular task. In the resolution, the student creates a symbol to differentiate the corresponding functions of a and b factors. The condition is that the symbol must correspond to the factor differentiating measurement procedures, indicator of unit division.

focus of the analysis is the intermediary measurement M, the functions of the factors are: b, repetition of unit E; and a, subdivision of unit M. Direct relation (E → C) is described as: $b.\bar{a}$. The schematic (Figure 21) represents these abstractions.

Figure 21 – Partial resolution of Task 3

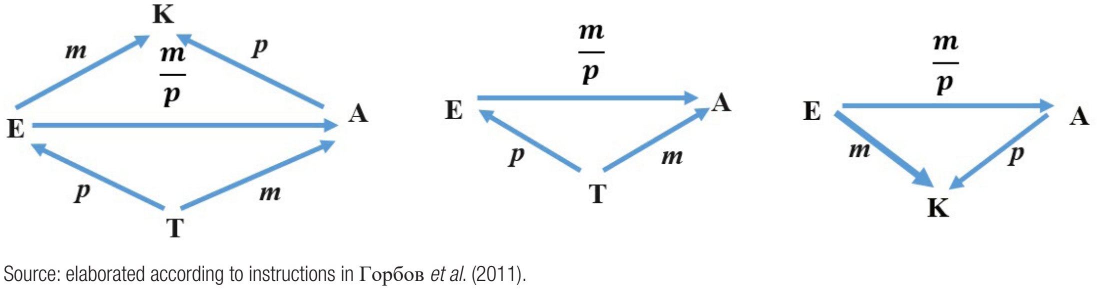


Source: elaborated according to instructions in Горбов *et al.* (2006).

The schematic above supports the observation that the direct relation (E → C) has two representations: $\bar{a}.b$ and $b.\bar{a}$. They express the relation of commutativity between factors a and b because, if their order is reversed, there is no alteration in the result of the measurement of quantity C. This is valid if the function of each factor is considered: $\bar{a}$ presents the division function and b the multiplication function, which imply a modification of steps in the process of measurement.

After the representation in the schematic of direct relation, Горбов *et al.* (2006) advise the adoption of the following register: $\left(\frac{b}{a}\right)$.[8] This is the literal representation, which characterizes the rational number named ordinary fraction (ГОРБОВ *et al.*, 2006). The $\left(\frac{b}{a}\right)$ register not only represents the number in the form of a fraction (as a result of the measurement of quantities) but also reveals the new method of measurement. In other words, it consists of the general law for the obtainment of rational numbers which manifests the internal relations of its development: subdivision and repetition of units (basic and intermediary), as shown in Figure 22, substituting m and p for a and b.

Figure 22 – Universal model of the concept of fractions



Source: elaborated according to instructions in Горбов *et al.* (2011).

8 - Record shows unification of factors – $\bar{a}.b$ and $b.\bar{a}$ – to become a simple $\left(\frac{b}{a}\right)$.

The letters in the schematic translate the meanings of relations between quantities:

> The number p, located below the fraction dash, is called a *denominator*. The denominator indicates the value of the division (of the basic unit) in equal parts. It shows how many equal times that quantity (E or K) has been divided into. The number m, above the dash, is called a *numerator*. The numerator indicates the value of repetition. It shows the number of times it is necessary to repeat part T or unit E. The separation line or dash in between the numerator and the denominator is called *bar*. (ГОРБОВ *et al*., 2011, p. 38, author's highlights).

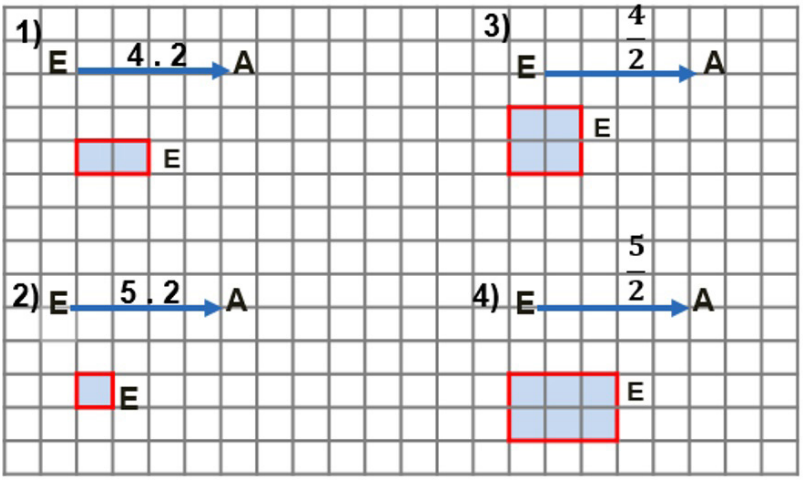
In this way, $\frac{m}{p}$ is translated into the universal model of the concept of fractions. It reflects in thinking the movement – of abstraction and generalization – of the essence revealed during the development of the system of particular tasks. According to Davýdov (1982), the universal model translates the essential interrelation of a given concept at the scientific level, which is the indispensable basis for the development of theoretical thinking.

With the revelation of the essential relation in its abstract form, $\frac{m}{p}$, thinking follows the movement of initial abstraction toward the study of the diversity of phenomena in order to reach a generalization of the concept of fractions. According to Rosa (2012, p. 51), this occurs with the identification of a "regular linking of the principal [essential] relation to its particular manifestations." In the process of resolution, departing from the general procedure, particular manifestations are analyzed in order to understand concrete forms of revelation of the universal basis. Such is the goal of the organization of the particular tasks presented below.

Task 4 addresses the theoretical generalization of the concept of fractions. It proposes a reflection on the new numeric field, in accord with the genesis of the concept of number.

Task 4 asks students to "construct area A by using unit E" (ГОРБОВ *et al*., 2011, p. 40), according to data presented in Figure 23.

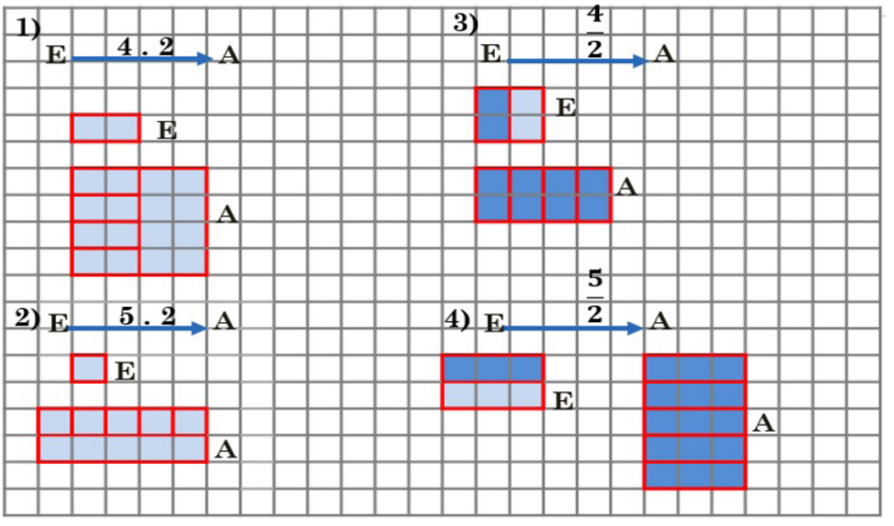
Figure 23 – Data corresponding to Task 4



Source: elaborated according to instructions in Горбов *et al.* (2011).

The basis for the analysis are the records (Figure 24) for the performance of the measurement processes.

Figure 24 – Resolution of Task 4



Source: elaborated according to instructions in Горбов *et al.* (2006).

At the stage of development in Task 4, according to Горбов *et al.* (2006), thinking moves mediated by abstractions articulated with schematic analysis which manifest the essential relation of the concept of fraction. This takes place, according to Rosa (2012, p. 104), because the schematic "is representative of the movement of the relation between quantities," mediated by graphical representation, that allows students to rise from the object plane to the mental plane. The task also proposes the representation of results by the formula $\frac{A}{E}=t$, corresponding to the literal expression of the universal model of the concept of number (ГОРБОВ *et al.*, 2006).

From data analysis (Figure 24) it is possible to find that the first two situations refer to the concept of multiplication whose register expresses the product of factors, i.e., the amount of times the intermediary unit (constructed by grouping the basic unit E) is repeated for the obtainment of area A. The results of the respective situations are: $\frac{A}{E}=8$ and $\frac{A}{E}=10$. In situation 3, despite the register in the form of a fraction, it can be verified that unit E can be contained 2 times in A, i.e., $\frac{A}{E}=2$. These first three situations result in integer measurement values (ГОРБОВ *et al.*, 2006).

However, situation 4 is configured among those which, within a process of ascension from the abstract to the concrete, requires an extrapolation of a given immediacy, in this case the measurement with real number characteristics in its natural singularity. ГОРБОВ *et al.* (2006) emphasize that the fourth situation brings back the problem of measurement resulting in non-integer measurements to represent the new number (fraction).

The construction of the area quantity A requires students to apprehend internal relations which were abstracted and generalized in previous tasks by means of the process of reduction from concrete to abstract. That apprehension constitutes the new method of measurement. According to Rosental (1962, p. 485), it is only after finding, by means of abstraction, the relation "which constitutes the essential basis and unit of all manifestations of the given thing that the process of ascension leading from this abstract moment to the concrete may begin." Kopnin (1958, p. 313, our translation) understands that, in the movement of ascension from the abstract to the concrete, "the concrete object itself is not created, it already existed before and independently of being known; what emerges is its concrete concept."

For Davýdov (1988), only through the analysis of the contents of a phenomenon – taken in its process of development – can thinking manifest its essence in such a way as to reflect the essential nexus that are intrinsic to the process of formation. As a consequence, the revelation of the essence expresses the universality of the phenomenon that leads thinking into the reproduction of the concrete in its integrity. That revelation, regarding the concept of fractions, occurs in those relations which are intrinsic to the universal model $\frac{m}{p}$ (where p corresponds to the subdivision of the basic unit and m to the repetition of the intermediary measurement) since they support the process of construction of area A. That is, the register $\frac{A}{E} = \frac{5}{2}$ indicates that the unit of measurement E must be subdivided into two equal parts, of which one constitutes the intermediary measurement which, being repeated 5 times, makes it possible to obtain area A. Thinking, at this stage of development, moves according to characteristics and properties apprehended in the movement of obtainment of a new method of measurement allowing for the verification that E can be contained $\frac{5}{2}$ times in A.

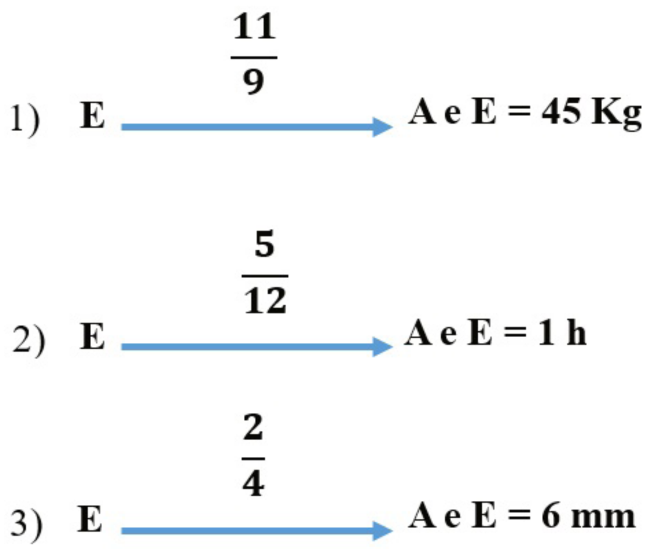
After the representations, the processes of analysis and synthesis in the task turn to the results of situations 3 and 4. The focus lies on the following detail: the relation between quantities may result in different numbers (integer and fractions). The necessity of generalizing these two types of numbers then emerges. That is, the set of numbers previously known by students – integers – now includes a new method of measurement expanding into another numerical singularity: fractions. Therefore, the set of rational numbers is created. This set is expressed by integer and fractional measurements, as are the cases of $\frac{4}{2}=2$ (integer) and $\frac{5}{2}$ (fraction).

The above specificity of Task 4 – adoption of a new method that creates the possibility of causing the two types of number to emerge in a single measurement process – reveals an additional characteristic of the Davydovian mode of organization of Mathematics teaching: the ascension of thinking to the thought-of concrete involves an overcoming of the method to expand it into another which unifies different numeric qualities. In this process, it is understood that thinking overcomes the transit leapt in definitions extracted from the object's exteriority in an immediate and contemplative manner. Ascending to the thought-of concrete makes thinking deliberate on connections and relations of both conceptual and procedural orders, however with the necessary and characteristic internal link (KOPNIN, 1978).

The next task is similar to the previous one since it proposes the obtainment of a quantity from the analysis of a schematic. However, the focus of the analysis turns to the numeric aspect corresponding to the basic unit of measurement instead of its generic aspect as unit of area.

Task 5 requests students to “find the value of A” from data shown in Figure 25 (ГОРБОВ *et al.* 2011, p. 40).

Figure 25 - Data for Task 5

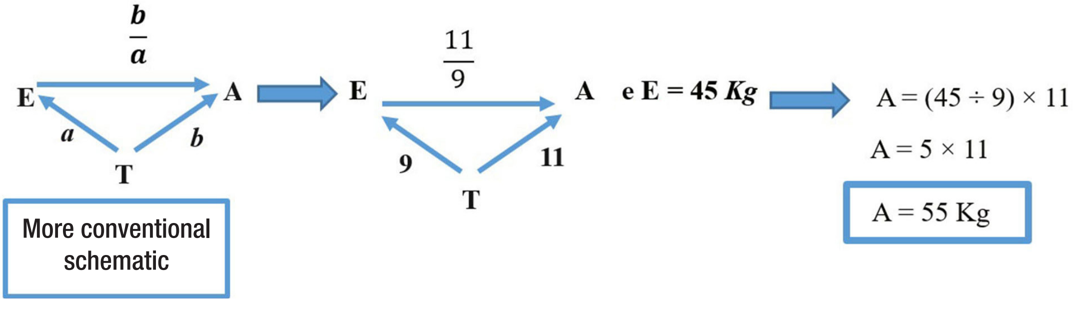


Source: elaborated according to instructions in Горбов *et al.* (2011).

It can be observed in Figure 25 that, in each situation, students have at their disposal the following registers: amount of times unit E can be contained in A, and the value of unit E. The adoption of standard units allows students to perform the operations – division and multiplication – in the mental plane, once their representation in sensory form is not necessary. The task consists of expressing, through arithmetic operations, the manifestations of the universal model.

In the first situation, the adopted unit is a particularity of the mass quantity: the kilogram (kg). Note that unit E is composed of 45 units of that mass and that quantity A corresponds to $\frac{11}{9}$ of the composition. To determine the value of A, it is necessary for students to adopt operations that are appropriate to the universal model of fractions; operations abstracted from the analysis of the numerator and denominator factors. One such operation is the division of 45 by 9 since it follows the movement of construction of the intermediary unit expressed in the model. Another operation is the repetition of this new unit (resulting from the division) 11 times to obtain quantity A, as shown in Figure 26.

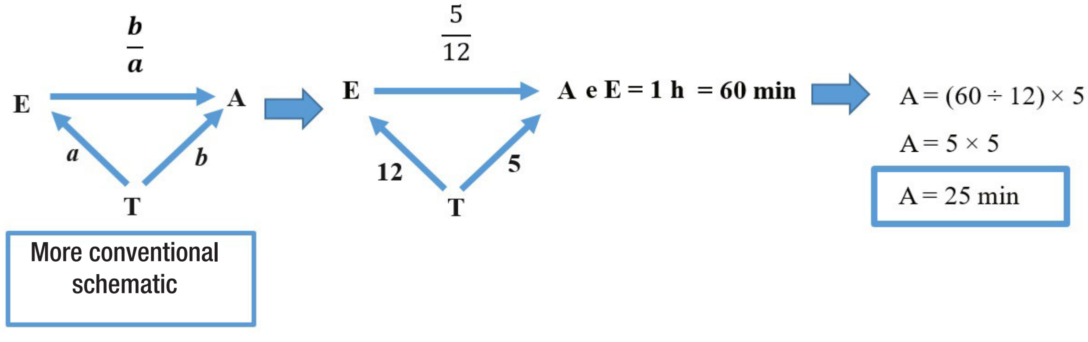
Figure 26 – Resolution of the measurement corresponding to situation 1



Source: elaborated according to instructions in Горбов *et al.* (2011).

The operations $45 \div 9 = 5$ and $5 \times 11 = 55$ translate the movement in the mental plane for the obtainment of the mass quantity in kilograms: A = 55 kg. The same process of abstraction occurs in situation 2 (Figure 27) to express the measurement of the quantity by means of standard units.

Figure 27 – Resolution of situation 2

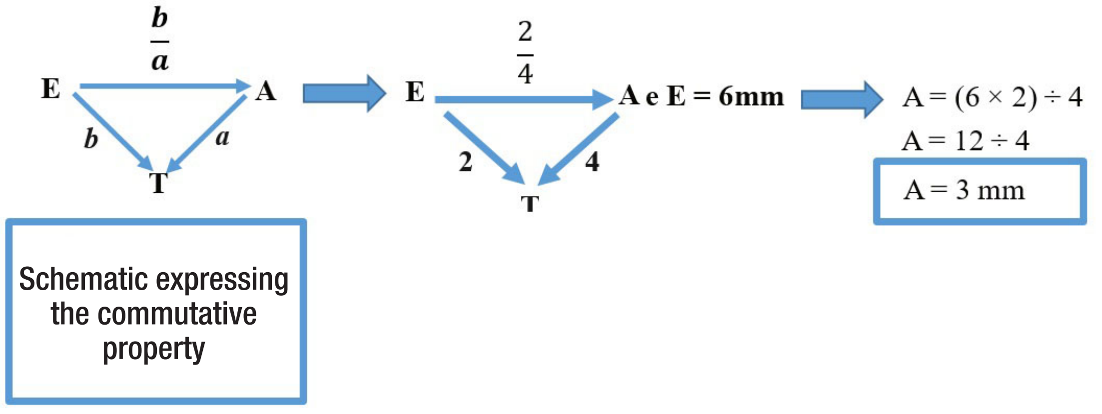


Source: elaborated according to instructions in Горбов *et al.* (2011).

As shown in Figure 27, situation 2 concerns the relation between unit E and register $\frac{5}{12}$. In order to resolve it, students must consider the transformation of units, i.e., 1h corresponds to 60 minutes. This is the case because the division of 1 by 12 (relation established by conventional schematic) does not result in an integer. Quantity A is obtained from the operations $60 \div 12 = 5$ and $5 \times 5 = 25$ given in the mental plane and guided by the arrow schematic. Ergo, $\frac{5}{12}$h of E corresponds to 25 min, the value corresponding to quantity A.

However, to resolve situation 3 it is necessary to adopt the commutative property. According to Горбов *et al.* (2006), instead of subdividing the basic unit it is initially preferable to repeat it, in order to obtain the intermediary unit. The latter is, in turn, subdivided into equal parts to obtain the measurement of quantity A, which is one of those parts (Figure 28).

Figure 28 – Resolution of situation 3



Source: elaborated according to instructions in Горбов *et al.* (2011).

The operations 6 × 2 = 12 and 12 ÷ 4 = 3 translate within thinking the acquisition of the commutative property (expressed by the schematic) for the obtainment of quantity A, which corresponds to 3mm in length.

The process of resolution of the task makes it possible to highlight two aspects that are relevant to the movement of ascension of the conceptual thinking of fractions. One is the point of departure consisting of the analysis of the universal model, characterized as initial abstraction, substantial (since it reflects the essence), the cause of the development of the concept of fractions. The other, the point of arrival, reveals the apprehension of the processes of analysis and synthesis mediated by abstractions and theoretical generalizations which, according to Davýdov (1988, p. 152), are two essential cognitive actions for the movement of thinking since they “find their expression in the theoretical concept serving as a procedure to deduce particular and singular phenomena from their universal basis.”

In this context, Task 5 as a whole brings the notion that the process of thinking, of ascension from the abstraction of the concept of fractions to its concretization (thought-of) has as a basis the revelation of the internal relations in singular events with determined specificities. The measurement is presented in predetermined units: kg, h and mm. These specifications allow for mental operations of a theoretical and conceptual nature by means of the universal model. According to Davýdov (1988), to mentally elaborate and transform an object constitutes its comprehension and revelation of its essence. Therefore, with each new task, Горбов *et al.* (2006) propose the reproduction of the concept of fractions in the context of a conceptual system involving number, multiplication, division, commutative property, measurement of quantities, among others.

Final remarks

This study brings the possibility of reflection – from the analysis of a set of particular tasks in the Davydovian proposal – on the movements of reduction and ascension for the development of the theoretical thinking of the concept of fractions. Departing from a study of the literature in a dialectical perspective, it is possible to find that the basis for the development of thinking consists of the processes of abstraction and generalization emerging from the procedures of analysis and synthesis. Mediations occur in the movement of reduction as much as in the movement of ascension. In reduction, thinking turns to the revelation of the initial abstraction. On that basis lies essence, which, as a singular source, determines the remaining particularities of the whole (DAVÍDOV, 1988). When thinking reaches the knowledge of essence, it moves from the abstract definition to the reproduction of a "system of nexus and relations characteristic of a given object as concrete integrity" (ROSENTAL, 1962, p. 496, our translation).

The possibilities of this movement of thinking, in the specificity of the concept of fractions, are revealed in the five analyzed particular tasks. Regarding the analysis of the three first tasks, the process of reduction from the concrete to the abstract is explained when the essential relation of the concept is revealed, emerging through the problem of measurement. Such relation presents itself in the situation in which the unit cannot be contained an integer number of times in the quantity to be measured. This impossibility creates the necessity to develop a new method of measurement to be modeled and acquired from the analysis of the methods previously adopted.

The new method brings as an essential characteristic the subdivision of the unit of measurement, which is transformed into an intermediary unit. The movement of transformation reveals the internal nexus of the concept of fractions: division of the basic unit and repetition of the intermediary unit (and vice versa), with the adoption of the commutative property of multiplication.

With the emergence of nexus, thinking turns to the generalization of the method. This is the modeling of the universal relation, with the adoption of representation systems in the following configurations: object, graphical and literal. Modelation translates the essence of the concept into law, which is revealed as a consequence of the emergence of the necessity for other generalizations: order and function of factors. These condition the identification of operations (multiplication and division) as well as the order of their performance. Therefore, the concept of fractions and the concept of multiplication and division of integers have the same universal model, with the difference being the intermediary unit constituting a part of the basic unit.

From the generalization of the two relations of measurement which are intrinsic to the commutative property, a law manifests which expresses both relations in a single literal register: $\left(\frac{4}{2} = 2\right)$. This register is a point of departure for the movement of ascension from the abstract to the concrete, as evidenced in tasks 4 and 5. In this case, the initial abstraction, originating in the process of reduction in thinking, is characterized as a future stage for the concretization of the concept. However, in accordance with Davýdov (1982), this abstraction is not empirical. It is concrete since it makes possible a search for the revelation of connections produced historically, as well as the essential contradictions characterizing the concept.

With the revelation of the essential relation in its abstract form, thinking follows the movement of the initial abstraction toward the study of the diversity of phenomena in order to reach a generalization of the concept of fractions. In the movement of ascension, thinking moves in search of evidence to characterize the appearance of its specificities. In the process of generalization of the concept, there appears evidence pointing to the possibility of representing an integer by way of a fraction. This occurs as long as the quantity of divisions be a submultiple of the quantity of repetitions . Evidence for this relation allows thinking to ascend to the set of rational numbers: integer and fractional.

It can be observed that, even departing from the sensorial concrete (geometric figures) by means of mediated processes, thinking seeks the abstraction and generalization of internal connections constituting the concept of fractions. The analyzed tasks serve as evidence to the claim that they manifest possibilities of developing, in students, the movement of dialectic conceptual thinking: reduction from the concrete to the abstract and ascension from the abstract to the concrete.

To arrive at such a level of development of conceptual thinking requires, according to Davýdov (1988), not only tasks with strong links in between them but also the collaboration of teachers with relevant guidance in order to create the conditions for the acquisition of actions and their corresponding capacities making their execution possible.

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Received on: 20.04.2020
Revised on: 30.08.2020
Approved on: 24.11.2020

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