Early mathematical concepts and language: a comparative study between deaf and hearing children[I]

Heloiza H. Barbosa[II]

Abstract

Research has shown that children who develop typically build mathematical concepts very early. This process of cognitive development seems to be closely connected with the development of verbal language. What happens to the mathematical development of children who have a different form of language such as the sign language used by deaf people? This question, and other questions about deaf students' low performance in mathematics documented by other studies guided the development of the study presented here. To answer these questions, experimental tests were carried out with deaf children (group 1), younger hearing children from public schools (group 2), older hearing children from public schools (group 3) and children from private schools (group 4). The results evidenced a clear distinction between mathematical cognitive skills more dependent and less dependent on linguistic stimuli, notifying that deaf children have the same performance, or in some cases even higher performance than hearing children in skills less dependent on linguistic stimuli. However, both deaf children and younger hearing children from public schools had a significantly lower performance in comparison to older hearing children from public schools and children from private schools. This result indicates that deafness is not a cause of poor academic performance in mathematics. Thus, it seems necessary to think of forms of pedagogical intervention which can ensure the successful learning of mathematics for both deaf children and hearing children who attend public schools in Brazil.

Keywords

Mathematics education – Deafness – Cognition – Counting.

[I] Study supported by CNPq.

[II] Universidade Federal de Santa Catarina, Florianópolis, SC, Brazil.
Contact: heloiza@hbarbosa.org

Conceitos matemáticos iniciais e linguagem: um estudo comparativo entre crianças surdas e ouvintes[I]

Heloiza H. Barbosa[II]

Resumo

Pesquisas têm demonstrado que as crianças que se desenvolvem tipicamente constroem conceitos matemáticos desde muito cedo. Esse processo de desenvolvimento cognitivo parece estar intimamente conectado com o desenvolvimento da linguagem verbal. O que acontece com o desenvolvimento matemático de crianças que possuem uma forma diferente de linguagem, como a língua de sinais utilizada pelos surdos? Essa pergunta, além de demais indagações sobre o baixo desempenho em matemática de alunos surdos documentado por outros estudos, orientou o desenvolvimento da pesquisa aqui apresentada. Para responder a tais questionamentos, foram realizados testes experimentais com crianças surdas (grupo 1), crianças ouvintes mais jovens da escola pública (grupo 2), crianças ouvintes mais velhas da escola pública (grupo 3) e crianças da escola privada (grupo 4). Os resultados evidenciaram uma clara distinção entre habilidades cognitivas matemáticas mais dependentes e menos dependentes do estímulo linguístico, notificando que crianças surdas têm o mesmo desempenho ou, em alguns casos, até mesmo um desempenho superior do que crianças ouvintes em habilidades menos dependentes do estímulo linguístico. Contudo, tanto as crianças surdas quando as crianças ouvintes mais jovens da escola pública demonstraram um desempenho significativamente baixo em relação às crianças ouvintes mais velhas da escola pública e às crianças da escola privada. Tal resultado indica que a surdez não é causa de baixo rendimento escolar na área da matemática. Assim, parece ser necessário pensar em formas de intervenção pedagógica que possam garantir uma aprendizagem de sucesso em matemática tanto para as crianças surdas, quanto para as crianças ouvintes que frequentam as escolas públicas brasileiras.

Palavras-chave

Educação matemática – Surdez – Cognição – Contagem.

[I] - *Estudo realizado com apoio do CNPq.*

[II] - *Universidade Federal de Santa Catarina,*
Florianópolis, SC, Brasil.
Contato: heloiza@hbarbosa.org

Research problem

Over the years, research focusing on the development of mathematical ideas has changed in a fundamental way our understanding of the quantitative and mathematical thinking of children (for a detailed review, see BARBOSA, 2008). Piaget and Szeminska (1952) innovatively promote a paradigm shift around the 1950s when studying mathematical thinking in children prior to their entry into formal schooling. Piaget, as well as many other researchers who followed him, argued that mathematical cognition – the construction of mathematical concepts – does not happen only when children are already able to operate with abstract symbols typical of formal learning, but that, on the contrary, the mathematical thinking of children begins prior to formal education, and is initially characterized by mental representations which require the concrete presence of entities and the transformations undergone by these entities. In other words, the suggestion is that mathematical cognition is, in principle, informal, because it operates with cognitive objects which are non-symbolic-formal and requires experience with the physical world.

Other studies suggest that these early mathematical concepts, informal in nature, seem to be important for the further development of more complex skills and understandings present in the higher grades of the educational system (BAROODY, 2000; BAROODY, 2003; MIX; HUTTENLOCHER, LEVINE, 2002; NUNES; BRYANT, 1996). Therefore, it is important to investigate the trajectory of cognitive development from the informal knowledge which is refined by means of the social, cultural and schooling experiences, leading to the construction of formal mathematical concepts and procedures. Regarding early mathematical knowledge, i.e, that present during the period of early childhood education – in which there is no formal teaching of mathematics – studies carried out with hearing children in this age group have shown that, before the beginning of formal schooling, children develop quantitative-numerical concepts with a non-verbal /non-symbolic basis, and with a verbal/ symbolic basis, which will later be involved in the acts of counting and calculating. For example, several studies by Kelly Mix (MIX, 1999; MIX; HUTTENLOCHER; LEVINE, 2002) on the development of the understanding of quantitative equivalence showed that, initially, even before they enter school and learn to count, 3-year-old children develop concepts for representing quantitative equivalence in a non-symbolic way. The quantitative equivalence is important to understand the cardinal value of numbers, because when judging that two sets – one with three small toy cars and the other with three apples – are numerically equivalent, children are abstracting numerical information and ignoring perceptual information. Mix, then, through her experiments showed that, at first, the child makes equivalence judgments based on perceptual data of similarity, i.e., the greater the similarity, the easier it is to realize equivalence – two black marbles and two black plums. Only later, around 4 and 5 years of age, do children begin to use both perceptual information and cardinality information to guide their judgments of equivalence.

Aside from these studies on the development of judging equivalent quantitave sets, there are numerous studies which show that children informally and gradually develop various skills and mathematical ideas, such as the procedures involved in the act of counting and the function of this act (BRIARS; SIEGLER, 1984; FUSON, 1988; FUSON; RICHARDS; BRIARS, 1982; FUSON; SECADA; HALL, 1983; FUSON, 2000; GELMAN; GALLISTEL, 1978; GALLISTEL; GELMAN, 1990; SHIPLEY; SHEPPERSON, 1990; SIEGLER; ROBINSON, 1982; WYNN, 1990, 1992), the ideas of quantification, the concepts of arithmetic and the additive and multiplicative logics (BAROODY, 1992, 2000, 2003; BISANZ; LEFEVRE, 1992; MIX; HUTTENLOCHER; LEVINE, 2002; NUNES; BRYANT, 1996; PIAGET;

SZEMINSKA, 1952). Additionally, they build concepts about ordinal relations and the nominal functions of numbers (WIESE, 2003).

As one can see, there is a vast complexity of mathematical knowledge which is developed during the period of early childhood education and which has been recorded by a huge volume of publications. These studies investigated quantitative-numerical skills in hearing children, who are the most represented population in early childhood education centers. However, there are minority groups of children with different cognitive and linguistic profiles, who are not represented in studies on the development of mathematical concepts. This is the case, for example, of deaf children who do not process auditory stimuli and produce and understand language in visual-spatial modality (sign language). To date, no studies on the development of mathematical concepts and procedures have been done with Brazilian deaf children at preschool age. Therefore, it seems necessary to investigate the trajectory of development of mathematical ideas in deaf children of that age, because there is a big gap in this area. This study, therefore, aims to remedy this gap through a comparative experimental investigation, which will be detailed later. Its investigative focus is the mathematical knowledge and procedures which deaf and hearing children informally have in the years of early childhood education.

This study is also justified by the data coming from academic research done in other countries which indicates a tendency of academic failure by deaf children in mathematics in the higher grades of primary education. The data in question are related to various studies and statistics of academic performance with the use of standardized tests (KLUWIN; MOORES, 1989; NOGUEIRA; ZANQUETTA, 2008; NUNES; MORENO, 1998; WOOD; WOOD; HOWART, 1983; TRAXLER, 2000), which showed that deaf children have a lower or below average performance in mathematics in comparison to hearing children of the same grade and age.

For example, Traxler (2000), when analyzing the performance of deaf students in the U.S. in the new edition of the standardized test Stanford Achievement Test (SAT 9th edition) – which was administered according to the level of each student, after a screening for detecting the appropriate level – found a much below average performance on the subtests of Mathematical Procedures and Mathematical Problem Solving. The performance levels of deaf students indicated a delay of two years at the age of 8 years (with a performance equivalent to that of 1st graders). This delay increases from three to four years at the age of 11 years (with a performance equivalent to that of 3rd graders), and six to eight years at ages between 17 and 18 years (with a performance equivalent to that of 5th graders).

Also, other studies have shown that difficulties in mathematics continue through university for deaf students, especially with regard to the solution of mathematical problems. For example, Kelly et al. (2003) detected a delay in the ability of deaf college students to solve arithmetic problems which involve comparison. In another recent study of the visual representation of mathematical problems, the results of Blatto-Vallee et al. (2007) showed that deaf secondary and college students use very little visual representation, compared to hearing secondary and college students. When using visual representation, deaf students create representations of pictorial and iconic aspects, which are, however, irrelevant to the solution of the problem. Another study conducted by Ansell and Pagliaro (2006) showed that deaf children aged 5 to 9 years have difficulties in solving mathematical problems which are presented in the context of stories in which they need to calculate differences, even when such problems are presented in sign language.

As these difficulties in mathematics occur and seem to pervade the schooling of deaf children, it is necessary to investigate whether the problems with mathematical knowledge are already present before formal schooling, i.e., in early childhood education. During early childhood education, do deaf children develop mathematical concepts and procedures informally following a temporality approximate to that of hearing children? Or are there time delays which may negatively influence the subsequent development? The dearth of studies on the development of mathematical concepts and procedures done with deaf children at preschool age leaves open this and many other questions. For example, the issues involved in the acquisition of the counting procedure, whose development begins informally in hearing children when they are approximately 2 years old. The few existing studies with that focus suggest that deaf children have difficulties in learning the numerical sequence used for counting (LEYBAERT; VAN CUTSEM; 2002; NUNES, 2004; ZARFARTY; NUNES, BRYANT, 2004). Such studies indicate that perhaps the difficulty in acquiring the numerical sequence can cause problems in the future development of mathematical skills which are important in the higher grades. There is, however, no conclusive evidence to say whether the difficulty in acquiring the numerical sequence occurs due to issues of cognitive processing (HITCH; ARNOLD; PHILIPS, 1983), or due to limited access to social and cultural experiences involving counting at home and at school (NUNES, 2004).

There are other issues related to the quantitative-numerical knowledge of deaf children which also need to be investigated. For example, we know that it is common for hearing children to make some coordination mistakes matching one-to-one (reciting the numeral and pointing at the same time) during the acquisition of the counting procedure. However, we know nothing about the counting errors of deaf children. It seems important that teachers know what are the types of counting errors most frequent among deaf children who use sign language for communication, because this way such professionals will be better prepared to organize a program of support and intervention to assist these children overcome such common difficulties. We also need to know how the knowledge of the numerical sequence influences the performance of deaf children in numerical tests. Moreover, it is necessary to have more information about the relationship between sign language and numerical knowledge in deaf children.

Regarding this latter point, researchers have been busy investigating how the language produced and understood in the visual-spatial modality (i.e., sign language) may contribute to the cognitive development of the deaf, considering the cognitive aspects which are more dependent or less dependent on linguistic stimuli. In cognitive functions less dependent on linguistic stimuli, deaf and hearing children seem to have a similar development. This hypothesis has been reiterated by several studies in the área, which showed that deaf children have a time and a trajectory of development similar or even superior to those of hearing children in non-linguistic cognitive functions such as face recognition, constructions with logic blocks, perception of motion, spatial memory and spatial localization (BEVALIER et al., 2006; BLATTO-VALLEE et al., 2007). Even though the emergence of these functions does not depend on linguistic stimuli, these researchers explain that the superiority in the development of such cognitive functions by deaf children was attributed to the use of sign language, which, for its visual-spatial characteristics, can contribute positively to the development of skills of handling information presented visually and spatially (BULL; BLATTO-VALLEE; FABICH, 2006; BLATTO-VALLEE et al., 2007). In this case, there are arguments showing the possibility of a close relationship between language and cognitive processes.

But there are several factors that complexify the relationship between language and cognitive processes in the case of deaf children. Among these factors, we can highlight the heterogeneity of profiles of the deaf (BARBOSA, 2009). For example, there are deaf individuals who were born in a deaf family and, consequently, have deaf friends and a deaf community around them. This family and social context ensures the exposure of those individuals from birth to a language code which is used by the members of their families and communities. Experts have argued that the fact that individuals are exposed from birth to linguistic stimuli has markedly positive effects in their development (QUADROS, 1997; MAYBERRY, 2002). On the other hand, there are deaf individuals who do not have access to linguistic stimuli during the first years of life, for social, cultural, familial and economic reasons. This second group represents the majority of the deaf population in Brazil (QUADROS, 1997). As an additional aggravating factor, the deaf population that has delayed access to sign language also has delayed access to appropriate education, and may thus have different profiles of development. Therefore, deaf children who are not exposed to linguistic stimuli and do not receive proper education at the appropriate age may not demonstrate the similarity and/or superiority in the development of certain cognitive functions documented in the studies already mentioned.

In summary, the record of a few studies in Brazil on the mathematical development of deaf children in early childhood education as well as the issues raised by recent research led to this research. This study is not intended to answer or exhaust all the issues raised here, but it intends to arouse interest in promoting good levels of academic performance in mathematics for all children.

Methodology

Experimental methodology

The present study aimed to investigate the performance of deaf and hearing children aged 5 and 6 years (early childhood education) through experimental tasks which include various cognitive aspects related to quantitative-numerical conceptualization. Among these issues are: a) mental representation of quantity; b) memorization and reproduction of an ordered sequence; c) spontaneous use of numerals in narratives; d) knowledge of numerical sequences; e) counting; f) understanding of cardinality; g) arithmetic; and h) knowledge of the number line.

The methodology was based on experimental clinical interviews with the use of tasks specially formulated for the investigation of early mathematical skills and procedures at issue in this study. There was a concern to develop and previously test experimental tasks which could be used with both hearing children and deaf children, so as not to undermine the comparative basis. That is, in the translation of the tasks into sign language, great care was taken not to convey numerical information through gestures, avoiding thus possible *facilitation* in the assignment of the tasks. In this article, the results of the experimental tasks mentioned will ground the discussions on general comparative aspects, such as the performance of the various groups in skills which are more dependent or less dependent on linguistic stimuli and the characterization of the counting errors observed. The hypothesis was that the numerical and quantitative aspects, which are not dependent on linguistic stimuli, could then present the same development among different groups of participants.

In comparative studies, it is sought to build roughly equitable bases of comparison between groups. In this study, in particular, because it involves children with different profiles of development – which is the case of the intrinsic heterogeneity of deaf children and hearing children –, creating groups for the control of important variables such as age, schooling, cognitive and language skills is a very complex task. I chose to pair the groups of deaf and hearing children based on age and schooling. Such option may have hampered the research, which will be pointed out later in this article.

Participants

As said, pairing deaf children with hearing children in experimental studies is always problematic due to the large diversity of cognitive profiles of children. For this reason, chronological age pairing seemed appropriate. However, even this option had problems due to the disparity found between deaf children in relation to their ages, their grades and time attending school. In other words, six-year-old deaf children who participated in the study were still starting their second year in early childhood education. In children's public centers, there were no longer six-year-old children because they had entered formal schooling. However, as the study was conducted in the year of transition of primary education to nine years, there was a public early childhood education center which retained 6-year-old children. Taking advantage of this opportunity, the research chose to test both a group of children one year younger than the deaf children and a group of the same age. Another variable that seemed important to research was the type of schooling, i.e., public or private. This is because research in Brazil has pointed to the disparity in academic performance between different social classes attending different school systems (PINTO; GARCIA; LETICHEVSKY, 2006).

Thus, forty-three (N = 43) preschool children participated in the study and were divided into four groups:

- group 1: eleven (N = 11) deaf children (profound deafness), averaging 6 years of age;
- group 2; eleven (N = 11) hearing children from public schools, averaging 5 years of age;
- group 3: ten (N = 10) hearing children from private schools, averaging 5 years of age;
- group 4: eleven (N = 11) hearing children from public schools, averaging 6 years of age.

As it can be seen, the composition of the groups of hearing children was used to control the variables of age (a year younger or the same age) and schooling type (public and private schools). For optimal viewing of the age groups, see Table 1.

Table 1 – Participants and average ages in months

Groups	N	Minimum	Maximum	Average	Standard deviation
Group 1: deaf child; 6 years old public school	11	61.00	90.00	73.54	8.58
Group 2: hearing child; 5 years old public school	11	59.00	68.00	63.09	3.33
Group 3: hearing child; 5 years old private school	10	61.00	71.00	66.40	4.11
Group 4: hearing child; 6 years old public school	11	69.00	80.00	73.72	3.03

For children to participate voluntarily in the study, their parents and/or guardians signed a consent form. Those who brought their children to be evaluated in the laboratory of the university were reimbursed for transportation expenses.

All the deaf participants attended public early childhood education centers and were educated in Libras. The knowledge of at least one year in Libras was set as the criterion for participation in the study.

None of the participants, deaf or hearing, received formal instruction in mathematics at school, and only practiced counting while playing.

Procedures

Each child participated individually in two sessions of approximately 40 minutes each, separated by an interval of one week. A deaf graduate student who uses Libras as her native language was trained in the experimental tasks of the study and conducted the sessions with the deaf children in Libras. The principal investigator conducted the sessions with the hearing children. All sessions were filmed to ensure greater accuracy of data collection and data analysis.

As shown in Table 2, the study was comprised of 14 experimental tasks: 1) nonverbal quantitative pairing, 2) reproduction of visible sequential order, 3) reproduction of invisible sequential order; 4) description of visual stimuli; 5) recitation of numerical sequence to the highest number known; 6) counting loose objects; 7) counting sets; 8) counting actions; 9) and 10) cardinality with homogeneous and heterogeneous objects; 11) equivalence of numerical transformation; 12) addition; 13) subtraction; and 14) knowledge of the number line. Tasks 1 through 5 were the first part of the study, in which non-symbolic quantitative knowledge and the knowledge of numerical sequence were investigated. Tasks 6 through 13 were the second part of the study, which focused on numerical knowledge, which has a high demand of language from the participants.

Table 2 – Games used in the research project

Skills	Games
<u>Session 1:</u> **Mental representation of quantity**	1- **Nonverbal production of the following quantities:** (1, 2) 3, 4, 6, 8 items *“Look at what I’m going to do.” “Do yours like mine.”; “Is yours like mine?” “What can you do for yours to be the same as mine?”*
	2 - **Reproduction of quantities following a serial memory:** (2) 3, 4, 6 items (*dinosaur, banana, truck, grape, airplane, frog, rabbit, boat, orange, sheep, car, button, bear,* etc. ...). Prepare six sets (three for children and three for the researcher) with the exact number of pieces, but only give the pieces to children when they are to reproduce the set shown. There will be no comparison in this activity.
	3- **Non-visible reproduction of quantities following a serial memory:** (2) 3, 4, 6 itens.
	4 - What do you see? The researcher shows the child a card each time containing stickers of objects, and asks: *What do you see?* The cards have two sets with two different conditions. In the first, there are six letters presented in a standard organization (S.O.), reminiscent of the organization present in the dice, and six cards in the random organization (R.O.). The aim of this experimental task is to investigate the use of vocabulary containing numerals in narratives and whether such use can be stimulated in S.O. or R.O formats.
	5- Count to the highest number you know. Counting data were used to create groups of knowledge of the number sequence: the basic group counted from 01 to 10; the intermediate group counted from 11 to 59; and the advanced group counted from 60 to 100. These count levels were correlated with other numerical skills in quantitative analysis.
<u>Session 2:</u> **Counting**	***6 - Count the objects:*** 3, 6, 10, 15 items. How many objects have you counted? ***7 - Count these pictures:*** 6, 10, 15, 30, arranged in a horizontal line. How many pictures have you counted? ***8- This puppet will jump a few times. Look. How many times has he jumped?*** Counting actions: 3, 4, 6, 10 hops.
Cardinality	9- ***Give me X***. With cubes of one color. ***10 - Give me X. With colorful bears:*** (1, 2) 3, 4, 6 e 10.
Arithmetic operations	***11- Where are there more (ou fewer)?*** Task with the marbles adapted from CMA. **12- Addition** with objects, but with non-visible results: 3+1; 4+2; 7+3; 1+3. **13- Subtraction** with objects, but with non-visible results: 3-2; 4-1; 7-3; 10-1.
Number line	***14- What number comes after X?*** (3, 7); *What number comes before X?* (4, 6).

The deaf children were also tested on their knowledge of Libras to correlate it with mathematical knowledge.

Data Analysis

Initially, this project involved 14 deaf children. However, three of them were eliminated and the remaining composed a group of 11 deaf children. The reasons for elimination were: a child had residual hearing and used oral language in communication; the other two, who were 6 years old, had no understanding of Libras and, therefore, had very deficient communication. In the groups of hearing children, there was no kind of exclusion from the study. Qualitative and quantitative analyzes were conducted. Children's performance on the tests was computed at two levels: (1) points for correct answers and (2) coding responses for qualitative analysis. The quantitative score was used in comparative analyzes of variables by means of ANOVA test, considering the four groups as independent variables and test performances as dependent variables.

Results

In the experimental tasks which focus on quantitative representation with a non-linguistic basis, there were no statistical differences between the groups of deaf and hearing children in *Non-Verbal Production of Quantities – 3, 4, 6 & 8*, $F(3, 39) = 1.81$, $p = .161$; *Reproduction of Visible Serial Order – 3, 4, & 6*, $F(3, 39) = .617$, $p = .608$; and *Reproduction of Invisible Serial Order– 2, 3, & 4*, $F(3, 39) = 1.59$, $p = .205$. Table 3 shows the means and standard deviation of the groups in these respects. This means that deaf and hearing children have the same level of numerical representation when the stimulus is non-linguistic.

As expected, there are no differences between deaf and hearing children in relation to non-symbolic quantitative skills. Thus, as regards the capacity to judge quantities as equivalent or to mentally represent and reproduce certain sets using perceptual information, there are no differences between deaf and hearing children of early childhood education. However, as can be seen in Table 3, there were differences between groups in the time taken to replicate a sequence, and deaf children made no mistake, but took longer to reproduce the sequence. This means that, during childhood education, deaf and hearing children demonstrated the same non-symbolic quantitative capabilities. Therefore, this result excludes the possibility that deaf children are cognitively deficient in forming their non-symbolic quantitative concepts.

Nonetheless, when the quantitative, numerical, symbolic knowledge was evaluated, i.e., when the use of symbolic numerical representation was measured by counting, arithmetic and number line tasks, there was a significant change. Deaf children performed well below average and statistically differently from some groups of hearing children, but not from all of them, as it is the case of five-year-old children from public schools.

It is noteworthy that the sores of the five-year-old hearing children from public schools scores in numerical tests were so low as the ones of the deaf children. These data, thus, suggest an uneven performance in both groups in comparison to the five-year-old children from private schools and six-year-old children from public schools. The implications of this result will be discussed later.

The data of the counting tasks (counting objects, pictures and actions) were combined to create the category *Counting*, which appears in Graph 1. ANOVA test revealed a difference between the four groups of participants, $F(3, 39) = 12.05$, $p < .001$. The profile of the differences is the same as seen previously, that is, deaf children do not differ from 5-year-old hearing children from public schools, because both groups have significantly lower performance compared to the other groups. All participants had

Table 3 – Results of ANOVA test

Experimental tasks	Average	*F*	Sig.
Nonverbal production of quantity	1.95	1.81	.161
Correct judgment of equivalence	2.19	1.55	.215
Number of times s/he employed the action of repairing the set	.46	1.52	.222
Reproduction of total visible serial order	.43	.617	.608
Average time of visbile serial order	292.80	5.11	.004
Reproduction of total invisible serial order	1.55	1.59	.205
Average time of invisbile serial order	133.57	4.43	.009
Number of times s/he used numerals (spontaneous language / standard organization)	24.15	5.25	.004
Number of times s/he used numerals (spontaneous language / random organization)	283.98	10.0	.000
Total: counting loose objects	10.76	9.83	.000
Total: counting pictures	12.71	10.2	.000
Total score: addition (max. 4)	7.21	6.02	.002
Total score: subtraction (max. 4)	4.97	3.41	.027

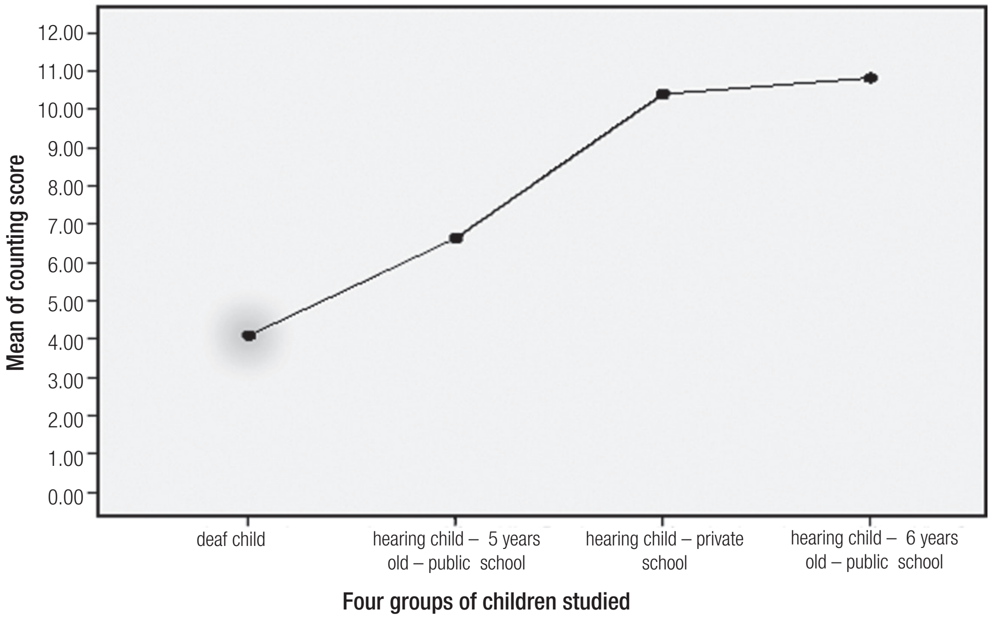
more difficulties in counting pictures than in counting objects. This might be justified by the high demand placed on the coordination between *pointing* and *counting* when there are fixed sets horizontally aligned.

In a still initial analysis of the data of counting errors, it was possible to perceive that deaf children make more mistakes related to numerical sequence. Furthermore, deaf children have a lower threshold for counting than hearing children. That is, in the present study, it was observed that the vast majority of six-year-old deaf children know how to count up to the numeral 10 making a one-to-one matching, i.e., children start counting with the hand closed and open the fingers, one at a time as they count. If the set to be counted has values greater than that expressed by the numeral 10 (limit of fingers on the hands), as it happened in one of the experimental tasks in which there was a picture with 30 pictures to be counted, deaf children who only know how to count to 10, when reaching this limit, stop and say *it's over* or recount three times up to 10, without adding the result at the end of counting to inform the cardinality of the set. This counting strategy is iconic and non-symbolic.

No error in the use of counting procedures was observed in children aged 5 from private schools, neither in children aged 6 years from public schools. But hearing children aged 5 from public schools made mistakes in all the forms of counting and all sets. Their mistakes, however, are more related to the coordination between counting and pointing as well as to cardinality, i.e., it was common for hearing children to count a set and inform a different cardinality from the one verbally counted. No deaf child made this type of cardinality mistake.

In general, the counting results suggest that both deaf and hearing children aged 5

Graph 1 - Average score between groups



Source: Reseach data.

from public schools seem to have difficulties in employing counting procedures. If not worked on in school, such difficulties can negatively influence the learning of mathematics (BAROODY, 2000; FUSON, 2000).

When analyzing the correlation between the linguistic knowledge of deaf children about sign language and their ability to count, I realized that there is a direct influence of language skills in the ability to count. Statistical tests revealed a positive correlation in which the children who have the most knowledge of LIBRAS in the group of deaf children are those who have the best performance counting F (1,9) = 7.73, p = .021, rs (9) = .68, p = .021. The correlation coefficient r ^{2} indicates that the knowledge of Libras explains 40% of the variation in counting scores.

In addition to counting, other quantitative-numerical skills of deaf children are strongly correlated to the knowledge that these children have of sign language. That is, children who have longer exposure to Libras and a greater degree of fluency also have higher performance on arithmetic and cardinality tests. This seems to demonstrate a relation between language and concept formation.

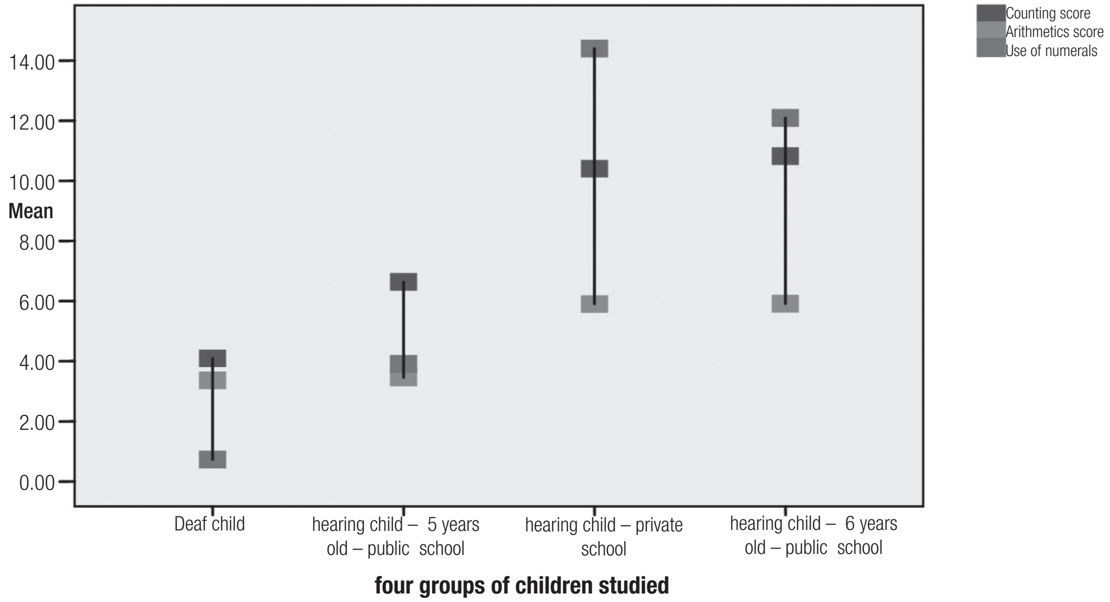
The same correlation was found among the hearing children. For example, the children with smaller numerical vocabulary documented by the task of using the numeral in the narrative were the children younger than 5 years from the public early childhood education center, and they had lower performance in the other tests. The children younger than 5 years from the private early education center had a large numerical vocabulary and their performance was significantly higher in all the tasks in comparison to the other groups.

In the arithmetic tasks of addition, there was a significant difference between groups, F (3,39) = 03.06, *p* = .002. The statistical post-hoc test Tukey HSD found that deaf children and hearing children aged 5 from the public schools performed similarly lower than children in the other groups. These two groups performed more poorly compared to 5-year-old hearing children from private schools (*p* = .004, d = 1.62 for the deaf child and *p* = .029, *d* = .52 for hearing children aged 5 years), and children aged 6 years from public schools (*p* = .028, d = 1:33).

In the arithmetic tasks of subtraction, there was a significant difference between groups, *F* (3,39) = 3:41, *p* = .027. But, interestingly, in the subtraction there was no difference between the performance of deaf children and that of the other groups. In general, deaf children found subtraction easier than addition. There was a difference between hearing children aged 5 years from public schools and older children aged 6 from public schools (*p* = .036, *d* = 1.36.)

In summary, these results suggest that deaf children and hearing children in early childhood education have the same skills in numerical and quantitative representation with a non-symbolic basis, but differ in skills which require quantitative numerical symbolic knowledge. For better visualization of these results, see Graph 2.

Graph 2 - Counting, arithmetic and vocabulary averages for numerals between groups



Fonte: dados da pesquisa.

Final Thoughts

The present study revealed no differences in mental quantitative non-symbolic representations of deaf and hearing children. That is, when the use of verbal counting or other knowledge of formal symbolic order are not required, both deaf and hearing children have the same abilities of representation of quantitative information. Regarding quantitative symbolic skills, the profile is more complex. Deaf children in general performed more poorly compared to hearing children one year younger (5 years) from private early childhood schools, as well as in relation to children of the same age (6 years) from public schools. But the performance of deaf children was equivalent to that of children of five years of public schools. These data are surprising and have very important implications for the teaching of mathematics in early childhood education and for the development of mathematical thinking in children.

One of the implications that can be drawn is that deafness is not a cause of low performance in mathematics (NUNES, 2004) because hearing children also showed low performance in the tasks assigned. In this case, the results confirm the hypothesis of Nunes (2004) that deafness can put children at risk of having difficulties in learning mathematics. However, it is critical to note that the data from this study showed that such risk is also experienced by children aged 5 of the working classes who attend public early childhood education centers, as shown in Graph 2. So what do the deaf and hearing children aged 5 from public schools who participated in the study have in common?

Results showed a lack of vocabulary to express mathematical and numerical information in both deaf and hearing children aged five years of the working classes. As this study did not aim to investigate the causes of a reduced mathematical vocabulary, I shall limit my analysis of the datum, which highlights two important factors to be considered. One is the close relation between mathematical thinking and language; the other is the sociocultural nature of language. Both have also been evidenced by other similar studies such as the research on the Amazonian Pirahã indigenous group. The Pirahã are Amazon Indians whose vocabulary does not have any form to accurately express quantities, not even the quantity *one*, but who have demonstrated they are able to represent numerical equivalence when the sets are physically present in a non-symbolic form, without a memory demand (GORDON, 2004; FRANK et al., 2008). The conclusion drawn by these studies suggests that one needs to have numerical vocabulary to accurately remember larger quantities, even if the concept of exact quantity is not created by language. According to such argument, numerical vocabulary works as a cognitive tool which helps the individual to control cardinal information of sets with a large number of items. Thus, we can realize the close connection between language and mathematical concepts.

It seems that the argument explains the results of this study with deaf and hearing children aged 5 of the working classes. That is, the lack of numerical vocabulary may have affected the performance of these children in tasks which require memory of the cardinal information of the number. Therefore, it seems important to invest in an education program which develops vocabulary to express mathematical ideas. Such vocabulary includes both the numerical sequence and the lexicon for expressing order (*first, second, third,* etc..), value (*more than or less than, greater than or less than*), equivalence (*equal to*) and other mathematical relations.

It is noteworthy that the data showed that even if hearing children from public schools experience difficulties in mathematics, they seem to overcome them with longer schooling, since the older children from public schools, aged 6 years, had a good performance. However, the performance of the older children from public schools seems to be a year down that of the children from private schools. This scenario is extremely worrying as it shows that there are different experiences of schooling in Brazil according to social classes.

Thus, to reduce the academic achievement gap in mathematics between deaf and hearing children and between children of different social classes, educational programs are needed in early childhood education and in the early grades of primary education to ensure conditions for the development of the informal quantitative numerical knowledge of these children. Huge efforts and investments need to be allocated to improve the mathematics education received by deaf and hearing children coming from disadvantaged social classes in order to meet the needs of the public early childhood education centers. The poor performance of the two groups shows the need for immediate action by the government to improve the learning and performance of these children, who are at high risk of academic failure. Data from this study suggest, for example, that deaf and hearing children would benefit from a program of teaching of mathematics that uses concrete and visual materials, which must be connected and grounded in their strong quantitative skills of non-symbolic basis. Deaf children would also benefit from education conducted in their native language, Libras. In addition, both groups would benefit from a program with emphasis on the acquisition of the quantitative-numeric lexicon, given the correlation documented here between language and concept formation. If children do not have the vocabulary to express mathematical ideas, their development in this area may be compromised. Therefore, it is essential that children be taught the quantitative numerical mathematical vocabulary in a meaningful way.

This study has provided some important information about cognitive areas in which children may be at greater risk of having difficultes in learning mathematics. However, more research on the mathematical cognition of deaf and hearing children are absolutely necessary for the elucidation of processes which may be affected by the lack of access to a language model. In future studies, it is also important to create and test methodologies which enable pairing and control by linguistic variation, because only then can we be more certain about the results of comparative studies carried out with deaf individuals.

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Received on September 13th, 2012

Approved on June 27th, 2013

Heloiza H. Barbosa is a researcher at *Universidade Federal de Santa Catarina* (UFSC), in the Graduate programs of Education (PPGE) and Scientific and Technological Education (PPGECT). She holds a PhD in education from Boston University. Her research was funded by CNPq.