



# Optimizing the production scheduling of fruit juice beverages using mixed integer programming models

## *Otimização da programação da produção de bebidas à base de frutas por meio de modelos de programação inteira mista*

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### Abstract

This study considers a production lot-sizing and scheduling problem of non-alcoholic fruit juice beverages. The problem is characterized by a multi-period planning horizon, a production process with different machines, capacity constraints and setup times independent from the production sequence, as well as special conditions regarding required machines clean-in-place (CIP) within production time periods. To deal with this problem, we propose a solution approach based on mathematical programming and using optimization software. The models are modifications of mixed integer programming models known in the lot sizing and scheduling literature. However, instead of considering multiple parallel machines, the proposed models explore other possible configurations of machines to properly represent the production processes involved in producing fruit beverages. The proposed approach is validated by a study carried out in a fruit juice company located in the interior of Sao Paulo State, Brazil, in which the solutions obtained by the models were tested and analyzed in real situations at the company. The results show that the approach is potentially good for practical applications.

**Keywords:** Non-alcoholic fruit juice beverage industry; Production planning; Mixed integer programming; Lot sizing and scheduling problem.

### Resumo

Neste estudo, considera-se um problema de dimensionamento e programação de lotes de produção de bebidas não alcoólicas à base de frutas. O problema é caracterizado por horizonte de planejamento com múltiplos períodos, processo de produção com máquinas distintas, restrições de capacidades de produção e tempos de preparação das máquinas independentes da sequência de produção, além de condições especiais de preparações, como limpezas obrigatórias das máquinas dentro de limitações de tempo de produção. Para tratar o problema, propõe-se uma abordagem de solução baseada em modelos de programação matemática e uso de *softwares* de otimização. Os modelos são modificações de modelos de programação linear inteira mista conhecidos na literatura de dimensionamento e programação de lotes de produção. Porém, ao invés de considerar múltiplas máquinas em paralelo, os modelos propostos exploram outras possíveis configurações de máquinas para representar apropriadamente os processos de produção envolvidos na produção de bebidas à base de frutas. A abordagem proposta é validada por meio de um estudo realizado em uma fábrica de sucos e néctares de frutas no interior do Estado de São Paulo, em que as soluções obtidas pelos modelos foram testadas e analisadas em situações realistas da empresa. Os resultados mostram que a abordagem tem bom potencial de aplicação prática.

**Palavras-chave:** Indústria de bebidas não alcoólicas; Planejamento da produção; Programação inteira mista; Dimensionamento de lotes.

## 1 Introduction

Over the last few years, there has been a significant growth in the production of non-alcoholic drinks in Brazil (ABIR, 2010; Diário Econômico, 2011). The potential increase in the consumption of these products, the

growth of the number of items manufactured by the industry, as well as the competition and demands of this market has resulted in companies being concerned about improving their production processes, especially

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in areas directly involved with production, such as production planning and control. In 2015, it was estimated that there will be a growth in the production of the entire drink sector, according to the Beverage Production Control System - SICOBE (SICOBE, 2005). One of the recurrent challenges in this sector is to obtain efficient production programs, which should consider the time available for production, the input availability, the demands of every period, the machines to be set up and the changeovers between items to be produced. It is also important to take into consideration the synchrony between the two main production phases: preparing syrups and packaging beverages. Some studies about non-alcoholic drink companies in Brazil, more specifically concerning the production of soft drinks and other carbonated drinks (Toledo et al., 2007; Ferreira et al., 2009) and fruit juice drinks highlight the difficulty in determining effective production plans (generally manually).

During the development of this research, various visits to typical companies of the beverage industry in the State of São Paulo were made. During these visits, the production managers and programmers of these companies reaffirmed the difficulty in taking lot sizing and scheduling decisions. Such challenges were also observed in fruit juice drink production lines, which are the object of this study. Having the collaboration of these companies, we chose one of these lines in order to meet the objectives of this study. The selected company has typical characteristics of other production lines of beverage companies. The fruit-based drink production line was selected because this type of drink has shown increasing trends over recent years (ABIR, 2010). The consumption of ready-to-drink fruit-based beverages is still below the consumption of soft drinks. On the other hand, this is seen by some authors as an indication of its growth potential (Pirillo & Sabio, 2009).

The lot sizing and scheduling problem in the fruit juice beverage industry consists of determining the lot size of different types of fruit juices (items) to meet the demands in each period of a finite planning horizon. Each beverage production line has its own characteristics. In the fruit juice line studied in this work, the setup time, for instance, is not sequence dependent, unlike other beverage production lines. In most studies, such as in carbonated drinks, the setup operations in the production lines are widely dependent on the sequence of production (Toledo et al., 2007; Ferreira et al., 2009). In addition, in fruit juice production, tanks and filling machines must be cleaned within production time constraints. It is mandatory to clean the lines (this is called CIP - Clean-in-Place) after a certain time of production, regardless of whether there is change of beverage flavor or not in the fruit juice line. If there is a change of flavor (item), the same cleaning is required, and the cleaning time does

not vary with the flavor of the product. A preliminary study in a fruit juice production line can be found in Leite (2008), where machine setups do not depend on the result of production batches.

The lot sizing problem and its extensions have been widely investigated by the scientific community and by consultants related to businesses, motivated by applications in industrial environments. Comprehensive reviews about lot sizing can be found in Drexl & Kimms (1997), Karimi et al. (2003), Jans & Degraeve (2008), Buschkühl et al. (2010) and Glock et al. (2014). Among the lot sizing models, linear models such as the Capacitated Lot Sizing Problem (CLSP) and the General Lot Sizing and Scheduling Problem (GLSP) stand out. CLSP is characterized by being big bucket, meaning that several items can be produced in each period. The problem considers that the limited resources are used for production and to set up machines. The CLSP with capacity constraints and setup times can be found, for example, in Trigeiro et al. (1989). The GLSP was proposed by Fleischmann & Meyr (1997) and Meyr (2002). It consists of determining lot sizes of several products and scheduling them in a single machine or parallel machines subject to capacity constraints. Each macro period is divided into smaller periods called micro periods. The duration of each micro period is so short that only one type of item can be manufactured in it. Hence, we obtain the sequence in which each item should be produced in each macro period.

The lot-sizing and scheduling problems have been widely used in several productive sectors (e.g., Araujo et al., 2008; Almada-Lobo et al., 2008; Luche et al., 2009; Toso et al., 2009; Santos & Almada-Lobo, 2012; Marinelli et al., 2007; Kopanos et al., 2010, among many others). Some studies are found in the production of non-alcoholic drinks such as soft drinks, water, tea and juice, but most of these studies consider soft drink production lines. For example, Toledo et al. (2007) presented an integrated model for a lot sizing and scheduling problem in a soft drink industry based on the GSLPPL (GSLP with parallel machines) from Meyr (2002). The production involves two interdependent levels with decisions concerning the storage of raw materials and bottling beverages. The aim was to determine the design and programming of raw materials in the tanks to produce syrups and bottle packaging in the lines where times and costs depend on the type of item previously stored and packaged. In the bottling lines, the setup time and cost depend on the type of previously stored and bottled item. Other related studies can be found in Toledo et al. (2009, 2014, 2015).

Clark (2003) explores several heuristic solution approaches for a mixed integer programming model, based on the CLSP, to address the production problem in a beverage company that fills cans and bottles,

and the process of filling the mixed liquid in cans was chosen for a pilot project. In Rangel & Ferreira (2003), a mixed integer programming model is also developed, nonetheless based on GLSP, for a lot sizing and scheduling problem in a soft drink company with sequence-dependent setup costs and setup times. In Ferreira et al. (2009), these studies have been extended to deal with cases in the soft drink industry with several filling lines and with several available tanks to prepare syrups. Regarding the problem, the production bottleneck alternates between the liquid preparation stage and the bottling one, therefore synchronization of the production between these stages is required. Other related studies appear in Ferreira et al. (2010, 2012) and Defalque et al. (2011).

Leite (2008) is the only study found in the literature review that deals with the production problem of a line of fruit beverages. A preliminary study was conducted in a beverage industry to find out about the particularities of the production lines involved, and a model based on Ferreira et al. (2009) and the GLSP were developed to analyze it. In this study, three identical filling machines serviced by a single tank were identified. Thus, the three machines can fill only one type of drink flavor at a time.

In this paper, we consider a lot sizing and scheduling problem in beverage plants which produce fruit beverages with some similarities to the problem studied in Leite (2008). However, although the characteristics of the product are the same (with setup time and setup cost that are non-dependent from the production sequence), the attributes of the production line are different as the process is different and involves various filling machines and more tanks, mixers and pasteurizing, as presented below. In this paper, the filling machines will be presented as packaging machines for the sake of simplification. The problem is initially represented by a mathematical model based on GLSPPL (Meyr, 2002), used to avoid the problem of maximum lot sizes in these types of companies, due to mandatory cleaning within production time constraints. The obtained solution by the model consists of determining the lot sizes of items and the production sequence, however these lots can be interchanged within each period without loss of generality, due to the fact that the setup is sequence independent of the production of the items. Considering the previous model, a simpler model is developed to represent the problem based on the CLSP. This new model also considers all characteristics of the production process such as mandatory cleaning, but simplifies decisions about scheduling lots. Both models are solved by CPLEX software using generated examples based on realistic data from the company with planning horizons of a few weeks.

The structure of the paper is as follows. The next section briefly describes the production process of

fruit beverages according to the reality of the studied company and also provides information about how the production planning is carried out in this company. The proposed mathematical models are presented in Section 3. The computational results are described in Section 4. Finally, in Section 5, the conclusions and perspectives for future research are discussed.

## 2 Production process

This research was supported by the cooperation of a factory in São Paulo. This factory produces soft drinks and fruit juice drinks. The need for a more effective production planning technique was observed due to the demand and type of product, especially on the line related to the group of non-carbonated and fruit juice products. This line is the market leader in its sector, and in this study it is renamed Line 1. Line 1 produces five different flavors of fruit juice drinks (pineapple, orange, passion fruit, strawberry and grape) and it is different from the other lines of the factory due to the fact that its setup time is sequence independent, unlike the soft drink lines, for example. On the other hand, the product has characteristics whereby the tanks and bottling machines need to be cleaned every 48 hours, which is not absolutely necessary for the other drinks. The cleaning process consists of different cycles that are recirculated through the tanks, pumps, valves and other equipment in the process flow. It is important to note that once the line has been cleaned, production should be started as soon as possible because the line is not perfectly hermetic. Production can be continuous, which means the production can be resumed in the last period.

The production of drinks generally has two main stages: liquid flavor preparation and bottling (packaging). In the first stage, the main raw material, the liquid flavor or syrup, is prepared in tanks of different capacities. The tanks should work with a minimum liquid quantity to ensure its homogeneity. To properly mix the necessary ingredients, the tank propeller must be completely covered. In the second stage, the liquid is bottled in the production lines. A production line is made up of a conveyor belt and machines that wash the bottles, fill them with a combination of liquid flavor and water (carbonated or non carbonated) and then seal, label and pack them.

The two stages are shown in Figure 1. The syrup and water are put into the mixer in order to homogenize the two components. The mixture is then put into the preparatory tank and, afterwards the buffer tank. The preparatory tank and the buffer tank maintain the mix homogeneous as both have helixes at the bottom of the tank. The purpose of the preparatory tank is to keep the buffer constantly full, therefore serving the purpose of an intermediate stock. In other words, although limited, it is as though the buffer tank had always enough capacity (infinite), because it is

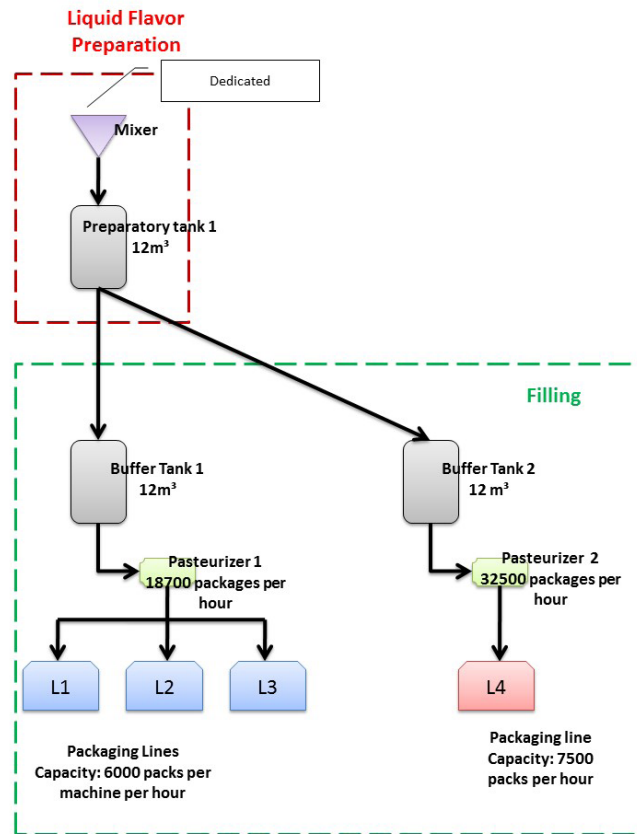


Figure 1. Line 1.

constantly provided by the preparatory tank. It takes approximately two hours to fill the preparatory tank for the first time and fifty minutes to fill the buffer. After that, the supply is constant. The bottleneck of this line production is the packaging machines. In Figure 1, the four packaging machines are called L1, L2, L3 and L4.

It can be observed in Figure 1 that the preparatory tank has a capacity of 12m<sup>3</sup> and it supplies two buffer tanks with capacities of 12 m<sup>3</sup> each. Buffer tank 1 only serves the L1, L2 and L3 machines, each one with a packaging capacity of 6,000 packages per hour, and buffer tank 2 supplies the L4 machine, with a capacity of 7,500 packages per hour. Pasteurizers 1 and 2 shown in the figure, whose function is to destroy bacteria, have various capacities. The maximum capacities are 18,700 packages per hour and 32,500 packages per hour, respectively. On the other hand, these maximum capacities are not reached because the L1, L2 and L3 machines have total nominal capacities of 18,000 packages per hour. The cleaning process of these packaging machines is generally simultaneous and it always takes more time than cleaning the tanks, therefore when the machines are ready to work, the buffer tank will be full.

In this study, we also studied a change in the productive process described in Figure 1 including

a new bottling machine, called L5, with a packaging capacity of 11,200 packages per hour, supplied by buffer 2 with machine L4. Buffer 2 also supplies the L4 machine. This process with the L5 machine went into the adaptation phase in the company while developing this study, therefore it is also considered in the study. However, not all the combinations of the machines are feasible for operation in Figure 1 or in the second configuration of this production process with machine L5. For example, with a limited capacity, it is not feasible to produce less than 12,000 packages/hour, which means that at least two of the machines L1, L2 or L3 must be connected (if machines L4 and L5 are off). Similarly, it is not feasible to produce only using machine L4 or at least machine L5 connected (if machines L1, L2 and L3 are turned off), both with smaller capacities than 12,000 packages/hour.

As can be seen, buffer tank 1 supplies the L1, L2 and L3 packaging machines, and buffer 2 supplies the L4 machine as shown in Figure 1 and the L4 and L5 machines in the modified configuration. It is worth mentioning that although the buffer tanks have the same capacity, the capacities of the pasteurizers are different: the pasteurizer connected to machines L1, L2 and L3 have a maximum capacity to process 18,700 packages of 200 ml per hour, while the pasteurizer connected to machines L4 and L5 have a processing capacity

of 32,500 packages per hour. The washing process of all the machines is generally done at the same time and takes four hours. According to the production programmer, it would take approximately two hours to clean only one machine, while the cleaning time of all the machines is four hours. Therefore, there was no reason to carry out the washing process in only one machine at a time.

The buffer tanks supply two different groups of packaging machines. Once we consider that the capacity of the tanks is large enough for the purposes of scheduling the production, the problem considered in this article can be considered as single-stage, considering that there is always enough syrup to supply the packaging machines. Hence, we approach the problem of lot sizing and scheduling the fruit based drinks considering a mixer and a preparatory tank that supply two buffers as shown in Figure 1.

The production scheduling in the company is done manually. The demand data of the 5 products are obtained through a second company (which is a partner to the same business group) that purchases products made by Line 1. Considering the demand data, the production programmer distributes the obtained demand of the month in weeks. These demand data are shared with the liquid preparation sector. These amounts are then passed to the syrup sector, which calculates the amount of kits needed to meet the demand. A relevant feature of Line 1 is that production lots generally last 48 hours. Production flexibility is achieved by changing the number of machines designated to that “batch” and by the speed of the pasteurizer. Stock quantities do not interfere in the production planning.

The stock created by the production planning is controlled by a partner of the business group that monitors the stock levels and labor distribution in the factory. Therefore, the task of optimizing lot sizing and scheduling is not affected by stock numbers. Consequently, the product programmer aims to meet the demand considering setup times without backlogging, but with no focus on inventory. On the other hand, it is advantageous to schedule lots that last at least 48 hours in order to avoid unnecessary setups.

The flexibility of the lot size production consists of sending one of the packaging machines for maintenance and reducing the speed of the pasteurizer, so that the product is not wasted by excessive cooking. Therefore, this results in the lot of 48 hours having a lower number of products. The programmer forwards the schedule to the production line supervisor, who places the quantities in lots of 48 hours. However, it is not always possible to meet the demand in lots of 48h. In this case, the production supervisor asks for the lowest nearest multiple demand and consults the programmer about the possibility of producing another lot.

Fractions of kits can be used in the plant, which is the measurement of syrup used. The minimum fraction is pre-set at 0.25 kit, as there must be a minimum amount of liquid in the tank so that the helix for the mixture can rotate. However, the fraction normally used by the production programmer is 0.5 kit, except for the strawberry and orange flavors. The correspondences can be seen in Table 1. These fractions are used because the tanks have a capacity of 12m<sup>3</sup>. If the calculated amount does not provide a whole number of fractions of kits or batches, the manager in charge of the syrup makes the largest whole number of batches contained in the calculated amount and consults the scheduler about making one more batch to cover the fraction or leave it for the following week, i.e. rounding it up or down.

For example, consider that the programmer sent a production of 120,000 boxes of grape flavor, which is equivalent to  $12 \times 120 \times 10^3$  units of 200mL = 1,440,000 packages of grape flavored drinks (288,000 liters) to the manager in charge of the syrup. According to Table 1, 14.4 kits would be needed. In this example, the manager in charge of the syrup would consult the programmer to know if they would use 14.5 or 14 kits of the grape flavor. In batches, this would mean 29 or 28 batches, respectively. For details of this fruit drink production process, please consult Pagliarussi (2013).

### 3 Problem modeling

In this section, the one-stage models to represent the fruit juice drink production planning are described. Model I is based on GLSPPL and Model II is based on CLSP. Model I can approach the production problem in order to maximize the contribution margin or minimize inventory, backlogging and setup costs. The two approaches are respectively called Model Ia and Model Ib. Model Ia is proposed taking into consideration the suggestions made by the company’s production manager, whilst Model Ib is more common in the lot sizing literature.

The main difference between Models Ia and Ib is the objective function. Model II also aims at minimizing production costs, similar to Model Ib. Model solutions can be interpreted as feasible production plans, optimizing a given criteria and providing lot sizes

**Table 1.** Flavor, batch and kit proportion.

Flavor	Amount produced by kit (L)	Batch (kit fraction)
Orange	12,000	1
Grape	20,000	0.5
Passion fruit	24,000	0.5
Pineapple	24,000	0.5
Strawberry	16,000	0.75

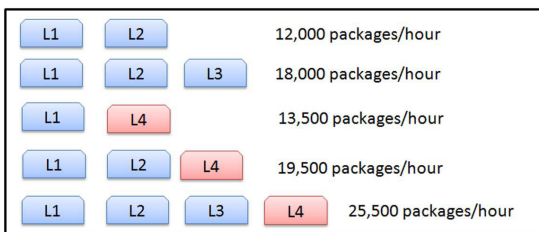
of each drink flavor in order to meet demands. Every model presented in this study takes into consideration sequent independent setup times.

It is mandatory to consider that it takes at least two packaging machines to make production viable in Line 1, such as shown in Figure 1. Therefore, in the following mathematical models, we consider parameter  $M$  as the set of all possible combinations of operating machines. Taking Figure 1 as an example, where machines L1, L2, L3 and L4 are presented, the viable combinations are shown in Figure 2. Every combination represents machines for which the sum of the packaging capacity is equal or more than 12,000 packages per hour. Other combinations including machine L5 are defined similarly. Since machines L1, L2 and L3 are equal, some of the combinations are symmetric and can be left out (i.e. any combination regarding two of the following L1, L2 or L3), without loss of generality. Thus, the cardinality of set  $M$  is 5, that is:  $M = \{(L1,L2); (L1,L2,L3); (L1,L4); (L1,L2,L4); (L1,L2,L3,L4)\}$ .

Another aspect that should be taken into consideration when developing models is the synchronization of production machines. According to the production programmer, all machines start working at the same time. However, there may be some breaks or pauses during the production runtime. L1, L2 and L3 are synchronized machines most of the time as they are connected to the same pasteurizer, but the timing of L4 and L5 machines with L1, L2 and L3 is adopted in the model as a combination for the sake of simplicity.

The model parameters and data inputs are:

$T$	number of micro periods, $t \in T$
$J$	number of liquid flavors, $j \in J$
$S_t$	set of micro periods in each period $t$ , $s \in S_t$
$M$	set of machine combinations.
$a_m$	processing time of one unit using machine combination $m \in M$ (not dependent on the flavor)
$Lmax$	maximum production limit, measured in time units
$d_{jt}$	demand for item $j$ in period $t$
$b_j$	changeover time for item $j$ to another



**Figure 2.** Viable combinations considering four packaging machines.

$q_j$	minimum lot size of flavor $j$ to ensure the liquid homogeneity
$Kt$	total time capacity in period $t$

### 3.1 Model I

Model I is based on the GLSPPL model (Meyr, 2002), but with some modifications in order to include the maximum production constraint. Model I represents a situation where there is one preparatory tank linked to two buffer tanks, connected to both pasteurizers. After the pasteurization process, the next stage is packaging, using machines L1, L2 and L3 connected to pasteurizer 1 and machines L4 and L5 connected to pasteurizer 2. At first, in this study, information obtained in the factory was used, such as not taking into consideration either inventory costs or idle machine costs. Backlogging costs were not available at that time. Hence, the fruit juice based drink production planning can be stated as follows. Define the lot sizes and production scheduling taking into account the fruit juice drink demands and the capacity of packaging machines, so that the profit margin is maximized (Model Ia), as suggested by the factory production manager. Afterwards, we altered the objective function in order to minimize the overall costs, which is discussed further (Model Ib)

Model Ia - additional parameters

$p_j$	profit margin of item $j$
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Variables

$x_{jmts}$	production quantity of item $j$ in machine combination $m$ in micro period $s$ of period $t$ ;
$I_{jt}$	inventory for item $j$ at the end of period $t$ ;
$z_{jmts}$	1, if item $j$ is produced in machine combination $m$ during micro period $s$ of period $t$ ; 0, otherwise.

Model Ia

$$\max \sum_{j \in J} \sum_{m \in M} \sum_{t \in T} \sum_{s \in S_t} p_j x_{jmts} \tag{1}$$

Subject to:

$$I_{j(t-1)} + \sum_{m \in M} \sum_{s \in S_t} x_{jmts} = I_{jt} + d_{jt} \quad \forall j \in J, \forall t \in T, \tag{2}$$

$$\sum_{m \in M} \sum_{j \in J} \sum_{s \in S_t} a_m x_{jmts} + \sum_{m \in M} \sum_{j \in J} \sum_{s \in S_t} b_j z_{jmts} \leq K_t, \quad \forall t \in T, \tag{3}$$

$$x_{jmts} \geq q_j z_{jmts}, \quad \forall j \in J, \forall m \in M, \forall s \in S_t, \quad \forall t \in T, \tag{4}$$

$$\sum_{m \in M} \sum_{j \in J} z_{jmts} \leq 1, \quad \forall s \in S_t, \quad \forall t \in T, \tag{5}$$

$$x_{jms} \leq \left( \frac{L \max}{a_m} \right) z_{jms} \quad \forall j \in J, \forall m \in M, \forall s \in St, \forall t \in T, \quad (6)$$

$$z_{jms} \in \{0,1\}, x_{jms} \geq 0, I_{jt} \geq 0, \forall j \in J, \forall m \in M, \forall s \in St, \forall t \in T. \quad (7)$$

The objective function (1) maximizes the total contribution margin. Constraints (2) are the inventory balance equations and together with  $I_{jt} \geq 0$ , for all  $j$  and  $t$ , they ensure that the demand is met without backlogging. Constraints (3) represent the capacity constraints in each period, that is, the time spent on production and on setup cannot exceed the available time in that period. Constraints (4) establish a minimum lot size of production to meet the minimum amount of liquid inside the preparatory tank for the helix to rotate. The choice of machines and the product to be produced in each micro period are ensured by constraints (5). Constraints (6) avoid the production time longer than  $Lmax$  and also ensures that production can occur if the combination of  $m$  machines is setup to item  $j$  ( $z_{jms} = 1$ ). When a flavor is produced for a period greater than  $Lmax$ , cleaning will be required after this period and then production can continue. These constraints also ensure cleaning when there is a change of products. The domain of the variables is defined by constraints (7).

In order to prevent symmetric (i.e., equivalent) solutions in the micro periods, the following constraints were added to the mathematical model:

$$\sum_{j \in J} \sum_{m \in M} z_{jmt(s-1)} \geq \sum_{j \in J} \sum_{m \in M} z_{jms} \quad \forall t \in T, s > 1. \quad (8)$$

Constraint (8) was proposed by Fleischmann & Meyr (1997) to avoid redundancy in the original GLSP: if a sequence of micro periods within a period is assigned to the same product, the production quantity of this lot or part of this could be distributed arbitrarily among these micro periods without changing the scheduling and the value of the objective function (idle micro periods distributed).

It is worth mentioning that in Model Ia, variable  $z_{jms}$  indicates, by constraints (4), that there will only be production if its value is equal to 1. However, despite being described as a binary variable which indicates production or not, in constraints (3) it is linked to the setup time, which ensures that every time there is production, the tanks will also be cleaned and the machine preparation time will be counted within the capacity.

To illustrate a solution to Model Ia, a small-size instance is considered. The instance contains 2 items, 2 machines with the same capacity, 2 combinations of machines, 3 periods with 10 micro periods each, and  $K = 144$  h. Since the two machines have the same capacity and considering that  $(L1, L2) = (L2, L1)$ , it follows that:  $M = \{(L1); (L1, L2)\}$ .

The parameter  $a_m$ , the time spent on production using the combination of machines  $m \in M$ , was calculated using the result of dividing number 1 by the nominal capacity of the combination of machines, in packages per hour, and then multiplying it by 12. The problem considers, as units of the decision variables, boxes with 12 units. As the values can be very small numbers, the value of  $a_m$  was multiplied by 1,000, i.e., the values correspond to the processing time of a thousand boxes of 12 units. Considering the first part of the set in Figure 2, the combination (L1, L2), the parameter  $a_2$  is determined, which is the time needed to produce one thousand boxes with 12 units each. For this combination, the processing time has a duration of 1 hour.

$$a_2 = \frac{1}{(6,000 + 6,000)} \times 12 \times 1,000 = 1h.$$

Thus, for Model Ia,  $(a_1, a_2) = (0.5; 1)$ . The solution for Model Ia for the illustrative example is shown in Figure 3. Note that the solution uses the full capacity (144 hours) and is represented in Figure 3. The graph contains the values of the main decision variables and the duration of the micro periods. The absolute value of variable  $x$ , in boxes with 12 units, is also presented. Then, its value, in units of time, is obtained by multiplying it 0.5, which is the selected processing time by Model Ia in this example.

We emphasize that although the model shows solutions with the lots in a defined sequence, the sequence can be interchanged in the period due to the fact that the setup is sequence independent. Note that the productions are placed in the first micro periods (1, 2 and 3, in this case), which is ensured by constraint (8) that prevents symmetric solutions. The objective function value in this example was 792 boxes of 12 units of 200mL of product each. This value can be conferred by adding all the productions  $x_{jms}$  shown in Figure 3.

The computational results reported in Section 4 with Model Ia show that many of the instances have no feasible solution, i.e. no production plans that meet all demands without backlogs, which led to the change of the objective function in Model Ia to allow backlogging (delay in delivering products), as discussed below.

### 3.2 Model Ib

As mentioned in Section 3, Model Ib was changed in order to minimize the overall production costs, such as setup, inventory and backlogging costs.

- $h_j$  inventory cost of item  $j$
- $g_j$  backlogging cost of item  $j$
- $cs_j$  changeover cost of item  $j$

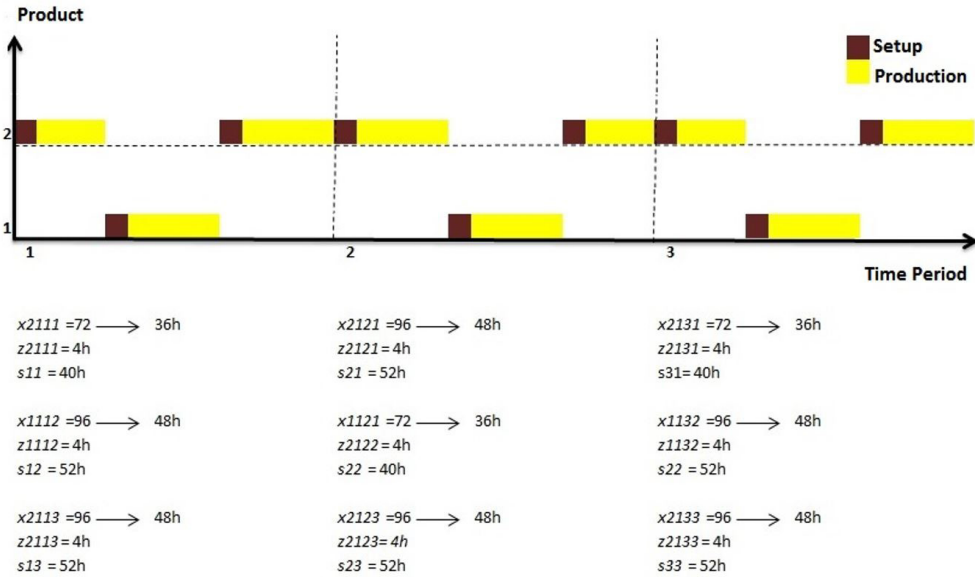


Figure 3. Solution for Model Ia.

Model Ib - additional variables:

$U_{jt}$  Backlog of item  $j$  at the end of period  $t$

Model Ib

$$\min \sum_{j \in J} \sum_{m \in M} \sum_{t \in T} \sum_{s \in S_t} cs_j z_{jmts} + \sum_{j \in J} \sum_{t \in T} (g_j U_{jt} + h_{jt} I_{jt}), \quad (9)$$

Subject to (3), (4), (5), (6), (8) and:

$$I_{j,t-1} - U_{j,t-1} + \sum_{m \in M} \sum_{s \in S_t} x_{jmts} = I_{jt} - U_{jt} + d_{jt}, \forall j \in J, \forall t \in T, \quad (10)$$

$$\begin{aligned} z_{jmts} \in \{0, 1\}, x_{jmts} \geq 0, I_{jt} \geq 0, U_{jt} \geq 0 \\ 0 \forall j \in J, \forall m \in M, \forall s \in S_t, \forall t \in T \end{aligned} \quad (11)$$

Note that the objective function (9) now minimizes the overall production costs such as setup, inventory and backlogging. Constraints (10) are the inventory balance equations, and now balance production, inventories and backlogs with demand. Constraints (11) describe variable domain.

### 3.3 Model II

Although the Models Ia and Ib show solutions with the lots in a defined sequence, the sequence can be interchanged in the period due to the fact that the setup is not sequence dependent, as mentioned before. Therefore, the next question is relevant: Why should a model based on the GLSPPL be used, which in the most general case, provides sequencing decisions, instead of basing it directly on a simpler model, for example, on the CLSP, which does not include such decisions? Thus, a simplified model was developed, representing the same problem, but disregarding the

sequencing decisions. The parameters and decisions variables of Model II remain the same as Model I, with the exception of:

Variables:

$x_{jmt}$  quantity of beverage  $j$  produced in the combination of machines  $m$  in period  $t$   $\left( x_{jmt} = \sum_{s \in S_t} x_{jmts} \right)$ ;

$z_{jmt}$  number of changeovers performed to produce the beverage  $j$  in the combination of machines  $m$  in period  $t$   $\left( z_{jmt} = \sum_{s \in S_t} z_{jmts} \right)$ .

In Model II, the concept of micro periods is not used, as the sequence in which the lots are produced in the period can be altered without loss of generality. Note that the variables in Model II can be seen as simple replacements of variables  $x_{jmts}$  and  $z_{jmts}$  of Model I.

Model II:

$$\min \sum_{j \in J} \sum_{m \in M} \sum_{t \in T} cs_j z_{jmt} + \sum_{j \in J} \sum_{t \in T} (g_j U_{jt} + h_{jt} I_{jt}), \quad (12)$$

$$I_{j,t-1} + \sum_{m \in M} x_{jmt} + U_{jt} = I_{jt} + d_{jt} + U_{j,t-1}, \forall j \in J, \forall t \in T, \quad (13)$$

$$\sum_{m \in M} \sum_{j \in J} a_m x_{jmt} + \sum_{m \in M} \sum_{j \in J} b_j z_{jmt} \leq K_t, \forall t \in T, \quad (14)$$

$$x_{jmt} \geq q_j z_{jmt}, \forall j \in J, \forall m \in M, \forall t \in T, \quad (15)$$



$$x_{jmt} \leq \left( \frac{L \max}{a_m} \right) z_{jmt} \quad \forall j \in J, \forall m \in M, \forall t \in T, \quad (16)$$

$$z_{jmt} \in \mathbb{I}, z_{jmt} \geq 0; x_{jmt} \geq 0, I_{jt} \geq 0, U_{jt} \geq 0 \quad \forall j \in J, \forall m \in M, \forall t \in T. \quad (17)$$

The objective function (12) is to minimize the sum of setup costs, inventory holding costs and backorder costs similar to Model I. Constraints (13) and (14) are the inventory balance equations and the capacity limitation of the machine, respectively. Constraints (5) are disregarded in Model II as there is a possibility of producing several products in the same period. The domain of the variables is defined by constraints (17).

In sequence, constraints (15) and (16) are discussed in more detail. When replacing the variables  $x_{jmt} = \sum_{s \in S^t} x_{jmts}$  and  $z_{jmt} = \sum_{s \in S^t} z_{jmts}$ , the minimum and maximum lot sizes constraints, (15) and (16) respectively, (similar to (4) and (6)), became either surrogate or substitute constraints of micro periods  $s$ . For this reason, these constraints are relaxations of the constraints of Model I and apparently do not ensure that each lot of product  $j$ , in the same period  $t$ , will have the minimum lot size  $q_j$  and the maximum size  $\left( \frac{L \max}{a_m} \right)$ . In this case, Model II could produce an infeasible solution to the problem defined in Section 2. However, in the computational experiments, a problem instance was not found where this happened. In fact, it is always possible to divide a large lot  $x_{jmt}$  obtained by Model II (if it exceeds the maximum lot) by the number of setups performed in this solution, that is,  $z_{jmt}$ , and, to obtain lots smaller than  $\left( \frac{L \max}{a_m} \right)$ . The same argument is valid for the constraint of the minimum lot ( $q_j$ ).

Consider the following numerical example:  $q_j = 2$  (minimum lot size of product  $j$ ) and  $\left( \frac{L \max}{a_m} \right) = 10$ .

Assuming a solution in which the lot size of item  $j$  is  $x_{jmt} = 24$  and the number of setups is  $z_{jmt} = 3$ , the result is the quotient of 8. This number is higher than the minimum lot size and lower than the maximum lot size, therefore it fulfills the constraints of the minimum and maximum lot sizes. Moreover, we must consider the interpretation of the programmer, who may choose to make two lots of size 10, not exceeding the maximum lot, and another smaller lot (equal to 4). That is, this simple post processing in the solution of Model II produces new lots, which will always be feasible from the point of view of minimum and maximum lots. It should be noted that Model II cannot be regarded as the classic CLSP, due to the fact that variable  $z_{jmt}$  is now integer, instead of binary, as it is considered in the CLSP.

Models Ia, Ib and II differ from the mathematical model in Leite (2008) because they represent a different production process and due to the fact that they treat machines as combinations of machines,

rather than just a configuration as in Leite (2008), thus enabling one or more machines to be stopped if it is more advantageous. Furthermore, unlike the model in Leite (2008), the filling machines are considered the bottleneck in production, while buffer tanks are assumed to have enough capacity to ensure continuous production, as discussed earlier. The model in Leite (2008) considers that the productions should be multiples of an integer value, referring to the kit size and how many liters of beverage the kit produces (Table 1). To address this issue, we consider the minimum lot size constraints in Models I and II.

## 4 Computational tests

Data concerning demands, changeover times between beverages in filling machines and tank capacity among other data necessary to solve the problem were kindly provided to this study by the factory. To perform the models I and II computational tests, we used a computer with an Intel® Core™ i5-2450M CPU 2.50GHz processor and 6GB RAM. The models were solved using IBM ILOG CPLEX 12.6 OPTIMIZATION STUDIO software, with time resolution limited to 3600s. To test Models Ia, Ib and II, a sample provided by the company was extrapolated to generate similar examples.

### 4.1 Data generation

The sample provided by the company had information about demand and production lot sizes and scheduling for two consecutive months. A random generator was implemented to vary the demand at a time horizon of up to 13 weeks, based on the company's sample values. During this time period, the demand varied from 110 thousand boxes containing 12 200mL units of fruit juice beverage to 216 thousand boxes. Therefore, 110,000 and 216,000 are the lower and upper bounds of the generator. As there were rarely more than 2 orders asking for different flavors in the same week, the generator is built to be as near to these conditions as possible, that is, non-zero demand to only 2 products in each period.

Examples were generated with a period time of up to 13 weeks, ( $T = \{3,4,5,6,7,8,9,10,11,12,13\}$ ), although a typical planning horizon is 8 weeks and up to 10 different items ( $J = \{3,4,5,6,7,8,9,10\}$ ). The examples in this study take into consideration 5 different flavors (items) and number of micro periods equal to 10 ( $S = 10$ ). The maximum lot size is limited to 48 h ( $L \max = 48$  h), the machine setup time is 4 hours ( $b_j = 4 \quad \forall j \in J$ ). The maximum period capacity is  $K_t = 144$  h,  $\forall t \in T$ , considering workdays as Monday to Saturday. The parameter  $q_j$  was calculated according to Table 1. The values in Table 1 were converted into one thousand boxes containing 12

200mL units of fruit juice beverage. Parameter  $a_m$  was calculated as described above.

The profit margin was considered equal to 1 for all items, for the sake of simplicity, ( $p_j = 1 \forall j \in J$ ) to test Model Ia. The company was unable to provide production, inventory and backlogging costs, which limited our analysis and comparison of the model solutions. For Models Ib and II, the inventory costs were considered equal to 1 for ( $\forall j \in J$ ). Backlogging costs were considered 10 times inventory costs. This multiplication factor was based on the difference between a remuneration rate of 1% (inventory cost) a month and a margin of contribution of 10% of the product value (backlogging cost). Setup costs were estimated based on the time the machines are paused, i.e., production equivalent to four hours (as an opportunity cost). Hence, every solution presented by Models Ib and II prioritizes minimizing backlogging costs, which are higher than inventory costs and setup costs. This meets the company’s production scheduling targets.

Considering the data, variables  $x_{jms}$  and  $I_{jt}$  are described in one thousand boxes containing 12 200mL units of fruit juice beverage. The computational study is organized as follows. In Section 4.1 we present the results of Model Ia and in Section 4.2, the results of Model Ib. Scenarios generated from the sample obtained in the factory are used in order to compare Models Ib and II in Section 4.3.

### 4.2 Model Ia

The computational tests for Model Ia were conducted taking into account three categories of examples, each containing 70 generated problem instances as shown in Table 2.

Initially, consider category I in a first packaging configuration, varying from 3 to 5 products, and 3 packaging machines (L1, L2, L3). The packaging capacity is 18,000 packages per hour, however without a decreasing demand.

In this category, the tests with the data using Model Ia showed that the solution for the problem is infeasible when using only three machines with a capacity of 6,000 packages per hour each, as would be expected. It is important to note that Model Ia assumes that every demand must be met without a delay. It must be highlighted, once more, that the bottleneck of the production is the packaging machines, as we consider

that the liquid flavor preparation tank capacity always meets the packaging machine demands.

As the solutions were infeasible in category I, we considered the L4 machine in category II, adding therefore a capacity of 7,500 packages per hour. However, it was still not possible to obtain feasible solutions with this configuration for the 70 sample tests. Hence, in category III, we included the L5 machine, adding a capacity of 11,200 packages per hour. In this category, the problem became feasible, meeting demands and providing optimal solutions within the imposed time limit.

Our results point to the fact that the factory expansion plan (i.e. adding one more machine) was beneficial in order to obtain production schedules that do not include any delays in meeting demands. The CPLEX solver solved all instances in category III, presenting optimal solutions within the time limit. This shows the potential of Model Ia to be used by the company on a daily basis. It is important to note that the computational time required to solve the model was less than one second, which means that these instances were easily solved by CPLEX.

### 4.3 Model Ib

The computer setting and the parameters used in this study are the same as the previous tests with Model Ia, as well as the random generator for the item demands. Owing to the fact that the Model Ia instances were solved quickly, the test categories for Model Ib were slightly different. In these new tests, we increased the number of items, surpassing 5. Furthermore, the time frame was increased to more than 8 periods per category, not exceeding a period of 13 weeks, equivalent to a planning horizon of 3 months. Although this study aims to support production scheduling decisions at an operational level (short term decisions), and is not intended to propose a tool for midterm decisions, the number of products and periods was increased in these tests to study the behavior of Model Ib as the number of variables and constraints increase. When the time frame and the number of items go up, the number of constraints and variables are also increased.

The Model Ib tests included instances with time periods varying from 6 to 13, items varying from 5 to 10 and the possibility of using all 5 packaging machines. For each possibility, 5 sample tests were generated, making a total of 65 categories. Within

**Table 2.** Categories of examples used in Model Ia.

Category	Periods	Products	Packaging machines	Instances for a configuration	Total Instances
I	$T=\{3,4,5,6,7,8\}$	$J=\{3,4,5\}$	L1, L2, L3	5	70
II	$T=\{3,4,5,6,7,8\}$	$J=\{3,4,5\}$	L1, L2, L3, L4	5	70
III	$T=\{3,4,5,6,7,8\}$	$J=\{3,4,5\}$	L1,L2,L3,L4,L5	5	70

each problem category, most of the test instances were solved optimally within the time limit of one hour. Categories with time periods 11 and 13 were not included in the previous statement. The fact that the tests with such time frames (when considering 5 different items) were not easily solved reinforced the potential for the model's practical use.

For every test sample for which the CPLEX solver could not find the optimal solution within the time limit, the Gap value was calculated by  $Gap = 100((OF - IL)/OF)$ , where OF is the objective function value of the best solution provided by the CPLEX solver and IL is the best lower bound as given by CPLEX in one hour time limit.

The tests were run with and without constraints (8), and only in 16 out of 65 instances, the solver solved the model more quickly with the constraint active. On average, the tests without constraints (8) were 45% faster. The solutions of the 130 tests, with and without these constraints, showed the same behavior. The gaps were relatively low. For the sake of comparison, this study was used for the 65 instances without constraints (8), as they were solved faster and optimally. Only two instances reached the maximum execution time and the biggest gap was 0.25%.

Table 3 shows the results of the tests using model Ib concerning the use of production capacity and backlog in relation to demand, as well as the stock concerning the demand. In the fifth column of the table, the percentage of capacity is shown occupied by the setup execution (%Setup), while (%Production) represents the percentage of the capacity used for the production of items. %Backorder is the backorder ratio related to the demand in the last period and %Inventory is the ratio of the stock in the last period and the demand in the last period.

Table 3 shows that the average use of the capacities did not exceed 70%, although there is a significantly small backorder ratio. The highest usage of capacity

was 72% in category 2 for one instance. It can be observed that because of the set of constraints (6), minimum lot, it is less expensive to have a minimum of delay that meets all the demand and stock at the end of the planning horizon. These results will be compared with the results from Model II, based on the CLSP, in the next section.

#### 4.4 Model II

In this section, we present a comparison between the test results obtained from Model II when using the problem instances shown in Table 3 of Section 4.2. The solutions achieved by Models Ib and II were similar when considering the objective function values, the utilized production capacities, the ratios between inventory and demand and the ratios between backorder and demand. The comparison between the models' performances is illustrated in Table 4. Note that the number of optimal solutions drawn from each model (N<sup>o\*</sup>) are shown, as well as the average gap and the average running time for each test category. The symbol (\*) indicates that the solver CPLEX was able to solve all test samples optimally.

Both models presented feasible solutions for all test samples. However, Model Ib obtained more optimal solutions than Model II. When the solutions of Model Ib were not optimal, the gaps were less than 0.35%. On the other hand, solutions from Model II could be found within the time limit and they presented a gap of 3.87%.

In category 10, Model Ib presented optimal solutions for all instances. In the same category, Model II was able to find only two optimal solutions and they presented an average gap of 1.80%. On average, Model II was solved three times slower than Model Ib. In general, both models yielded solutions of good quality. However, when we take into account the average time and the number of optimal solutions, the performance of Model Ib was better than Model II.

**Table 3.** Model Ib test results - Solution characteristics

	J	T	N <sup>o</sup> example.	%Setup	%Production	%Backorder	%Inventory
1	5	6	5	2.0	58.37	0.47	0
2	5	7	5	2.3	68.51	0.04	0
3	5	8	5	2.18	59.50	0.67	0
4	5	9	5	2.40	57.44	0.35	0
5	5	10	5	1.98	56.71	0.36	0
6	5	11	5	1.98	57.39	0.64	0
7	5	12	5	2.01	57.43	0.26	0
8	5	13	5	2.04	56.90	0.38	0
9	6	13	5	1.99	57.33	0.19	0
10	7	13	5	2.10	57.33	0.00	0
11	8	13	5	2.06	57.40	0.17	0
12	9	13	5	2.01	56.90	0.41	0
13	10	13	5	2.03	57.17	0.38	0

**Table 4.** Model performance comparison.

	$ J $	$ T $	N° example.	N°* model Ib	N°* model II	Gap GLSP(%)	Gap CLSP(%)	Time GLSP(s)	Time CLSP(s)
1	5	6	5	5	5	*	*	<b>23.99</b>	11.04
2	5	7	5	5	5	*	*	15.99	<b>22.59</b>
3	5	8	5	5	5	*	*	29.51	<b>49.92</b>
4	5	9	5	5	5	*	*	<b>74.87</b>	53.87
5	5	10	5	5	5	*	*	113.60	<b>299.06</b>
6	5	11	5	4	5	0.05	*	<b>731.96</b>	267.21
7	5	12	5	5	5	*	*	<b>345.15</b>	137.03
8	5	13	5	4	5	0.02	*	<b>743.53</b>	365.36
9	6	13	5	5	5	*	*	30.59	<b>212.42</b>
10	7	13	5	5	2	*	1.80	66.74	<b>3,150.72</b>
11	8	13	5	5	3	*	0.11	72.44	<b>1,095.52</b>
12	9	13	5	5	5	*	*	71.91	<b>444.80</b>
13	10	13	5	5	3	*	0.48	313.68	<b>1,837.71</b>

\* indicates that the solver CPLEX was able to solve all test samples optimally.

This was unexpected as Model Ib is less simple because it considers scheduling decisions based on the GLSPPL, unlike Model II which is based on the CLSP.

It is worth mentioning that Model II still requires a post-processing step to review the sizes of the obtained lots and ensure that the solution is feasible from the point of view of the minimum and maximum allowable lots. This post-processing step consists of simply dividing the big lot,  $x_{jmt}$ , (if it exceeds the maximum lot) by the number of performed setups, that is,  $z_{jmt}$ , and therefore, obtaining smaller lots ( $L_{max}/a_m$ ). Model Ib does not need a post-processing step and since the resolution of the instances with it was faster and sometimes resulted in better quality solutions (within the time limit) than Model II, it is concluded (on the basis of the results of this study) that Model Ib is the most recommended one to deal with this problem.

The results obtained from Models Ia, Ib and II were shown to the factory production manager and to the production programmer. As we mentioned earlier, the objective functions of Models Ib and II prioritize minimization of the overall production costs, such as inventory, setup and backorder. However, the objective function suggested by the manager was the one presented in Model Ia.

The solutions from Model Ia were the best according to the production programmer, because they were closer to what was already done in the factory. In the case of Model Ia, the objective function gives priority to maximizing the profit margin and does not consider the costs, therefore its solutions avoid idle time. However, the programmer stated that, considering the solutions in Model Ia, these lots could be too small, and therefore it would be better not to produce these products during these periods. Therefore, it might be better not to make these lot sizes in such time frames. A simple alternative to

avoid this type of solution would be to revise the parameters of Model Ia related to the minimum lot size corresponding to  $q_j$ , for some of these products.

It should be noted that the models presented here were solved by the solver Cplex using the branch-and-cut method and requiring only a few seconds for these samples, which are as large as the ones we were given by the factory. On the other hand, the production programmer takes at least 40 minutes to come up with a feasible production plan. Both the production manager and the production programmer found that the solutions produced by Models Ia and Ib are potentially good to be used in practice. Any adjustments to adapt the models to the specific operational conditions might be of use, especially to obtain a production plan of each week of the company. Interesting future research would be to develop an effective validation of the models using them in the company on a daily basis to better evaluate the advantages and disadvantages of the production schedules generated by the models in comparison with the schedules used in the factory.

### 5 Concluding remarks

This paper presented optimization models based on mathematical programming and applied methods of solution to solve them in order to support the process of taking decisions regarding production planning and scheduling of fruit juice drink lines. A case study was undertaken in a production line of a typical company of this sector. The proposed models used a theoretical framework for the lot sizing problems and to schedule production lots, which are well known and explored in the literature. Considering the results obtained, it can be concluded that the presented models and the tested methods are potentially good to support short term production planning decisions in this industry, for example, considering time frames of 8 or 9 weeks. We believe

that these approaches can be adopted successfully in the company under consideration and also in other companies in this industry, as well as other industrial sectors whose processes are similar. It is important to notice that the criteria for optimizing the lot sizing and scheduling could be different when considering costs related to preparation, inventories and delays, time limits, and minimum and maximum lots size capacities, among other factors.

Interesting future research would be to develop a user-friendly version of Model Ia, so that the production programmer of the company could test and evaluate the efficiency of its solutions on a daily basis. Having such results available, it would be possible to develop an effective case study and proceed to validating this model in real life situations. In this case, it would be interesting to evaluate the performance of the model in production scheduling and rescheduling, using rolling planning horizon approaches. Another motivating line of research would be to investigate effective heuristic methods to solve Models Ia, Ib and II in situations where optimization software, based on branch-and-cut algorithms, such as the one of the CPLEX solver, were not capable of providing good solutions within acceptable computer times.

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