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ORIGINAL ARTICLE

Optimization models as applied to equipment replacement problems: review and trends

Modelos de otimização aplicados ao problema de substituição de equipamentos: revisão e tendências

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Abstract: Equipment replacement is a definitive and relevant engineering decision. The aim of this work was to identify and organize the mathematical optimization proposals and solution search techniques that have contributed toward solving this problem. As a result, we classified the bibliographic materials we identified into seven distinct types of approaches. The work also provides an integrative overview of the level of complementarity of the categories we identified. The network visualization approach represented about 57% of the selected works and is still in use today. Nonetheless, since 2000 other approaches, such as fuzzy logic, real options, and machine learning have increased by 40% and become relevant current trends.

Keywords: Equipment replacement policy; Mathematical programming; Optimization.

Resumo: A substituição de equipamentos é uma relevante decisão de engenharia. O objetivo deste trabalho foi identificar e organizar as propostas de otimização matemática e as técnicas de busca de resultados que têm contribuído para a solução deste problema. Como resultado, classificamos os materiais bibliográficos identificados em sete tipos distintos de abordagens. O trabalho também fornece uma visão integradora do nível de complementaridade das categorias identificadas. A abordagem de visualização em rede representou cerca de 57% dos trabalhos selecionados e ainda está em uso. No entanto, desde 2000 outras abordagens, como lógica fuzzy, opções reais e aprendizado de máquina aumentaram 40% e se tornaram tendências atuais relevantes.

Palavras-chave: Política de substituição de equipamentos; Programação matemática; Otimização.

1 Introduction

To improve the productivity of a company's facilities in order to gain market share and increase profits, one or more pieces of its equipment may be replaced to keep pace with technological advances and the actions of competitors. Keeping or replacing a piece of equipment is common in companies and for people. The equipment replacement decision is usually irreversible, and companies may incur significant costs for several years (Abensur, 2015; Valverde & Resende, 1997).

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The equipment replacement problem (ERP) in particular analyzes how long an asset will work over a defined planning horizon (*H*) until it has to be replaced by another. The solution to the problem, therefore, defines a keep (*K*) or replace (*R*) sequence along the planning horizon that maximizes the economic benefits to the owner (e.g.: *K*, *K*, *K*, *R*).

Figure 1 shows a sequence of replacement decisions and a planning horizon *H*. The asset purchase value (including installation) B_i generates operating costs (maintenance, materials, depreciation, human resources) C_{ij} during its service life until the optimal replacement moment T_i when it will have a residual value VSB_i (selling value, taxes). Operating costs, purchase cost, and residual values are the parameters, while the replacement interval of time is the variable to be determined. The minimum attractive rate is also another significant economic parameter.

From the perspective of mathematical programming, we can regard ERP as a multi-stage economic assessment because the replacement decision can occur at any moment. Multiplicity not only occurs in the sequence of linked periods but also in the number of analyzed assets (e.g., one asset in use versus two substitute assets). Thus, ERP is a multi-stage solution search problem that involves the cash flow manipulation of two or more assets simultaneously. All business fields (aviation, maritime, railway, agriculture, computing, telecommunications, and medical centers) have examples of the application of ERP (Abensur, 2015; Abensur et al., 2019; Altalabi et al., 2020a; Altalabi et al., 2020b; Alves, 2020; Chenna, 2010; Espiritu & Coit, 2008a, b; Ezekafor et al., 2015; Fawcet & Forero, 2019; Feng & Figliozzi, 2012, 2014; Grano & Abensur, 2017; Leung, 1983; Leung & Tanchoco, 1986; Marques et al., 2005; Paolanti et al., 2018; Schwartz et al., 1971; Zvipore et al., 2015).

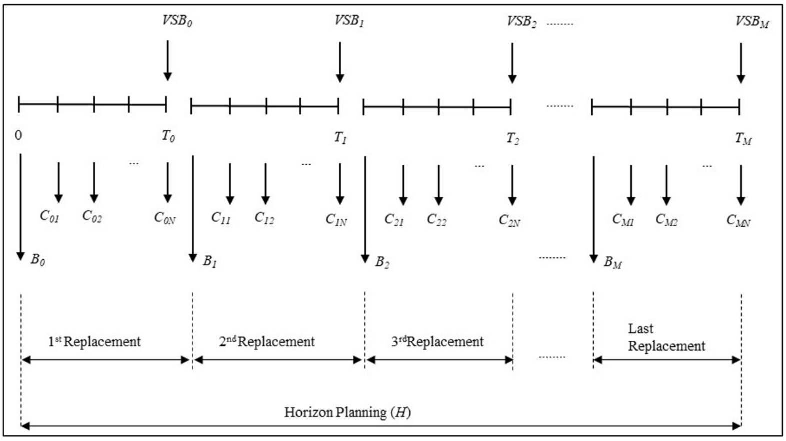

Figure 1. General equipment replacement model.
Source: adapted from Abensur (2015).

Preinreich (1940) published the first approach to the economic decision to replace equipment showing that the economic life of equipment cannot be determined without considering the economic life of the assets in the chain of any future replacements during the planning horizon. Since then, researchers have developed other innovative approaches for improving this topic.

Due to the relevance and frequency with which ERP occurs, procedures based on discounted cash flow (DCF) techniques, such as net present value (NPV), internal return rate (IRR), and equivalent annual cost (EAC) have been customized for making this decision.

Considering operating costs to be the main factor in the replacement analysis of an asset with an estimated working life *n*, the EAC converts all cash flow values into net present values based on Equations 1 and 2, which enable a comparison to be made between assets with different ages by determining the equivalent annuity (EA) (Brigham et al., 2016).

$$NVP_{n,i} = \sum_{j=1}^{n} \left[\frac{-OPC-(MV_j-MV_{j-1})-iMV_j+IR(OPC+Dep)\pm IR(MV_n-(PV-nDep)}{(1+i)^j} \right] \quad (1)$$

$$EA = NVP \left[\frac{[(1+i)^n-1]}{(1+i)^n i} \right] \quad (2)$$

Where:
OPC = the equipment's operating costs
MV = market value of the analyzed assets
IR = income tax rate
PV = the equipment's purchase value
i = minimum attractive rate
Dep = depreciation
n = working life

Nonetheless, DCF techniques evaluate only one replacement event during the planning horizon, so there are significant operational difficulties when it comes to assessing multiple replacements. Besides, each parameter that is changed (e.g., operating costs) requires that the entire method be restarted, thereby making manual replication an exhausting or even unfeasible task.

Bellman (1955) developed the dynamic programming technique (DP) that divided the ERP into a sequence of stages, each one representing a small part of the problem with only one variable. At each stage there are only two alternatives: (i) keep the equipment for one more period; or (ii) replace the existing equipment. DP became the ERP structure that is more comprehensible and with which the analyzed parameters are easier to manipulate. DP also created the appropriate conditions for computing routines.

Since then, due to its multidimensional characteristic, the ERP has been addressing many scientific areas, such as economic engineering, and operations research with sophisticated computing tools, and has become a topic of great interest in mathematical modeling and of considerable combinatorial complexity.

1.1 Objectives and contributions of the study

This work has identified, analyzed, and organized various mathematical models and search algorithms applied to ERP in diverse business areas. It was not the aim of this review to compare the chosen approaches; the strengths and contributions of this work are:

- It identifies mathematical optimization models that expand the traditional economic approach of ERP;
- It classifies the chosen approaches in accordance with the mathematical treatment and the type of the applied search algorithm;
- It organizes and classifies the studies in a comprehensible and accessible way;
- It guides and rationalizes future research efforts;
- It identifies the characteristics, complementarities, and trends of the subject.

Most of the bibliographic databases we researched were freely accessible but did not allow full access to the content of the articles. We assessed many works, therefore, by way of their abstracts. While this was a quantitative restriction of the study, it did not compromise the quality of our conclusions.

Our article differs significantly from Hartman & Tan's (2014) review, mainly because of the incorporation of 34 new citations since their publication. These additional citations enhance the comprehensive treatment of mathematical models in our study, including visual representations. Furthermore, we introduce innovative approaches, such as machine learning applied to predictive maintenance policies, providing a broader perspective for a robust and current analysis of the ERP problem. Together, the two reviews contribute to advancing research in this field.

2 Methodology

Figure 2 shows the steps of this work. As we said, this work aims to identify and organize the mathematical optimization models and solution search techniques that have contributed to ERP. We searched, therefore, for available articles, theses, and dissertations that are regarded as contributing to the topic. Based on the subject definition and the main idea of the research, we selected the criteria we used to search for and select the papers. We used the following databases: Scopus, Web of Science, Google Scholar, and UMichigan library.

Since Bellman (1955), ERP publications can be easily found using the English expression "equipment replacement policy". Other languages, such as Spanish or Portuguese, use "*reemplazo de equipo*" and "*política de substituição de equipamentos*", respectively. We used them, therefore, as search expressions to obtain papers in English, Spanish, and Portuguese.

Mathematics is timeless because old breakthroughs do not change over time (e.g., Pythagoras' theorem). Thus, time restriction was not an additional search criterion. We focused on availability, and whether experts had already assessed them (articles, conferences, dissertations, theses). Originality was also another relevant criterion we used.

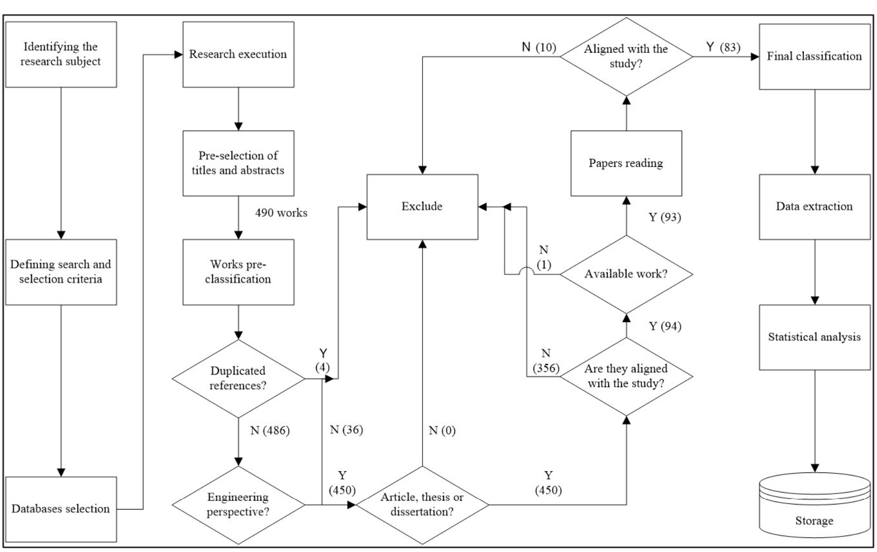

Figure 2. Flow of the steps of the work.
Source: prepared by the authors.

The study's approach should adopt an engineering perspective, focusing on mathematical optimization models and utilizing computational tools, rather than the more traditional management perspective that emphasizes investment analysis. We also considered the replacement of individual equipment components based on mathematical models of preventive maintenance. According to this line of thinking, we excluded papers based exclusively on net present value (NPV), internal rate of return (IRR), or equivalent annual cost (EAC) techniques.

Due to the special conditions of work during the Covid-19 pandemic, we carried out the research without using any special bibliographic search routines (robots). The initial filters of titles and abstracts helped us in determining the alignment of the papers with our study. In cases of positive alignment, we classified them immediately, while the remaining papers were evaluated in the subsequent phase.

Finally, we classified doubtful papers in the last step. We read and classified all works that satisfied the established conditions. We also selected some works for further reading to use them as examples of ERP applications.

3 Results

We classified eighty-three works (88% articles, 7.2% theses, and 4.8% dissertations), the vast majority in English (86.8%). From the perspective of mathematical optimization models using computational tools, the 1950s presented the first results that were influenced by Bellman´s work.

Since then, there has been an increase in the number of published papers because of the dissemination of the topic and advances in computing that have allowed the incorporation of new techniques and data into the ERP (e.g., genetic algorithms). Figure 3 presents the selected papers distributed by year of publication. The Portuguese and Spanish studies started in 2000; before 2000 we found only papers in English.

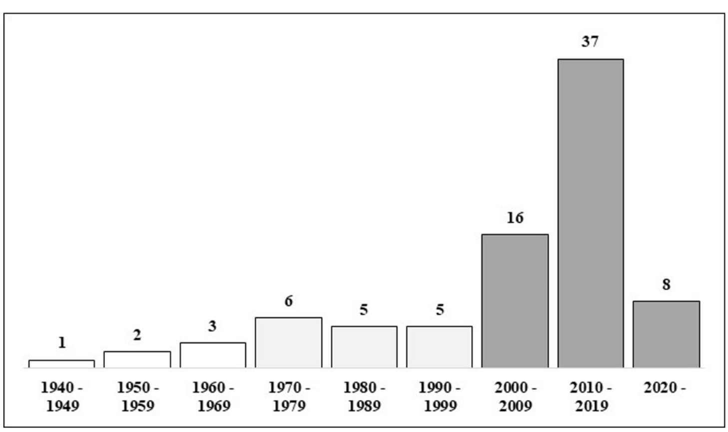

Figure 3. Distribution of the selected works.
Source: prepared by the authors.

Considering the ERP, we have selected the most productive authors. The vast majority of the 157 authors we researched are the authors or co-authors of only one article. The twenty-one authors with more than one published paper are shown in Figure 4 below.

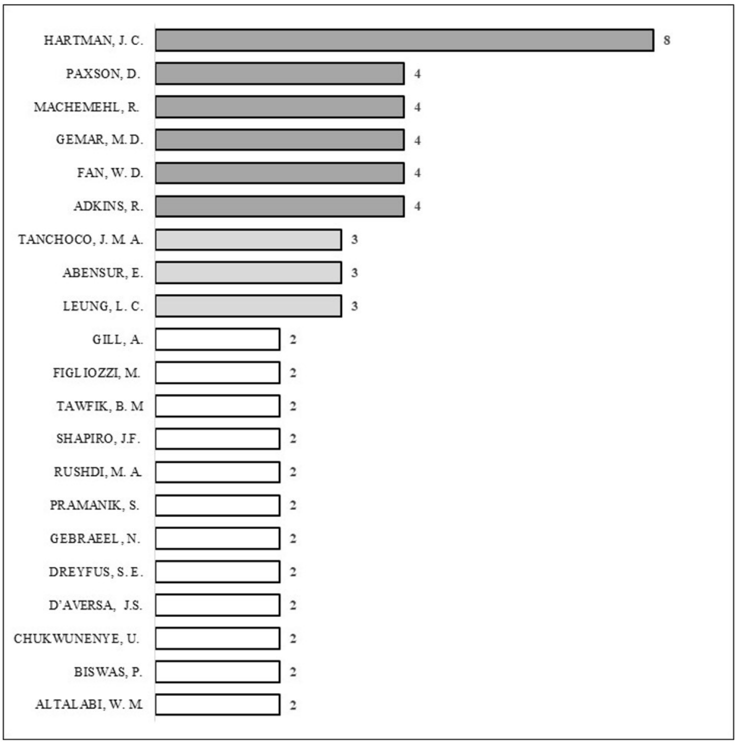

Figure 4. Most productive authors.
Source: prepared by the authors.

Joseph C. Hartman, the author with the most papers published, focuses on dynamic programming and linear programming as applied to the ERP (Hartman, 1998, 1999, 2000, 2001; Hartman & Murphy, 2006; Tan & Hartman, 2010; Hartman & Rogers, 2006; Hartman & Hartman, 2001).

Of the sixty-five searched journals and digital repositories, Figure 5 presents the thirteen with the highest number of published works. The sources comprise a vast variety of areas such as operations research, transportation, applied mathematics, technology, economics, and optimization. The sources with the highest number of works in the databases we searched are: (i) The Engineering Economist; (ii) Journal of the Operational Research Society, and (iii) IIE Transactions.

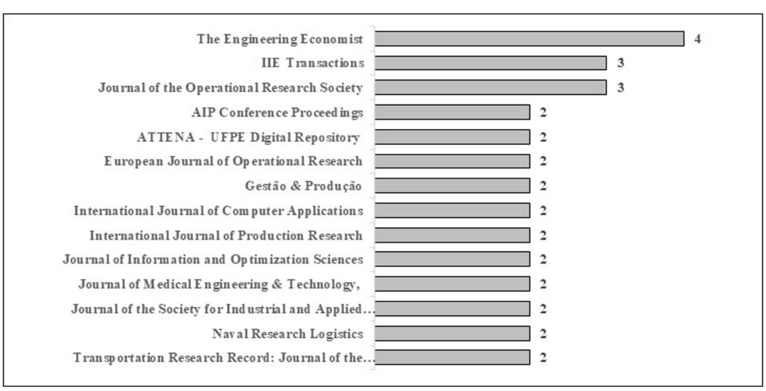

Figure 5. Sources with the highest number of published works (more than one selected work).
Source: prepared by the authors.

We divided the main search techniques into seven categories as follows:

- Network visualization: comprising DP, linear programming (LP), and the shortest path technique;
- Multi-criteria analysis;
- Simulation: comprising metaheuristics (e.g., genetic algorithms) and the Monte Carlo method;
- Real options;
- Continuous functions;
- Fuzzy logic;
- Machine learning: comprising data collection situations in real time.

Figure 6 shows the distribution of works by category that we considered in this review. Some works were classified into more than one category because they applied more than one ERP technique. More than 57% of the studies used the network visualization approach, which has encompassed dynamic programming since 1950. Furthermore, we have identified fourteen distinct application types, as illustrated in Figure 7. It is clear that the automobile and manufacturing industries together account for over 50% of the overall ERP applications.

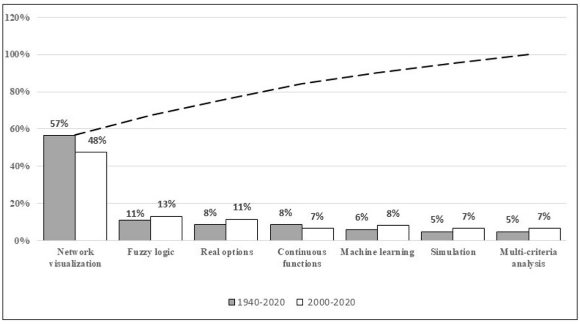

Figure 6. Classification of the selected works.
Source: prepared by the authors.

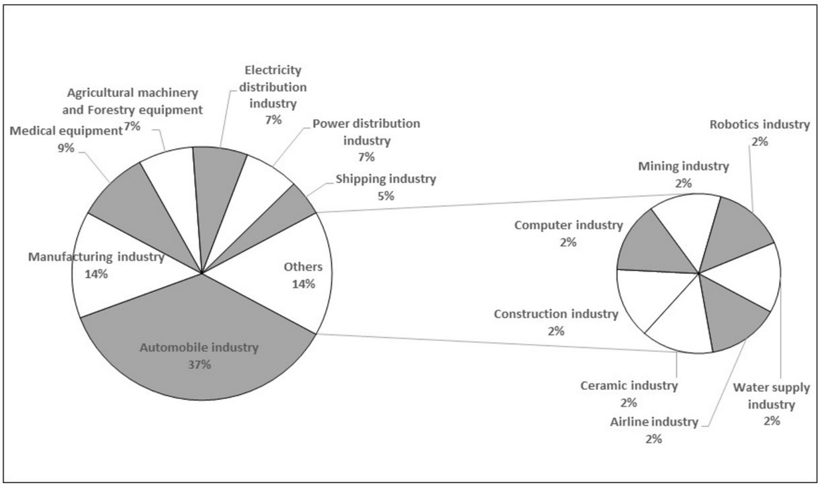

Figure 7. ERP applications.
Source: prepared by the authors.

Figure 8 presents a chronological summary of the evolution of mathematical models applied to ERP. Researchers have been applying dynamic programming for sixty years and it is still one the most important approaches used. Since 2000, new techniques have been applied to ERP and some of them have been combined with DP and LP

Due to computational advances, the collection and treatment of real-time operational data have improved the replacement decision process. These are recent approaches that have been applied since 2000.

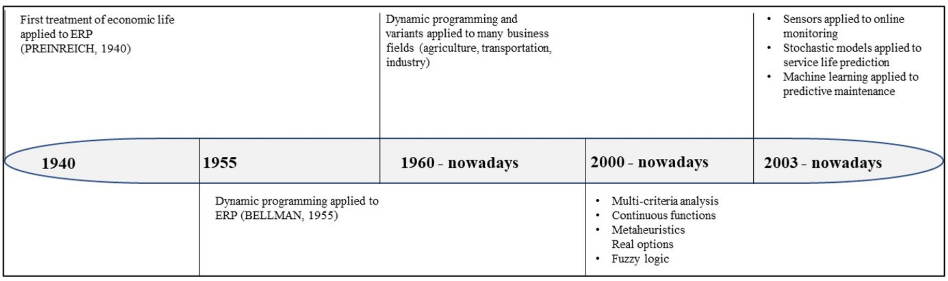

Figure 8. Timeline of the evolution of mathematical models applied to ERP.
Source: prepared by the authors.

The following sections give brief summaries of the proposed categories and provide illustrative examples of each one. Chart 1 describes the symbols used in the selected mathematical models.

Chart 1. Description of symbols used in some mathematical models.

Symbol	Description
A	Net cash flow of the asset at the end of a period
C	Acquisition and installation costs of the purchased asset
S	Residual value of the asset at the end of a period
t	Asset age
K	Keep the existing equipment
P or R	Replace the existing equipment
n	Stage n = 1, 2, 3, …,N
i	Minimum attractive rate
$f_n(t)$	Function that represents the discounted present value of all cash flows related to an asset of age t and work life n

Source: prepared by the authors.

3.1 The network visualization approach

This category gathered proposals that can be better understood in a net representation. In general, the intermediate nodes represent the retention and replacement possibilities while the arcs between nodes represent the transition values between distinct stages (e.g., costs).

DP fits perfectly into this category. Bellman's objective function is presented in recursive Equation 3 as follows. Figure 9 shows a net representation example with the arcs representing the decision costs. We analyzed the optimal replacement policy of an asset in use with an initial age of four years. In the first stage (node 4), we can keep (K) for one more year, thereby reaching five years old at the end of this arc (4-5), or (ii) replace (R) for one more year, thus attaining one year old at the end of the arc 4-1. The sequence of arcs at the lowest cost is the optimal solution.

$$f^{*}n(t) = max \begin{Bmatrix} f_n\,(t, K) = A(t) + (1+i)^{-1} f^{*}{}_{n+1}\,(t+1) \\ f_n\,(t, P) = C - S(t) + A(0)(1+i)^{-1} f^{*}{}_{n+1}\,(t) \end{Bmatrix} \tag{3}$$

$$f_{n+1}(t) = S(t)$$

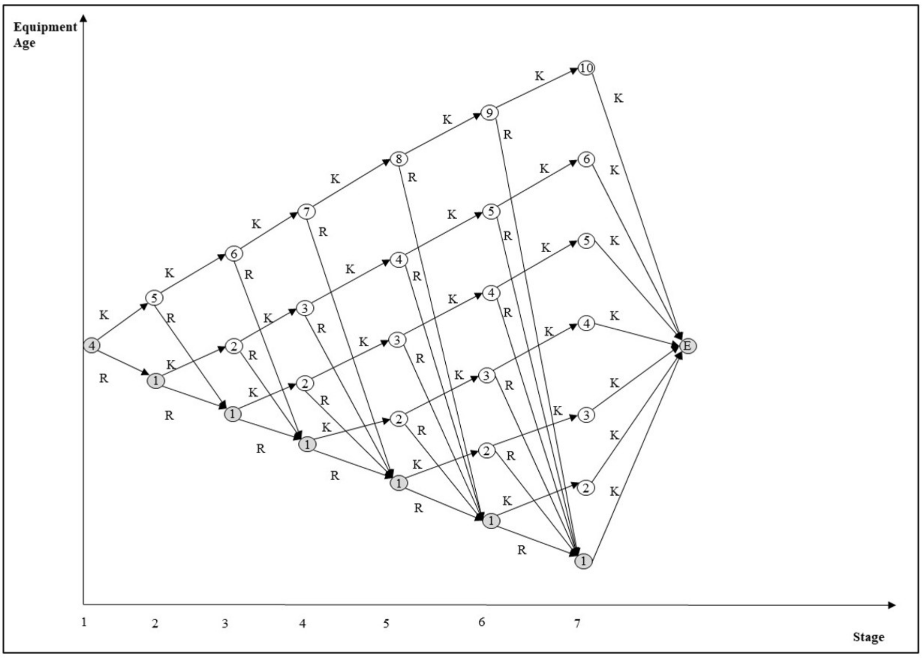

Figure 9. Net of ERP problem. Source: prepared by the authors.

Over time, the logic of the net visualization combined with DP has been applied to a large variety of equipment replacement applications (Abensur, 2015; Abensur et al., 2019; Adil & Gill, 1994; Ahmed, 1973; Altalabi et al., 2020a; Altalabi et al., 2020b; Bector et al., 2013; Bohner, 1994; Chukwunenye, 2016a, b; D'Aversa & Shapiro, 1978; Dreyfus, 1957, 1960; Espiritu & Coit, 2008a, b; Fan et al., 2011, 2012, 2013, 2014; Fawcet & Forero, 2019; Feng & Figliozzi, 2012, 2014; Gress et al., 2012; Hartman, 1998, 1999, 2000, 2001; Hartman & Hartman, 2001; Hartman & Murphy, 2006; Hartman & Rogers, 2006; Leung, 1983; Leung & Tanchoco, 1983, 1986; Marques et al., 2005; Rahmawati & Shahab, 2019; Schwartz et al., 1971; Swathi, 2020; Tanchoco & Leung, 1987; Waddell, 1983; Whitin, 1968; Zvipore et al., 2015).

3.1.1 The knapsack variant

Hartman & Murphy (2006) presented a new DP formulation for ERP in an analogy with the classic knapsack problem. According to this classic problem, a student should select the items he would carry in his knapsack to maximize his total utility but restricted to its available volume.

In this proposal, knapsack size is the planning horizon. The assets' working life defined by their work service are the items to be put into the knapsack. As an example, if an asset can be kept for more than two periods and the planning horizon is three, then the knapsack's size is three, and the items (assets) of ages 1 and 2 can be put into it. The DP recursive equation is shown in Equation 4 as follows.

$$f_i(t) = min_{m_i:t-m_i n_i \geq 0} \left\{ \begin{matrix} \alpha^{t-m_i n_i} \sum_{j=1}^{m_i} \alpha^{(j-1)n_i} p(n^*_i) + f_{i-1}(t - m_i n_i^*) \\ t = 0,1,2, \dots, T \end{matrix} \right\} \quad (4)$$

Where:
n_i = estimated working life of asset i
p = net present value of the equivalent annuity
α = discount rate of the period
m_i = number of times that the asset is used by n_i periods
T = planning horizon

3.1.2 Vehicle routing applied to ERP (RVPSE)

Figure 9 shows two ERP characteristics: (i) they are unidirectional, in other words, the decision moves from the beginning to the end with no return; and (ii) the replacement nodes are at well-defined places in the network (in this case, the grey nodes with the number 1). These characteristics have influenced the development of the RVPSE approach (Abensur, 2015; Grano & Abensur, 2017).

The proposal searches for solutions by adapting a popular vehicle routing algorithm (Fisher & Jaikumar, 1981). This algorithm considers that the optimal policy behaves like a vehicle that describes a path from the origin to the end of the network: it can make some stops (replacements) to reduce the total cost. The mathematical Equations 5-8, available in Grano & Abensur (2017), were adapted to fit the adopted symbols used in this study as follows.

$$f^*n(t) = min \left\{ \begin{matrix} f_n(t,K) = A(t) + (1+i)^{-1} f^*_{n+1}(t+1) \\ \\ f_n(t,P) = C - S(t) + A(0)(1+i)^{-1} f^*_{n+1}(t) \end{matrix} \right\} \quad (5)$$

$$f_{n+1}(t) = S(t)$$

Subject to:

$$\sum_{i=1}^{W} x_{ik} - \sum_{i=1}^{W} x_{ki} = y_i \text{ (Continuity restriction)} \quad (6)$$

$$\sum_{i=1}^{H} \sum_{w=1}^{W} y_w = 1 \text{ (Number of nodes per stage)} \quad (7)$$

$$\sum_{i=1}^{H} \sum_{w=1/w\in\{M\}}^{W} y_w \leq T_i \text{ (Maximum number of replacement nodes per stage)} \quad (8)$$

$x_{ik} \in \{0,1\}$
$y_i \in \{0,1\}$

Where:

x_{ik} = arc from i to k, chosen to be part of the solution

y_j = indicates node i as the solution

W = number of nodes in the network

M = set of replacement nodes

S = number of substitute assets under analysis

H = number of stages

T = number of replacements

N = longest service life estimated from the assets analyzed

3.2 Multi-criteria analysis

The ERP approaches in this study focus on only one optimization criterion (e.g., minimum cost). Nonetheless, other relevant criteria can be considered during the ERP decision (Feldens et al., 2010). Multi-criteria analysis regards two or more criteria simultaneously, based on expert opinions for defining the weight or the relevance of each equipment performance characteristic.

Sabino (2009) applied the PROMETHEE II method to support the replacement decision for a system exposed to extreme impacts, in other words, when the system's failures occur due to a disturbance that exceeds a threshold. He developed a model based on two different criteria: (i) the maintenance cost C1 (the function decreases to a minimum point and then increases); and (ii) the value of the last operating time before equipment replacement C2 (descending function). The optimal C1 is the minimum value of the function. The optimal C2, however, is the maximum value of its respective function. This characteristic makes it unfeasible to find a common optimal point, but both criteria are relevant to the analyzed problem. He ranks the alternatives (number of failures based on which the substitution must be carried out) according to the degree of global preference (a, b), estimated from the performance of each option in the two criteria.

The expert's opinion is taken into consideration when defining the objective function, the extreme values, and the weights of each criterion. Another relevant point of the PROMETHEE method is that it uses relative differences between the results and not absolute values, which avoids scale problems (Cavalcante, 2005; Sabino, 2009).

Cavalcante (2005), applied PROMETHEE I and PROMETHEE II methods to define the optimal replacement policy in two cases and considering three criteria: (i) expected cost C_m; (ii) reliability R, and (iii) total downtime D. A common optimal point cannot be found simultaneously using these criteria. In the first case, he applied PROMETHEE II to rank the best and worst alternatives for equipment replacement. In the second case, the repair time becomes a random variable with an exponential distribution whose parameter can be defined by an expert. The PROMETHEE I method follows the same steps as PROMETHEE II, but in the aggregation phase it generates a previous alternative selection to be chosen by the decision maker.

Alves (2020) applied Hierarchical Process Analysis (HPA) to hospital medical equipment replacement. He considered equipment obsolescence, safety risks, and technical and operational performance as the criteria.

3.3 Simulation

In this section, we have grouped together various techniques such as Metaheuristics and Monte Carlo simulation because they are both optimization methods for complex problems like ERP and can complement each other in the search for better and more robust solutions.

3.3.1 Metaheuristics (genetic algorithms)

Metaheuristics are a class of heuristics formed by flexible and adaptable optimization methods to overcome search failures and escape from local optimal values, even without the guarantee of a global optimal solution (Arroyo, 2002).

Genetic algorithms (GA) are an example of metaheuristics. GA carry out simulations based on genetics and the natural selection process of the species. The chromosome is a feasible solution composed of a set of data (Chenna, 2010). Over generations of chromosome populations, crossovers and mutations can occur. The optimal solution is the chromosome that achieves the best objective function result or the fitness function.

In this category, we considered the works of Zong et al. (2017) and Chenna (2010) to show how genetic algorithms can be applied to ERP. Both works consider stochastic processes to estimate failures in the generation and distribution of energy systems.

Zong et al. (2017) applied genetic algorithms to the generation and distribution of energy systems under environmental uncertainties and climate variations that can cause external impacts. The model considers an impact δ, in other words, equipment failure occurs if the interval between two consecutive shocks is less than δ. The objective is to minimize a total cost function considering a restriction of minimal availability. The total cost function is the sum of the maintenance and replacement costs minus the operational income. The authors also defined the initial algorithm parameters such as population size, crossover and mutation probabilities, and the stop condition. The results show the best moment to replace or repair the equipment from the relationship between the long-term average cost and the number of repairs done.

Chenna (2010) applied genetic algorithms to replace components of a power distribution system to minimize the total cost function. The model was applied to two different situations with budget restrictions over a planning horizon. The total cost function considered maintenance costs, unavailability, and the purchase price of the component. The costs were estimated in accordance with the component's age. Uncertainties were related to component failure. He defined the initial algorithm parameters such as the size of the population, crossover, mutation probabilities, stop condition, and the annual budget.

3.3.2 Monte Carlo simulation

The Monte Carlo simulation (MCS) is a statistical technique based on random numbers and simulations to estimate the optimal solution of a problem. MCS focuses on the uncertainties of the problem and can be combined with other techniques.

Plizzari (2017) developed a MCS applied to ERP combined with DP. Optimization is achieved by maximizing the net income during the planning horizon. He defined the cash flow variables such as: the initial investment, the equipment's average income, the discount rate, market value, and the average equipment replacement cost.

For these variables, MCS generates random numbers that follow a normal distribution and define confidence intervals for each one.

Calculation is based on the average values of the variables and their respective coefficients of variation. MCS uses the approximations as an input for the DP calculations, and so the treatment of the uncertainties is incorporated in the model.

3.4 Real options

Real Options have emerged as an alternative to incorporate managerial flexibility (abandonment, postponement, anticipation) into traditional capital budgeting models (NPV, IRR, EAC), which assume that the initial conditions of the investment project remain unchanged throughout the planning horizon (Brigham et al., 2016; Ross et al., 2002).

According to ERP, costs are divided into acquisition (purchase of the new asset), operational (human resources, maintenance, materials, depreciation) and residual values (sales value of the asset in use). Uncertainties with regard to the values can fall on any of these groups (e.g., the residual values) and, consequently, on the estimation of the economic life of the analyzed assets.

The use of binomial decision trees combined with search algorithms (e.g., exhaustive enumeration) represent alternative real options for ERP. From the perspective of real options, the replacement of equipment is equivalent to an option to purchase a substitute asset (or to sell the asset in use) with impacts on the final present value of the project (Adkins & Paxson, 2008; Adkins & Paxson, 2013b; Lee et al., 2016; Park et al., 2015; Zambujal-Oliveira & Duque, 2011).

Figure 10 presents a simplified binomial tree for ERP. At each decision step, there is a probability *p* and 1-*p* of the success or failure, respectively, of keeping or replacing the equipment.

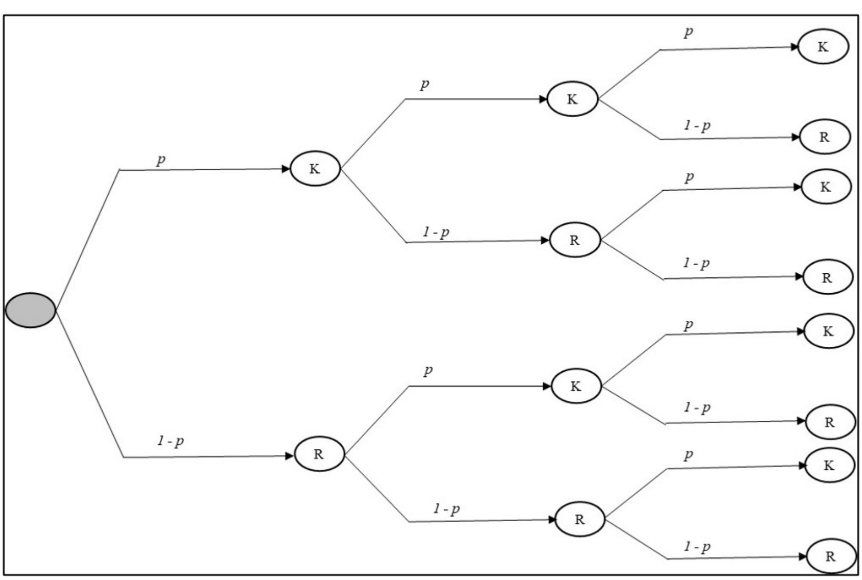

Figure 10. Binomial tree. Source: adapted from Park et al. (2015).

3.5 Continuous functions

This section is considered an approach because it uses specific differential calculus techniques to find more accurate solutions for the ERP. This approach carries out mathematical manipulations based on systems of differential equations to find the best solution for ERP considering continuous and not discrete time (e.g., number of years) (Adkins & Paxson, 2011; Adkins & Paxson, 2013a; van den Boomen et al., 2019; Cheng, 1992; Corrêa & Dias, 2016; Jin & Kite-Powell, 2000; Rogers & Hartman, 2005; Sivazlian, 1973; Starbuck, 1961; Tapiero & Venezia, 1979).

Cesca (2018) transformed the discrete model equations used to calculate equivalent cost of capital, equivalent cost of maintenance and equivalent cost of ownership into non-differentiable continuous functions. Based on mathematical manipulations and non-smooth analysis, he demonstrated that the optimal time for replacement is unique, and, in some cases, there is no optimal replacement time. He proposes a calculus to define the time for replacing the equipment by way of a continuous time model that considers four variables: the growth rate of the maintenance cost of the equipment, the purchase value, the devaluation rate and the minimum attractiveness rate.

Ezekafor et al. (2015) developed a continuous model that uses functions to obtain an approximation of the optimal time to replace a piece of industrial equipment. The authors used the operating and capital costs of the equipment and the residual value.

3.6 Fuzzy logic

The fuzzy numbers manage vague information in mathematical problem modeling, and since 1965 it has been applied in many areas (Biswas & Pramanik, 2011b; Çakir & Ulukan, 2020; El-Kholi & Abdelalim, 2016; Mummolo et al., 2007; Nivatha & Varadharajan, 2019; Sundari & Saranya, 2020; Vitanov et al., 1996).

Biswas & Pramanik (2011a) consider that the total cost of the equipment is the result of the sum of the capital and maintenance costs, discounting the residual value of the equipment. These three values are fuzzy numbers. The method finds alternative values (indexes) for fuzzy numbers using a ranking method to convert a fuzzy model into a classic equivalent. Such key figures are used in total cost calculations. The model does not consider the variation in the value of money over time, which was later included in the model by Biswas & Pramanik (2011b).

Balaganesan & Ganesan (2020) used triangular fuzzy numbers to deal with cost uncertainties. The calculation of the total cost is based on the same costs as in Biswas & Pramanik (2011a), but the maintenance cost is considered in terms of the number of hours. In this approach, time is also considered discrete, and the value of money remains the same over time. Nonetheless, they proposed a model in which conversion to a classical model does not occur, in other words, costs are kept as fuzzy numbers in the average annual cost calculation to determine the optimal replacement time.

3.7 Machine learning

Machine learning uses statistical algorithms (decision trees, k-Nearest neighbors, naive Bayes, neural network, random forest, regression, support vector machine - SVM) to learn from the data instead of programming the computer with detailed rules for each situation. Therefore, understanding data behavior is the origin of the rules and not vice versa (Elwany & Gebraeel, 2007; Gebraeel, 2003).

In terms of ERP, machine learning works focus on predictive maintenance. Predictive maintenance or online monitoring is based on estimating the future values that define the system being studied (machine, facilities, production process) by way of specific mathematical models for forecasting potential failures (Grall et al., 2002).

In general, predictive maintenance systems deal with a lot of data that are monitored in real time and are highly cost-intensive (monitoring, maintenance, failure). Since sensors are integrated into vital components such as engines, bearings, spindles, batteries, and valves, it is possible to leverage the vast amount of available data to prevent unnecessary equipment replacements and minimize unscheduled downtime (Paolanti et al., 2018; Susto et al., 2015).

Figure 11 shows a generic machine learning flow to ERP. The data collected by the sensors are pre-processed according to the operational characteristics (features) that are defined - or not - by the analyzed system. Features such as pressure, temperature, and interval between failures are determined in this phase. The collected data are separated into training and statistical tests. The algorithms which give the best performance are chosen. In terms of ERP, as an example, the selected algorithm can predict the optimal replacement time of a component before it fails.

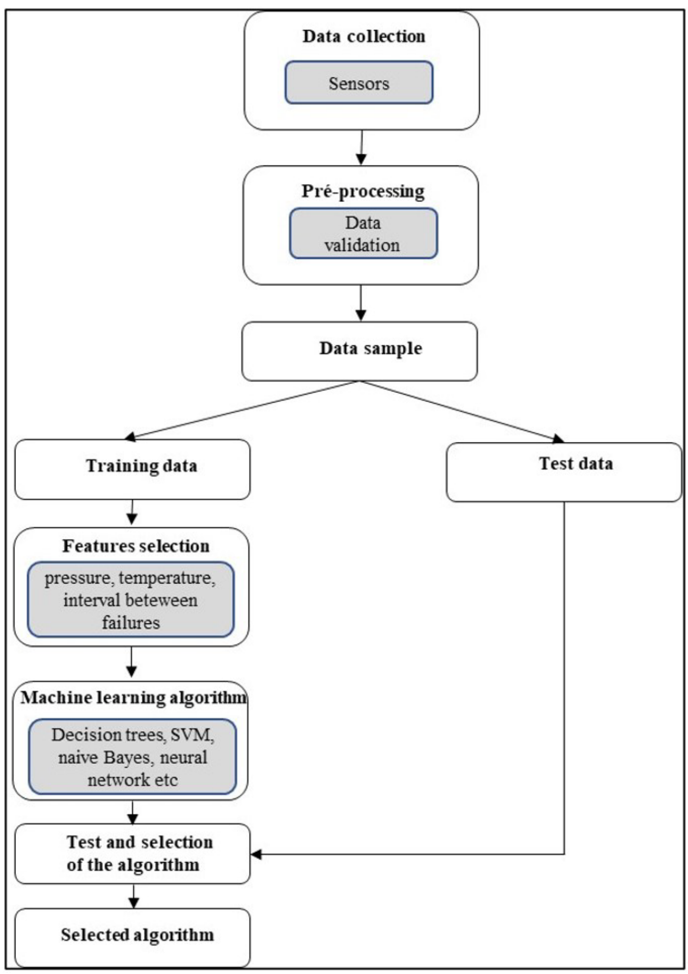

Figure 11. Machine learning flow to ERP.
Source: prepared by the authors.

Chart 2 presents a descriptive summary of the selected categories, their contributions to ERP, and the main techniques observed in each one. We used the symbols * and ** to identify those categories with a high level of complementarity. The machine learning category is independent of all other categories.

Chart 2. Descriptive summary of the selected categories.

Category	Description	Main observed techniques	Contributions to ERP
Network visualization approach *	A recurrence formula (transition) is defined to calculate the changes in variable values between different states of the analyzed asset (retention or replacement) at specific stages (years)	Deterministic dynamic programming, Stochastic dynamic programming, Integer linear programming and Mixed linear programming	A logical structure that is easily comprehensible and adaptable to various mathematical programming techniques, facilitating the determination of the optimal moment of replacement in discrete time
Multi-criteria analysis	Decision-making models based on two or more criteria, such as costs, reliability, and time between stops, rely on the behaviors of variables that influence the ERP cash flow. The limit values and weights assigned to each criterion are defined by managers or experts	HPA Promethee, Promethee II	More decision criteria other than costs. There is the possibility of defining a common optimal solution that would not be possible because of the individual behavior of the analyzed variables
Simulation *	The variables that impact the ERP cash flow, including acquisition value, operating costs, market value, and discount rate, are estimated using the selected simulation technique	Monte Carlo simulation Metaheuristics (e.g., genetic algorithms)	The ERP cash flow variables are subjected to randomness
Real options **	The option to buy or sell equipment is determined through a stochastic calculation involving operating costs, asset residual value, and ERP cash flow	Geometric Brownian motion	The ERP cash flow variables are subjected to randomness
Continuous functions **	ERP expenses such as operating costs (maintenance, depreciation), capital, and market value are converted into mathematical functions	Differential calculation	The optimal moment for replacement in continuous time and identification of assets that do not require replacement due to the absence of an optimal point in their mathematical functions
Fuzzy logic	ERP expenses (capital, operating, residual value) are converted to fuzzy numbers with a variation within limits established by a formation rule	Triangular fuzzy numbers	Uncertainty with regard to ERP expenses
Machine learning	The data define the guidelines for action in each situation, and rule updates are based on recorded data	Data regression statistical algorithms (e.g., neural network, decision trees, random forest, SVM, naive Bayes)	Predictive maintenance applications using online-collected data to estimate the remaining service life of crucial equipment components

* high level of complementarity; ** high level of complementarity.
Source: prepared by the authors.

4 Final considerations

This review provides a classification of the equipment replacement problem, considering mathematical optimization techniques that have been utilized since the creation of dynamic programming in the last century and continue to be employed today. As part of this study, eighty-three selected papers were grouped into seven categories of analysis.

There has been a significant increase in the number of works over time that apply mathematical optimization models. To complement the overview of the subject, we conducted analyses of the number of works by author and by source (journals and university repositories) in order to identify where there is the most material and who has written on the subject. The most influential sources include international journals that focus on engineering, operational research, transport, and computing.

Net visualization is the most widely applied approach, representing 57% of all the selected works, while dynamic programming is still the most commonly used technique. Since 2000 we have noticed a trend toward the application of the other approaches we considered in this review, which together now represent 52% of the total number of applications. Advances in the computing field, which are essential for greater data processing, have contributed to these approaches being more widely used and more widespread. In some cases, these more recent methodologies have been combined with dynamic programming.

As a suggestion for future work, more search terms and databases can be used in order to include more works for comparison and validation purposes aiming to identify the most commonly used approach.

References

Abensur, E. O. (2015). A substituição de bens de capital: um modelo de otimização sob a óptica da Engenharia de Produção. *Gestão & Produção*, 22(3), 525-538. http://dx.doi.org/10.1590/0104-530X1690-14.

Abensur, E. O., Pertinhez, A. K., Zilber, S. N., & Silveira, F. F. (2019). Um simulador em software livre para suporte à decisão de substituição de máquinas agrícolas. In *Anais do Encontro Nacional de Engenharia de Produção* (pp. 1-12). Rio de Janeiro: ABEPRO. http://dx.doi.org/10.14488/ENEGEP2019_TN_STO_296_1672_36843.

Adil, G. K., & Gill, A. (1994). An efficient formulation for an equipment replacement problem. *Journal of Information and Optimization Sciences*, 15(1), 153-158. http://dx.doi.org/10.1080/02522667.1994.10699175.

Adkins, R., & Paxson, D. (2008). An analytical real option replacement model with depreciation. In *The 26th Annual International Real Options Conference* (pp. 1-43). Durham University Business School, United Kingdom. Retrieved in 2023, July 7, from http://www.realoptions.org/papers2008/Adkins%20Roger%20-%20Article%20B%20revD.pdf

Adkins, R., & Paxson, D. (2011). Renewing assets with uncertain revenues and operating costs. *Journal of Financial and Quantitative Analysis*, 46(3), 785-813. http://dx.doi.org/10.1017/S0022109010000815.

Adkins, R., & Paxson, D. (2013a). The effect of tax depreciation on the stochastic replacement policy. *European Journal of Operational Research*, 229(1), 155-164. http://dx.doi.org/10.1016/j.ejor.2013.01.050.

Adkins, R., & Paxson, D. (2013b). Deterministic models for premature and postponed replacement. *Omega*, 41(6), 1008-1019. http://dx.doi.org/10.1016/j.omega.2013.01.002.

Ahmed, S. B. (1973). Optimal equipment replacement policy: an empirical study. *Journal of Transport Economics and Policy*, 116(1), 28-47. Retrieved in 2023, July 7, from https://www.jstor.org/stable/20052308

Altalabi, W. M., Rushdi, M. A., & Tawfik, B. M. (2020a). Optimisation of medical equipment replacement using stochastic dynamic programming. *Journal of Medical Engineering & Technology*, 44(7), 411-422. http://dx.doi.org/10.1080/03091902.2020.1799096. PMid:32886020.

Altalabi, W. M., Rushdi, M. A., & Tawfik, B. M. (2020b). Optimization of medical equipment replacement using deterministic dynamic programming. *Journal of Engineering and Applied Sciences*, 67(2), 467-486. Retrieved in 2021, November 10, from https://www.researchgate.net/profile/Waleed-Altalabi/publication/341538974_OPTIMIZATION_OF_MEDICAL_EQUIPMENT_REPLACEMENT_USING_DETERMINISTIC_DYNAMIC_PROGRAMMING/links/5f5b8879a6fdcc11640983e7/OPTIMIZATION-OF-MEDICAL-EQUIPMENT-REPLACEMENT-USING-DETERMINISTIC-DYNAMIC-PROGRAMMING.pdf

Alves, N. R. (2020). *Substituição ou compra de equipamentos médico hospitalares pela aplicação do método AHP* (Master dissertation). Universidade Federal do Rio Grande do Norte, Natal. Retrieved in 2023, July 7, from https://repositorio.ufrn.br/handle/123456789/32855

Arroyo, J. E. C. (2002). *Heurísticas e metaheurísticas para otimização combinatória* (Doctoral thesis). Universidade Estadual de Campinas, Campinas. https://doi.org/10.47749/T/UNICAMP.2002.242717.

Balaganesan, M., & Ganesan, K. (2020). Fuzzy environment replacement model. *IOP Conference Series. Materials Science and Engineering*, 912(6), 062021. http://dx.doi.org/10.1088/1757-899X/912/6/062021.

Bector, C. R., Hawaleshka, O., & Gill, A. (2013). The equipment replacement problem: a simple solution technique. *Journal of Information and Optimization Sciences*, 14(3), 257-288. http://dx.doi.org/10.1080/02522667.1993.10699155.

Bellman, R. E. (1955). Equipment replacement policy. *Journal of the Society for Industrial and Applied Mathematics*, 3(3), 133-136. http://dx.doi.org/10.1137/0103011.

Biswas, P., & Pramanik, S. (2011a). Application of fuzzy ranking method to determine the replacement time for fuzzy replacement problem. *International Journal of Computers and Applications*, 25(11), 41-47. http://dx.doi.org/10.5120/3154-4359.

Biswas, P., & Pramanik, S. (2011b). Fuzzy approach to replacement problem with value of money changes with time. *International Journal of Computers and Applications*, 30(10), 28-33. http://dx.doi.org/10.5120/3676-5151.

Bohner, W. (1994). *Exhaustive and efficient search algorithms for determining optimal equipment replacement policies* (Doctoral thesis). University of Southern California, Los Angeles. https://doi.org/10.25549/usctheses-c36-189439.

Brigham, E. F., Gapenski, L. C., & Ehrhardt, M. C. (2016). *Financial management: theory and practice*. Boston: Cengage Learning.

Çakir, E., & Ulukan, Z. (2020). An interval type-2 fuzzy dynamic approach to replacement of server equipment. In *2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)* (pp. 1-7). Nova York: IEEE. http://dx.doi.org/10.1109/FUZZ48607.2020.9177554.

Cavalcante, C. A. V. (2005). *Modelagem de decisão multicritério no planejamento da manutenção abordando problemática de escolha e classificação* (Doctoral thesis). Universidade Federal de Pernambuco, Recife. Retrieved in 2023, July 7, from https://repositorio.ufpe.br/handle/123456789/5221

Cesca, I. G. (2018). Desdobramentos da tomada de decisão em problemas de substituição de equipamentos por meio de funções contínuas e análise não suave. *Revista Produção Online*, 18(3), 850-874. http://dx.doi.org/10.14488/1676-1901.v18i3.2977.

Cheng, T. C. E. (1992). Optimal replacement of ageing equipment using geometric programming. *International Journal of Production Research*, 30(9), 2151-2158. http://dx.doi.org/10.1080/00207549208948142.

Chenna, V. D. (2010). *Component replacement analysis for electricity distribution systems using evolutionary algorithms* (Master dissertation). University of Texas, El Paso. Retrieved in 2023, July 7, from https://www.proquest.com/openview/f0755faf4fd9c2d9cf8c5d0bb932daa8/1?pq-origsite=gscholar&cbl=18750

Chukwunenye, U. (2016a). An algorithm for global optimal strategies and returns in one fell swoop, for a class of stationary equipment replacement problems with age transition perspectives, based on nonzero starting ages. *Advances in Research*, 7(4), 1-20. http://dx.doi.org/10.9734/AIR/2016/26667.

Chukwunenye, U. (2016b). Sensitivity analysis of time horizon for a class equipment replacement problem with stationary pertinent data. *Journal of Basic and Applied Research International*, 16(1), 80-87. Retrieved in 2023, July 7, from https://www.ikppress.org/index.php/JOBARI/article/view/3820

Corrêa, R. F., & Dias, A. (2016). Modelagem matemática para otimização de periodicidade nos planos de manutenção preventiva. *Gestão & Produção*, 23(2), 267-278. http://dx.doi.org/10.1590/0104-530x2001-15.

D'Aversa, J. S., & Shapiro, J. F. (1978). Optimal machine maintenance and replacement by linear programming and enumeration. *The Journal of the Operational Research Society*, 29(8), 759-768. http://dx.doi.org/10.1057/jors.1978.164.

Dreyfus, S. E. (1957). A note on an industrial replacement process. *The Journal of the Operational Research Society*, 8(4), 190-193. http://dx.doi.org/10.1057/jors.1957.30.

Dreyfus, S. E. (1960). A generalized equipment replacement study. *Journal of the Society for Industrial and Applied Mathematics*, 8(3), 425-435. http://dx.doi.org/10.1137/0108029.

El-Kholi, A. M., & Abdelalim, A. M. (2016). A comparative study for fuzzy ranking methods in determining economic life of equipment. *International Journal of Construction Engineering and Management*, 5(2), 42-54. Retrieved in 2021, November 10, from https://www.researchgate.net/profile/Ahmed-Abdelalim-2/publication/338356040_A_Comparative_Study_for_Fuzzy_Ranking_Methods_in_Determining_Economic_Life_of_Equipment/links/5e0ec09d299bf10bc38c3222/A-Comparative-Study-for-Fuzzy-Ranking-Methods-in-Determining-Economic-Life-of-Equipment.pdf

Elwany, A., & Gebraeel, N. (2007). Sensor-driven decision models for equipment replacement and spare parts logistics. In *Proceedings of the 2007 Industrial Engineering Research Conference* (pp. 1488-1493). Institute of Industrial Engineers Norcross, USA. Retrieved in 2021, November 10, from https://www.proquest.com/docview/192457025?pq-origsite=gscholar&fromopenview=true

Espiritu, J. F., & Coit, D. W. (2008a). A component replacement model for electricity distribution systems. *The Engineering Economist*, 53(4), 318-339. http://dx.doi.org/10.1080/00137910802482279.

Espiritu, J. F., & Coit, D. W. (2008b). Component replacement analysis for complex electricity distribution configurations. In *IIE Annual Conference and Expo 2008* (pp.170-175). Institute of Industrial Engineers Norcross, USA.

Ezekafor, S. C., Okolli, O. C., & Agunwamba, J. C. (2015). On the existence and uniqueness of approximation of optimal time replacement of industrial equipment for a truncated continuous model. *International Journal of Advanced Multidisciplinary Research Reports*, 1(1), 1-8. Retrieved in 2023, July 7, from http://rex.commpan.com/index.php/ijamrr/article/view/44

Fan, W. D., Gemar, M. D., & Machemehl, R. (2013). Equipment replacement decision making: opportunities and challenges. *Journal of the Transportation Research Forum*, 52(3), 79-90. Retrieved in 2022, November 22, from http://journals.oregondigital.org/index.php/trforum/article/view/4180

Fan, W. D., Machemehl, R., & Gemar, M. D. (2012). Optimization of equipment replacement: dynamic programming-based optimization. *Transportation Research Record: Journal of the Transportation Research Board*, 2292(1), 160-170. http://dx.doi.org/10.3141/2292-19.

Fan, W. D., Machemehl, R., & Kortum, K. (2011). Equipment replacement optimization: solution methodology, statistical data analysis, and cost forecasting. *Transportation Research Record: Journal of the Transportation Research Board*, 2220(1), 88-98. http://dx.doi.org/10.3141/2220-11.

Fan, W. D., Machemehl, R., Gemar, M. D., & Brown, L. (2014). A stochastic dynamic programming approach for the equipment replacement optimization under uncertainty. *Journal of Transportation Systems Engineering and Information Technology*, 14(3), 76-84. http://dx.doi.org/10.1016/S1570-6672(13)60137-3.

Fawcet, R. A. B., & Forero, C. A. D. (2019). *Diseño de políticas de reemplazo y mantenimiento de aires acondicionados de expansión directa basadas en programación dinámica y análisis de ciclo de vida de activos* (Master dissertation). Universidad del Norte, Barranquilla. Retrieved in 2023, July 7, from http://hdl.handle.net/10584/9267

Feldens, A. G., Muller, C. J., Filomena, T. P., Kliemann, F. J. No., Castro, A. S., & Anzanello, M. J. (2010). Política para avaliação e substituição de frota por meio da adoção de modelo multicritério. *Revista ABCustos*, 5(1), 61-91. http://dx.doi.org/10.47179/abcustos.v5i1.86.

Feng, W., & Figliozzi, M. A. (2012). Bus fleet type and age replacement optimization: a case study utilizing King County Metro fleet data. In *Conference on Advanced Systems for Public Transport (CASPT)* (pp.1-14). Israel Institute of Technology, Tel Aviv. Retrieved in 2023, July 7, from https://archives.pdx.edu/ds/psu/8869

Feng, W., & Figliozzi, M. A. (2014). Vehicle technologies and bus fleet replacement optimization: problem properties and sensitivity analysis utilizing real-world data. *Public Transport*, 6(1-2), 137-157. http://dx.doi.org/10.1007/s12469-014-0086-z.

Fisher, M. L., & Jaikumar, R. A. (1981). Generalized assignment heuristic for vehicle routing. *Networks*, 11(2), 109-124. http://dx.doi.org/10.1002/net.3230110205.

Gebraeel, N. (2003). *Real-time degradation modeling and residual life prediction for component maintenance and replacement* (Master dissertation). Purdue University, West Lafayette. Retrieved in 2023, July 7, from https://www.proquest.com/openview/a86727f358e594af411f4c387b587394/1?pq-origsite=gscholar&cbl=18750&diss=y

Grall, A., Dieulle, L., Berenguer, C., & Roussignol, M. (2002). Continuous-time predictive-maintenance scheduling for a deteriorating system. *IEEE Transactions on Reliability*, 51(2), 141-150. http://dx.doi.org/10.1109/TR.2002.1011518.

Grano, C., & Abensur, E. (2017). Optimization model for vehicle routing and equipment replacement in farm machinery. *Engenharia Agrícola*, 37(5), 987-993. http://dx.doi.org/10.1590/1809-4430-eng.agric.v37n5p987-993/2017.

Gress, E. S. H., Arango, O. M., Armenta, J. R. C., & Reyes, A. O. O. (2012). A reward functional to solve the replacement problem. *Intelligent Control and Automation.*, 3(4), 413-418. http://dx.doi.org/10.4236/ica.2012.34045.

Hartman, J. C. (1998). A linear programming formulation with integer solutions for solving an equipment replacement problem with multiple assets: the concave demand case. *International Journal of Systems Science*, 29(11), 1225-1234. http://dx.doi.org/10.1080/00207729808929611.

Hartman, J. C. (1999). A general procedure for incorporating asset utilization decisions into replacement analysis. *The Engineering Economist*, 44(3), 217-238. http://dx.doi.org/10.1080/00137919908967521.

Hartman, J. C. (2000). The parallel replacement problem with demand and capital budgeting constraints. *Naval Research Logistics*, 47(1), 40-56. http://dx.doi.org/10.1002/(SICI)1520-6750(200002)47:1<40::AID-NAV3>3.0.CO;2-T.

Hartman, J. C. (2001). An economic replacement model with probabilistic asset utilization. *IIE Transactions*, 33(9), 717-727. http://dx.doi.org/10.1080/07408170108936868.

Hartman, J. C., & Hartman, R. V. (2001). After-tax economic replacement analysis. *The Engineering Economist*, 46(3), 181-204. http://dx.doi.org/10.1080/00137910108967572.

Hartman, J. C., & Murphy, A. (2006). Finite-horizon equipment replacement analysis. *IIE Transactions*, 38(5), 409-419. http://dx.doi.org/10.1080/07408170500380054.

Hartman, J. C., & Rogers, J. (2006). Dynamic programming approaches for equipment replacement problems with continuous and discontinuous technological change. *IMA Journal of Management Mathematics*, 17(2), 143-158. http://dx.doi.org/10.1093/imaman/dpi032.

Hartman, J. C., & Tan, C. H. (2014). Equipment replacement analysis: a literature review and directions for future research. *The Engineering Economist*, 59(2), 136-153. http://dx.doi.org/10.1080/0013791X.2013.862891.

Jin, D., & Kite-Powell, H. L. (2000). Optimal fleet utilization and replacement. *Transportation Research Part E, Logistics and Transportation Review*, 36(1), 3-20. http://dx.doi.org/10.1016/S1366-5545(99)00021-6.

Lee, D. J., Kim, K. T., & Park, S. (2016). Optimal strategy for equipment replacement: an application to R&D equipment case in Korea. In *Proceedings of Real Option Conference* (pp.1-13). University of Oslo, Norway. Retrieved in 2023, July 7, from https://realoptions.org/openconf2016/data/papers/21.pdf

Leung, L. C. (1983). *An economic equipment replacement model for flexible manufacturing systems* (Doctoral thesis). Virginia Polytechnic Institute and State University, Blacksburg. Retrieved in 2023, July 7, from http://hdl.handle.net/10919/74673

Leung, L. C., & Tanchoco, J. M. A. (1983). Replacement decision based on productivity analysis--an alternative to the MAPI method. *Journal of Manufacturing Systems*, 2(2), 175-187. http://dx.doi.org/10.1016/S0278-6125(83)80030-2.

Leung, L. C., & Tanchoco, J. M. A. (1986). Multiple machine replacement within an integrated system framework. *The Engineering Economist*, 32(2), 89-114. http://dx.doi.org/10.1080/00137918608902957.

Marques, G. M., Silva, M. L., Leite, H. G., & Fontes, A. A. (2005). Application of dynamic programming in equipment substitutions. *Revista Árvore*, 29(5), 749-756. http://dx.doi.org/10.1590/S0100-67622005000500010.

Mummolo, G., Ranieri, L., Bevilacqua, V., & Galli, P. (2007). A fuzzy approach for medical equipment replacement planning. In *Proceedings of the 3rd International Conference on Maintenance and Facility Management* (pp. 229-235). Retrieved in 2022, November 22, from https://s3.amazonaws.com/publicationslist.org/data/bevilacqua/ref-33/034.pdf

Nivatha, G., & Varadharajan, R. (2019). A study on fuzzy replacement model using octagonal fuzzy numbers. *AIP Conference Proceedings*, 2112(1), 020099. http://dx.doi.org/10.1063/1.5112284.

Paolanti, M., Romeo, L., Felicetti, A., Mancini, A., Frontoni, E., & Loncar, J. (2018). Machine learning approach for predictive maintenance in industry 4.0. In *2018 14th IEEE/ASME International Conference on Mechatronic and Embedded Systems and Applications (MESA)* (pp. 1-6). New York: IEEE. https://doi.org/10.1109/MESA.2018.8449150.

Park, S.-J., Lee, D. J., Kim, K.-T., & Lee, W.-H. (2015). An assessment of the economic life of research equipment using real option. *Journal of Advanced Management Science*, 3(3), 250-255. http://dx.doi.org/10.12720/joams.3.3.250-255.

Plizzari, R. (2017). *Modelo para avaliação da vida útil econômica de máquinas e equipamentos utilizando a programação dinâmica e o método de Monte Carlo* (Master dissertation). Universidade de Caxias do Sul, Caxias do Sul. Retrieved in 2023, July 7, from https://repositorio.ucs.br/handle/11338/3085

Preinreich, G. A. D. (1940). The economic life of industrial equipment. *Econometrica*, 8(1), 12-44. http://dx.doi.org/10.2307/1906860.

Rahmawati, D., & Shahab, A. (2019). Replacement scheduling of brine heater desalination plant using binary integer programming method. *AIP Conference Proceedings*, 2187(1), 030013. http://dx.doi.org/10.1063/1.5138317.

Rogers, J. L., & Hartman, J. C. (2005). Equipment replacement under continuous and discontinuous technological change. *Journal of Management Mathematics.*, 16(1), 23-36. http://dx.doi.org/10.1093/imaman/dph027.

Ross, S. A., Westerfield, R. W., & Jaffe, J. F. (2002). *Corporate finance*. New York: McGraw-Hill.

Sabino, E. R. C. (2009). *Política de substituição de um sistema sujeito a choques* (Master dissertation). Universidade Federal de Pernambuco, Recife. Retrieved in 2023, July 7, from https://repositorio.ufpe.br/handle/123456789/5513

Schwartz, A. N., Sheler, J. A., & Cooper, C. R. (1971). Dynamic programming approach to the optimization of Naval aircraft rework and replacement policies. *Naval Research Logistics*, 18(3), 395-414. http://dx.doi.org/10.1002/nav.3800180310.

Sivazlian, B. D. (1973). On a discounted replacement problem with arbitrary repair time distribution. *Management Science*, 19(11), 1301-1309. http://dx.doi.org/10.1287/mnsc.19.11.1301.

Starbuck, W. H. (1961). A generalization of Terborgh's approach to equipment replacement. *International Journal of Production Research*, 1(3), 29-38. http://dx.doi.org/10.1080/00207546108943087.

Sundari, M. S., & Saranya, V. (2020). A novel method to solve replacement problem under fuzzy environment. *AIP Conference Proceedings*, 2187(1), 090015. http://dx.doi.org/10.1063/5.0026484.

Susto, G. A., Schirru, A., Pampuri, S., McLoone, S., & Beghi, A. (2015). Machine learning for predictive maintenance: a multiple classifier approach. *IEEE Transactions on Industrial Informatics*, 11(3), 812-820. http://dx.doi.org/10.1109/TII.2014.2349359.

Swathi, N. (2020). Age replacement optimization of various types of buses using integer programming model. *Malaysia Journal of Matematik*, S(2), 1385-1387. Retrieved in 2023, July 7, from https://www.malayajournal.org/articles/MJM0S200376.pdf

Tan, C. H., & Hartman, J. C. (2010). Equipment replacement analysis with an uncertain finite horizon. *IIE Transactions*, 42(5), 342-353. http://dx.doi.org/10.1080/07408170903394363.

Tanchoco, J. M. A., & Leung, L. C. (1987). An input-output model for equipment replacement decisions. *Engineering Costs and Production Economics*, 11(1), 69-78. http://dx.doi.org/10.1016/0167-188X(87)90030-9.

Tapiero, C., & Venezia, I. (1979). A mean variance approach to the optimal machine maintenance and replacement problem. *The Journal of the Operational Research Society*, 30(5), 457-466. http://dx.doi.org/10.1057/jors.1979.106.

Valverde, S. R., & Resende, J. L. P. (1997). Substituição de máquinas e equipamentos: métodos e aplicações. *Revista Árvore*, 21(3), 345-351. Retrieved in 2021, November 10, from https://books.google.com.br/books?id=qj6aAAAAIAAJ&printsec=frontcover&hl=pt-BR&rview=1&lr=/#v=onepage&q&f=false

van den Boomen, M., Leontaris, G., & Wolfert, A. R. M. (2019). Replacement optimization of ageing infrastructure under differential inflation. *Construction Management and Economics*, 37(11), 659-674. http://dx.doi.org/10.1080/01446193.2019.1574977.

Vitanov, V., Mincoff, N., & Vladimirova, T. (1996). An application of goal geometric programming to equipment replacement under fuzziness. In M. Tamiz (Ed.), *Multi-objective programming and goal programming: theories and applications* (pp. 331-345, Lecture Notes in Economics and Mathematical Systems, 432). Berlin: Springer. http://dx.doi.org/10.1007/978-3-642-87561-8_22.

Waddell, R. (1983). A model for equipment replacement decisions and policies. *Interface: a Journal for and About Social Movements*, 13(4), 1-7. http://dx.doi.org/10.1287/inte.13.4.1.

Whitin, T. M. (1968). Dynamic programming extensions to the theory of the firm. *The Journal of Industrial Economics*, 16(2), 81-98. http://dx.doi.org/10.2307/2097794.

Zambujal-Oliveira, J., & Duque, J. (2011). Operational asset replacement strategy: a real options approach. *European Journal of Operational Research*, 210(2), 318-325. http://dx.doi.org/10.1016/j.ejor.2010.09.011.

Zong, S., Chai, G., & Su, Y. (2017). Determining optimal replacement policy with an availability constraint via genetic algorithms. *Mathematical Problems in Engineering*, 2017, 8763101. http://dx.doi.org/10.1155/2017/8763101.

Zvipore, D. C., Nyamugure, P., Maposa, M., & Lesaoana, M. (2015). Application of the equipment replacement dynamic programming model in conveyor belt replacement: case study of a gold mining company. *Mediterranean Journal of Social Sciences*, 6(2 S1), 605. http://dx.doi.org/10.5901/mjss.2015.v6n2s1p605.

Authors contribution

Eder Oliveira Abensur, Bruna Pereira Santos, and Anselmo Alves Bandeira collaborated on the conceptualization of this project. Eder Oliveira Abensur and Bruna Pereira Santos were responsible for the theoretical and methodological approach, literature review, data collection, data analysis, and manuscript writing. Eder Oliveira Abensur was also in charge of the final manuscript revision.