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Mathematical modeling for optimization of periodicity in the preventive maintenance plans

Modelagem matemática para otimização de periodicidade nos planos de manutenção preventiva

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Abstract: This study aims to present the development of a model for optimization of periodicity in the preventive maintenance plans of industrial assets, through the study of the lifespan of systems justified by use, time, condition, and costs. The mathematical modeling used was implemented computationally using the MATLAB® software. The aim of this model is to provide increased reliability to the facilities, in line with the financial results of the business. The line of research is integrated into the Reliability Centered Maintenance (RCM) management process.

Keywords: Preventive maintenance; Periodicity; Lifetime; Mathematical modeling; Residual cost; Maintenance cost.

Resumo: *Propõe-se, com este trabalho, apresentar o desenvolvimento de um modelo de otimização de periodicidade dos planos de manutenção preventiva de ativos industriais por meio do estudo da vida útil dos sistemas, fundamentado pelo uso, tempo, condição, custos. A modelagem matemática utilizada foi implementada computacionalmente por meio do MATLAB. O objetivo do modelo é proporcionar maior confiabilidade às instalações, alinhadas ao resultado financeiro do negócio. A linha de pesquisa está integrada ao processo de gerenciamento de manutenção centrada em confiabilidade (MCC).*

Palavras-chave: *Manutenção preventiva; Periodicidade; Vida útil; Modelagem matemática; Custo residual; Custo de manutenção.*

1 Introduction

The maintenance cost is decisive factor on the operational viability of an equipment or process. In the industrial context, the maintenance cost has come to represent, on average, 20% of the fixed cost of products.

A given published in ABRAMAN (2011) shows that the maintenance cost of the Brazilian industry represents, on average, 3,95% of the Brazilian GDP.

Espinosa Fuentes (2006) and Biasotto (2006) presented maintenance strategies that were employed in industrial complexes, having, as highlighted, management models that seeks preventive actions such as TPM (Total Productive Maintenance), RCM (Reliability Centered Maintenance), preventive based on condition, in time, in impairments, among others. Methodologies to manage well these management models were presented by Waeyenbergh (2005) and Rigone (2009). According to Smith (1993), the great challenge for optimization of the cost in these strategies is on the "what to do" and "when to do"; i.e., what scope and with what periodicity. The correct definition of a periodicity defines the cost in all technical preventive measures.

This approach is very important for companies encouraging several studies. Christer (1998) addressed the issue of optimizing the frequency of preventive maintenance, from the failure rate of equipment; Ferreira (2010) addressed the mathematical modeling using method of approval, Bayesian network, to optimize the use of the most appropriate maintenance techniques to a given preventive/corrective process. Haicanh et al. (2014) addressed the mathematical modeling with genetic algorithm to check the dependence of components that suffers preventive maintenance and that affects positively and negatively the maintenance cost.

Given the importance of the periodicity optimization in the preventives maintenances, the objective is, with this work, develop a mathematical model that assists in the dimension of periodicity in the preventive

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maintenance plans (PM) and answer questions of research such as:

- √ What is the influence of the difference between corrective MTTR and preventive MTTR in the maintenance cost?
- √ What is the financial impact of the preventive maintenance in accordance with the cost of the outgoing time?
- √ What is the influence of residual cost (due to premature component exchange) in maintenance cost?
- √ What is the preventive periodicity that provides lower maintenance cost over the lifetime of the process?

Premature exchange is a term used in this work to designate replacement of the item before it reaches the end of life.

2 Periodicity in the PM plan

Although the preventive maintenance enable anticipation of correction of damage, before the fault occurs, it also generates unavailability in the process, because for each maintenance event there is the need to stop the process, making setup of the periodicity and the execution time of a preventive maintenance becomes complex, due to this and other factors such as:

- The periodicity of preventive maintenance of each equipment should be combined with all the equipment of the process, to generate a better use of the stop time of the process.
- Difficulty to define which components will be swapped, from the knowledge of the useful life of the same.
- Dimensioning of labor for the execution of activities.
- Concentration of the largest possible number of activities to enjoy the impairment of the process.

The definition of great periodicity, that provides lower cost and higher reliability in systems, is one of the challenges of preventive maintenance.

2.1 Definition of the periodicity on PM

Act in a conservative manner in relation to the reliability generates a high cost in maintenance due to premature component exchange and the excessive use of maintenance labor. The experience of one of the authors, by 17 years in the industrial maintenance, in mechanical level of corrective and preventive maintenance, as a planner, analyst, engineer, coordinator and maintenance manager, allowed to experiment decisions that, for increased reliability, demanded questions, such as: "intensify the preventive maintenance, it increases the scope or reduces the periodicity"?

By acting in a conservative manner in relation to the cost of maintenance, with objective of obtaining a good use of components to make maximum use of its useful life, you can also compromise the reliability of the system due to the uncertainty that exists on the useful life of each component, (region of periodicity 17 to 20 in Figure 1). Consequently, there will be the possibility to reduce the cost of maintenance and raise the cost of the process, due to the low reliability, generating unavailability in the process.

As shown on Figure 1, the region of great periodicity (between 9 and 12), that provides better financial result of the system, depends on many factors, such as: cost of preventive maintenance, profitability of the process and especially the knowledge of the useful life of the systems. For this it is necessary an in-depth statistical control of failures and time of occurrence.

Still according to Figure 1, the reduction of periodicity provides better reliability, but can derail the profitability of the process due to the increase in maintenance costs.

2.2 Useful life in preventive maintenance

For Smith (1993, 2004), preventive maintenance is the operation of the services or tasks of inspection that has been planned for the achievement of specific points in time and preserves the function of the operation of the equipment or systems.

For Bertsche (2008), preventive maintenance is a maintenance method, where the tasks are performed preventively; that is, to a predetermined time, or after a specified periodicity or a quantity of operating hours these activities are performed.

For both authors, the preventive maintenance can be based on time, condition or failure.

It is based on time, when it`s set a determined time of use or a number of cycles for the execution of certain repairs, adjustments or replacement of components.

It is based on condition when applied techniques of visual inspection, routine or more depth as techniques of vibration analysis, thermo graphic analysis, analysis of oil and ultrasound, also defined as predictive.

It is based on failure, when the repair occurs after detecting the fault. Whereas the failure won`t damage other components and won`t generate consequences to safety and the environment, planning tools, parts and labor and looks forward to the occurrence of the failure to perform the repair.

These preventive actions are ways to predict the moment of equipment failure, that is, predict the end of it`s useful life. Failure analysis techniques can be seen in Kumamoto & Henley (1996), Dias et al. (2011) and Dias (2012).

As shown on Figure 2, the higher the intensity of inspection the lower the uncertainty of the estimation of component life, to the point at which they can act in the exact moment of the failure "based on the failure", when, then, there is a use of 100% of component life.

Leaving the component fail, in view of the cost of maintenance, has a better exploitation of the component, with the use of 100% of it`s useful life. In this sense, there would be no premature component cost, due to this recovery; however, in most situations, the predominant maintenance policy would be corrective.

According to Souza (2009), normally the assessment of useful life of components is based on past experience and on statistical data provided by the manufacturers. On the incompatibility of adjustments with the production program, many equipment can`t be reviewed at certain times, sacrificing components that could be in good conditions if done the exchange in the right time. These are the reasons that generate

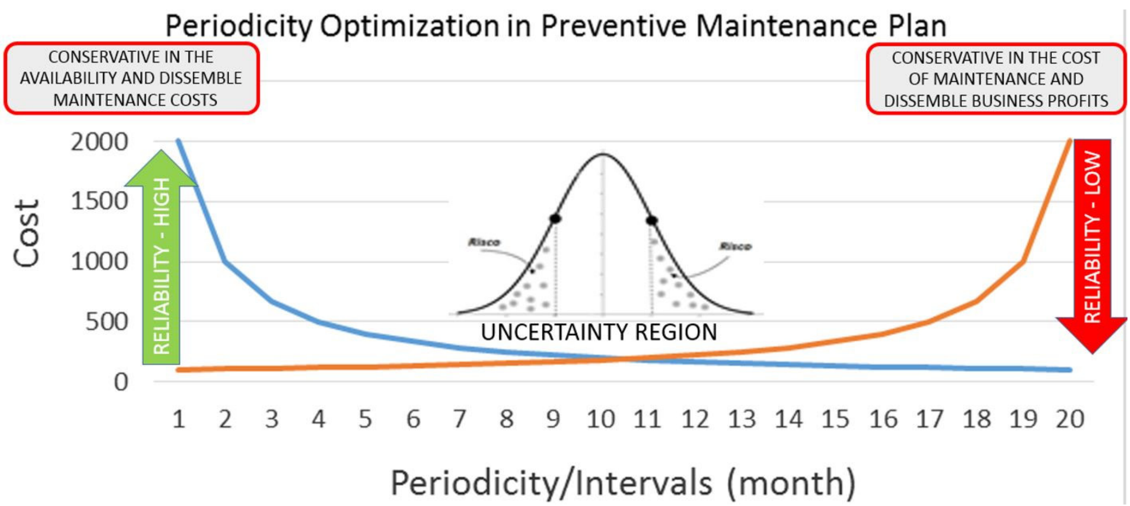


Figure 1. Cost effects on the periodicity of preventive maintenance (Corrêa & Dias, 2014).

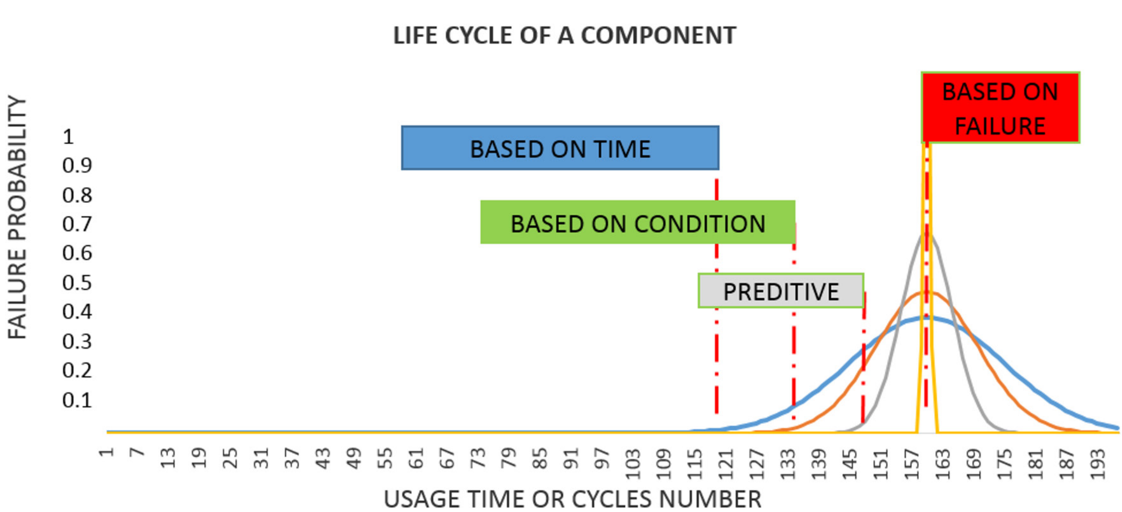


Figure 2. Definition of periodicity in function of useful life (Corrêa & Dias, 2014).

the main criticisms of the preventive maintenance policy. It is observed, on Figure 3, the statistical distribution of 10 systems for the sliding equipment; you may ask: how can you define the periodicity of an intervention of equipments from the distribution of the useful life of its system?

Can the periodicity of preventive maintenance of this equipment be defined, based only on knowledge of the useful life of each system?

In case the answer was "yes", there would be the definition of impairments of the equipment in function of time, generating a stop for each "mode" of the distribution that represents the useful life of each system, as shown on Figure 4.

Figure 4 shows 27 stops on the equipment, on a period of 25 months. As in this example is not being considered the MTTR (medium time to repair) and nor the hours cost of outgoing process to which the equipment is inserted, the cost generated by stopping the process is not significant; that means, the maintenance cost is generated only by the cost of parts and labor.

It`s important to observe in this figure, that some systems are replaced more than once, during the

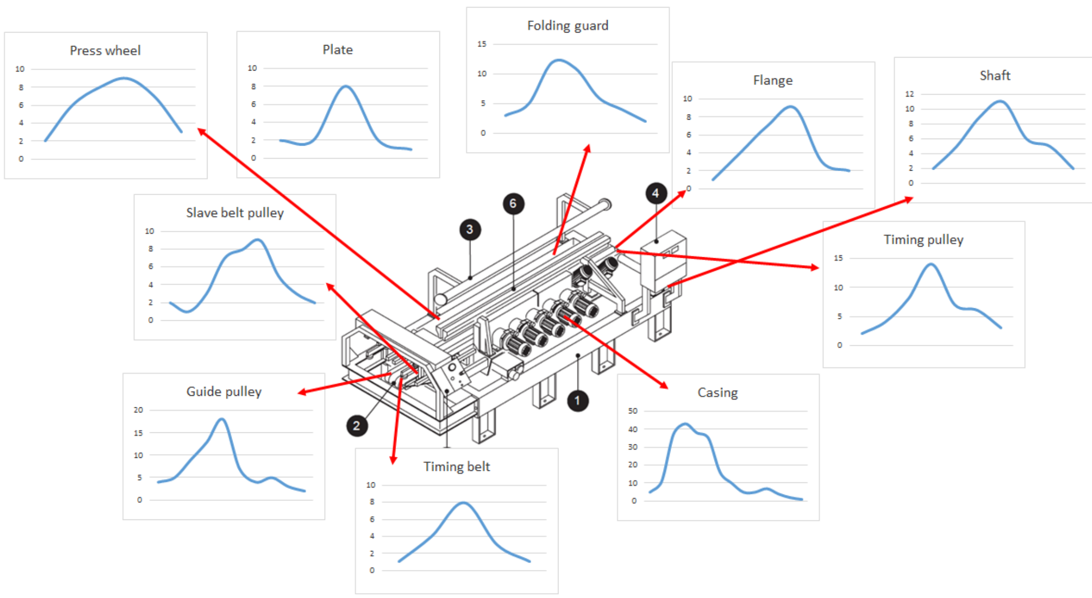


Figure 3. Distribution of the function of "useful life" for the equipment systems (Corrêa & Dias, 2014).

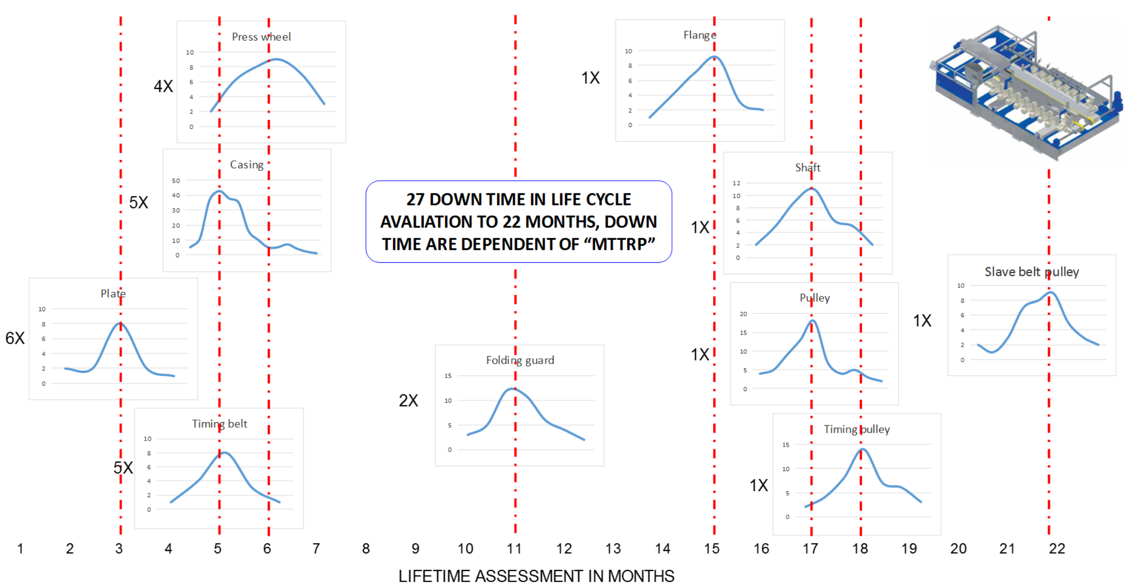


Figure 4. Distribution of function of "useful life" of systems, distributed in time (Corrêa & Dias, 2014).

useful life of the evaluation, that means, the system that has an average life of 4 months for a useful life of 25 months of evaluation is necessary to be replaced 6 times, generating 6 impairments to process.

This way, the definition of periodicity more favorable financially is to act on a corrective form, making repairs on each component at the end of its useful life, despite having a high unavailability due to the large number of impairments.

2.3 Evaluation of periodicity

To better understand the comments made earlier, note the evaluation of a hydraulic press, represented in six systems with their respective MTTR, useful life and cost of repair, as Table 1.

Each system has an estimated life from statistical data with an uncertainty for each value displayed: each useful life was estimated from a probabilistic distribution and may be normal, lognormal or Weibull.

It was, also, considered the residual cost of component, i.e., the value of the component that was replaced without having been used its useful life in full. The likely total useful life less the effective life of work. This approach is the process of maintenance cost as the following Equation 1 and 2:

$$CM = CCn + CRn + CP \quad (1)$$

and: CM – Cost of maintenance (R$); CCn - Cost of component "n" (R$); CRn – Residual cost of component "n" (R$); CP – Cost of impairment (R$).

$$CP = (MTTRn \times CHC) \quad (2)$$

and: MTTRn – Medium time to repair system "n" (h); CHC – Cost of outgoing time of system (R$/h).

The residual cost seen in Figure 5 is represented by means of the function of a descending straight line. Whenever a component is replaced before the end of its estimated useful life, the cost of the repair is being added to the residual cost of the component replaced.

In the example in Figure 5, the component "COMP1" has an estimated useful life of 20,000 hours. Opting to replace it preventively before of 20,000 hours, (as the example in Figure 5 that shows an exchange at 10,000 hours), it has a residual cost. When you choose to replace a component or make a repair of a system before the end of its useful life, the cost of maintenance will be: the cost of a new component (necessary parts for the repair) plus the residual cost of the system or component, that still didn`t reach the end of its useful life.

Imagine, now, that the hydraulic press shown as an example on Table 1 is insert in a process in which the cost of outgoing time of process is R$ 0.00 for hour of impairment. In this case, you can stop the process at any time without financial effect on the process. From the point of view of cost of maintaining more economical, the option would be replace each component, only on the end of its useful life, having, then, a better use of systems and lack of residual cost for maintenance.

For this situation, there would be 14 impairments on the process, totalizing 120 hours stopped, as shown in Figure 6: by the end of each component useful life, has a stop in the process for actuation of maintenance.

Table 1. Representation of a hydraulic press divided in systems (Corrêa & Dias, 2014).

HYDRAULIC PRESS			
SYSTEM	MTTR (h)	USEFUL LIFE (h)	COST OF COMPONENT
Filtering	4	3,000	R$ 3,500.00
Hydraulic fluid	10	5,000	R$ 13,000.00
Master cylinder	30	15,000	R$ 40,000.00
Pumping	6	20,000	R$ 38,000.00
Hydraulic command	5	25,000	R$ 20,000.00
Hydraulic booster	12	30,000	R$ 12,000.00

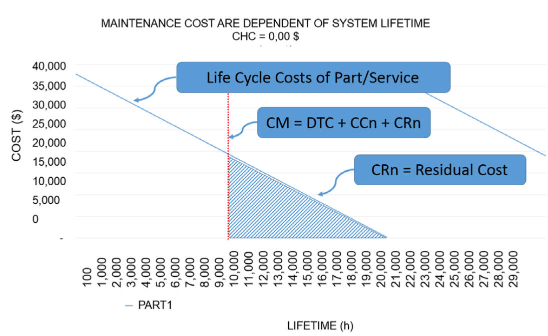


Figure 5. Representation of residual cost for a system (Corrêa & Dias, 2014).

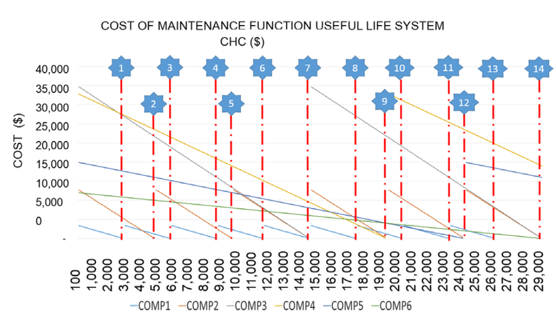


Figure 6. Representation of residual cost of the six systems of a press (Corrêa & Dias, 2014).

For the example shown in Figure 6, operates according to the policy of corrective maintenance (MC), taking in consideration that the fault generated on the equipment due to the use of the comprehensive life of each equipment does not cause any side effect, such as: damage to other systems, security or the environment. From the economic point of view, for the scenario presented previously, act as a corrective action would be the most viable for the profitability of the process.

There are other factors that should be considered in this analysis such as; availability of labor to act correctively, specialty of labor, tooling, spare parts and MTTR, which will be exemplified with more details to follow.

You can make a new analysis for the same equipment, this time it is installed in a process whose hour cost of resigning is \$ 7,000.00 for hour of impairment. In this case, obviously, the strategies should be other.

To better understand it will be compared both systems, represented in Figure 7, by COMP3 and COMP5.

When replaced the component COMP3 with 15,000 hours that represents its useful life, the maintenance cost will be, cost of component COMP3 + cost of impairment (CP).

If in this intervention is also chose to replace the component "COMP5", in order to take advantage of impairment of the process, the cost of this maintenance would be: cost of components COMP3 and COMP5 + impairment cost (CP) + residual cost of the component COMP5 (Crcomp5). According to the data shown on Table 1, the cost of this maintenance would be:

$$CP = MTTRmax \times CHC = 15 \times 7{,}000 = R\$\ 105{,}000.00.$$

$$CM = Cccomp3 + Cccomp5 + CP + Crcomp5$$

$$CM = 13{,}000 + 20{,}000 + 105{,}000 + 8{,}000$$

$$CM = R\$\ 146{,}000.00$$

Another option would be replace each component by the end of its useful life. For this situation there would be two impairments on the process, as represented by Figure 8.

In the example above, there would be an impairment in instant 15,000 hours, and another impairment in instant 25,000 hours.

For 15,000 hours has the CM as:

$$CP = MTTRcomp3 \times CHC = 10 \times 7000 = R\$\ 70{,}000.00.$$

$$CM = Cccomp3 + CP$$

$$CM = 13{,}000 + 70{,}000 = R\$\ 83{,}000.00$$

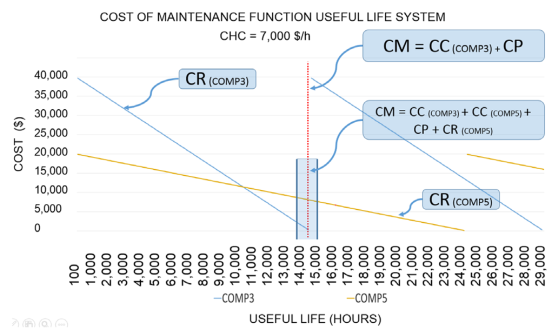


Figure 7. Cost of maintenance with CHC greater than zero and residual cost (Corrêa & Dias, 2014).

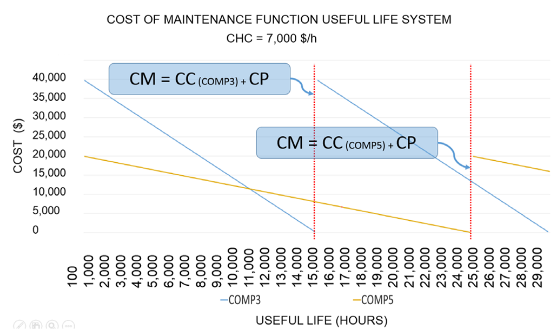


Figure 8. Cost of maintenance with CHC greater than zero without residual cost (Corrêa & Dias, 2014).

For 25.000 hour has the CM as:

$$CP = MTTRcomp5 \times CHC = 15 \times 7{,}000 = R\$\ 105{,}000.00.$$

$$CM = CCcomp5 + CP$$

$$CM = 20{,}000 + 105{,}000 = R\$\ 125{,}000.00$$

In the period of 25,000 hours has a cost of maintenance of two impairments, totalizing in R\$ 208,000.00.

When is compared both scenarios presented in Figure 7 and 8, you can observe: on Figure 7, in the period of 25,000 hours, has only one impairment in the process totalizing a maintenance cost, for the period, of R\$ 146,000.00. Now in Figure 8, in the same period of 25.000 hours, has two impairments, totalizing a cost of maintenance of R\$ 208,000.00.

It can be observed, by means of example cited, that the scenario represented in Figure 7 has a lower cost of maintenance, consequently more profitable for the business; therefore, the replacement of components or premature revisions of systems, depending on the profitability of the process, are necessary.

3 Modeling

The mathematical modeling has as objective the structuring of systems variables to be studied in order to obtain calculations optimization of the periodicity of preventive maintenances.

For this modeling, the model takes into consideration some characteristics of the systems:

1 –All intervention of preventive maintenance takes into consideration the replacement of all the evaluated components.

2 –The time of execution of the preventive maintenance is based on the component that has the largest time of preventive repair MTTRp, within the evaluated system.

3 –The events of corrective maintenance occurs so that it does not cause any harm to safety, environment or damage the underlying component; i.e., the damage caused in the corrective event is mainly the component, causing only financial impact.

4 –All the systems are represented by a RBD (Block Diagram of Reliability) in series being that any failure in a single component requires that stops all the process.

The model is based in the estimated useful life, medium time to repair and the outgoing time of the process of each component to be evaluated. To have a result that is consistent with this model, should only be evaluated systems for which it possess information about the useful life of components well defined.

In the productive processes (systems) can exist a multitude of components, but this model is proposed to make a great assessment of periodicity from most significant components that has a greater representativeness in the cost of repair and the time of impairment of the process.

3.1 Declarations of variables

To start the modeling, first shall be informed all the variables involved in the model, as follows:

MTTRpn = Medium time to repair the preventive of component "n".

MTTRcn = Medium time to repair the corrective of component "n".

MTTRmax = Maximum medium time to repair the preventive of system.

Vun = Average useful life of component "n".

VUTn= Use of the component life "n".

Vumax = Maximum useful life of the system.

Vumin = Minimum useful life of the system.

Vus = Useful life of evaluation of the system.

CCn = Cost of component "n".

CRn = Residual cost of the useful life of component "n".

CRT = Total residual cost of the useful life of the system.

CHC = Cost of outgoing time of the system.

CRPp = Cost of preventive repair.

CRPc = Cost of corrective repair.

CMVu = Cost of maintenance for the evaluated useful life.

Nc = Number of systems.

Wp = Periodicity of preventive.

Wpot = Periodicity of preventive optimized.

CR(w) = The total residual cost of the useful time of the system in accordance with the periodicity.

3.2 Equations

As previously mentioned, the residual cost of the component represented by the variable CRn, is a periodic descending function as a function of time. Each period represents the component replacement.

To model this event, the function that best represents is a series of Fourier, saw-tooth type, according to Equation 3.

$$y(t) = \frac{2L}{\pi}\sum(-1)^{(n-1)} \times \sin\frac{n \times \pi \times t}{L} \quad (3)$$

To model the CRn (Residual cost of useful life of component "n"), in the series, was necessary to perform some adjustments in the original equation, as will be shown in the Equation 4.

$$CR_n(t) = \frac{CC_n}{1.5} + \frac{CC_n}{2}\sum_{n=1}^{1000}\frac{(-1)^n}{n} \times \sin\left[n \times \frac{\pi}{0.5 \times Vun} \times \left(t + \frac{Vu_n}{2} + VUT_n\right)\right] \quad (4)$$

It is observed, initially, that the original Equation 1 is a growing series and the necessity here is decreasing. It was also necessary to insert an additional term, to move the amplitude of the equation that varies positively and negatively around the point "zero". In this modeling is necessary only positive values that represents the CR_n (t). The number of terms for modeling each series is present in the value of 1 to 1000, varying in one unit. It was added the terms $\frac{Vu_n}{2}$ and *VUTn* that represents the phase angle in the function, by moving all the function to the beginning of the useful life of each evaluated component and adjusting the life of system utilization in function of the moment to be assessed.

An example of this function can be observed in Figure 9.

After the definition of the equation that represents the cost of each component in funtion of its useful life, to obtain the cost of useful life of all the systems to be evaluated, applies the Equation 5.

$$CR_T = \sum CR_n(t) \quad (5)$$

From obtaining the cost of the useful life of the system in function of time, is needed to know the cost of preventive repair in function of time, that is, what is the cost of repair for a determined moment. This may be calculated by means of the Equation 6.

$$CRP_p(t) = \sum CC_n + MTTR_{max} \times CHC + CR_T \quad (6)$$

It is recalled that the cost of instantaneous repair considers the exchange of all the components.

For the present study, no matter the instantaneous cost to perform a certain repair, but what is the cost of the repairs in function of the periodicity.

The system will simulate many frequencies in function of a determined useful life, denominated Vus (useful life of evaluation of the system). The Vus is the life that absorbs the maximum useful life of the component of a determined system. Therefore the Vus will be two times higher than the maximum life of a determined component of the evaluated system, or defined by the user, and may be entered its value during the entry of data in the program.

The life of the initial assessment, i.e., the lower periodicity evaluated, is defined by a quarter of the value of the smallest life of a determined component of the system evaluated. According to Equation 7.

$$W_p = F \times Vu_{min} \Rightarrow \left[\frac{Vu_{min}}{4}, 2 \times Vu_{max}\right] \quad (7)$$

For each periodicity evaluated within the range defined in the Equation 7, you get the cost of preventive repair in function of the periodicity, according to Equation 8. The periodicities simulated for the Equation 8 arise from the Equation 7, that increments a "F" factor equal to 0.5.

$$CRP_p(W_p) = \sum CRP_p \quad (8)$$

It is observed that as the periodicities are incremented, the cost of maintenance gets another variable, generated by the corrective events. Thus, for each periodicity that exceeds the time of the useful life of a determined component, has a corrective event which can be calculated by the Equation 9.

$$CRP_C(w_p) = \sum [(CC_n + MTTR_{cn} \times CHC) \times \left(\frac{2 \times Vu_{max}}{Vu_n}\right)] \quad (9)$$

The total cost of maintenance for the life of evaluation of the system in function of the periodicity can be calculated by the Equation 10.

$$CM_{Vu} = CRP_p(W_p) + CRP_c(W_p) \quad (10)$$

With the function obtained in the Equation 8, is possible to define the periodicity great for the evaluated system, which is the lowest value of the function generated through the Equation 8.

4 Numerical applications

The modeling was developed through the software MATLAB, according to Willian (2013).

The main objective of the program is to provide the experts and managers of the maintenance area the ease of evaluation of various scenarios of the industrial process that comprise the cost of preventive maintenance. In addition, gives the visibility to the economic viability of the projects of industrial processes.

4.1 Data entry

To evaluate the mathematical modeling and the program implemented, will be used a simple model of a ceramic industrial process, as Figure 10. The variables of entry can also be exemplified in this figure.

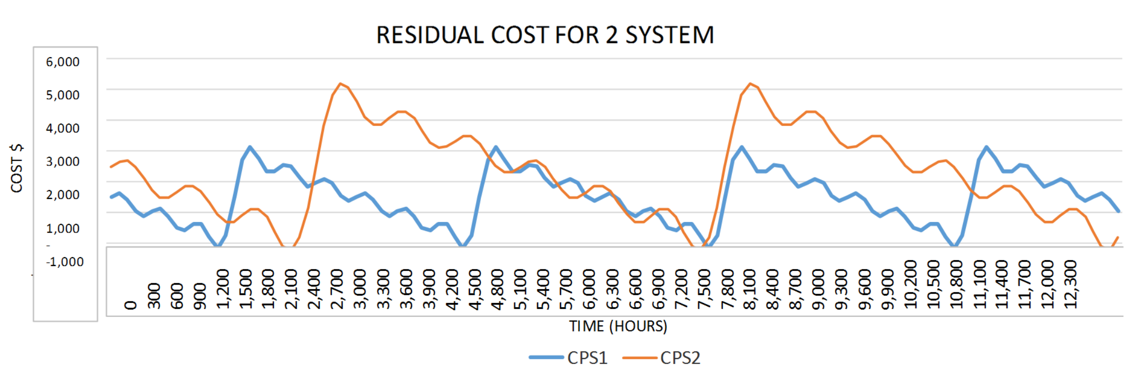


Figure 9. Residual cost of two systems for the number of terms in the Fourier systems, n=5.

In Figure 11, it is possible to observe the data entry page of the designed program named POPMP (Program to Optimize Preventive Maintenance of Periodicity).

4.2 Results

In Figure 12 can be observed the function of the residual cost of each component of the system in function of its useful life that was modeled in the series saw-tooth Fourier, according to the Equation 2.

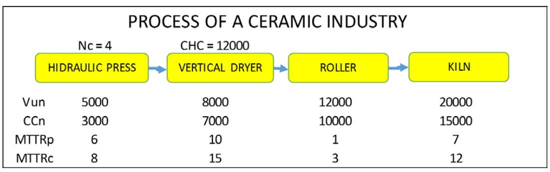
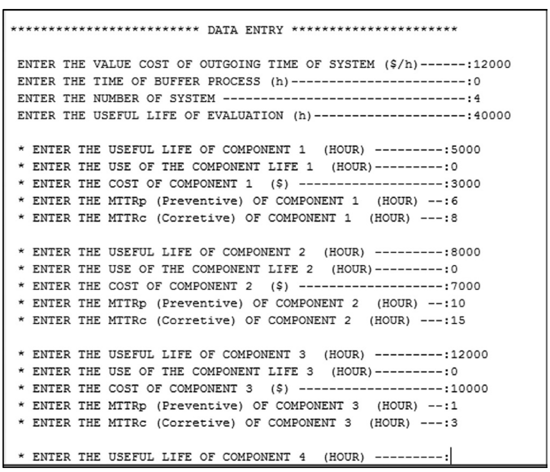


Figure 10. Model of a system for simulation of modeling in MATLAB.



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************************* DATA ENTRY **********************

ENTER THE VALUE COST OF OUTGOING TIME OF SYSTEM ($/h)------:12000
ENTER THE TIME OF BUFFER PROCESS (h)-----------------------:0
ENTER THE NUMBER OF SYSTEM --------------------------------:4
ENTER THE USEFUL LIFE OF EVALUATION (h)--------------------:40000

* ENTER THE USEFUL LIFE OF COMPONENT 1  (HOUR) ---------:5000
* ENTER THE USE OF THE COMPONENT LIFE 1  (HOUR)---------:0
* ENTER THE COST OF COMPONENT 1  ($) -------------------:3000
* ENTER THE MTTRp (Preventive) OF COMPONENT 1  (HOUR) --:6
* ENTER THE MTTRc (Corretive) OF COMPONENT 1  (HOUR) ---:8

* ENTER THE USEFUL LIFE OF COMPONENT 2  (HOUR) ---------:8000
* ENTER THE USE OF THE COMPONENT LIFE 2  (HOUR)---------:0
* ENTER THE COST OF COMPONENT 2  ($) -------------------:7000
* ENTER THE MTTRp (Preventive) OF COMPONENT 2  (HOUR) --:10
* ENTER THE MTTRc (Corretive) OF COMPONENT 2  (HOUR) ---:15

* ENTER THE USEFUL LIFE OF COMPONENT 3  (HOUR) ---------:12000
* ENTER THE USE OF THE COMPONENT LIFE 3  (HOUR)---------:0
* ENTER THE COST OF COMPONENT 3  ($) -------------------:10000
* ENTER THE MTTRp (Preventive) OF COMPONENT 3  (HOUR) --:1
* ENTER THE MTTRc (Corretive) OF COMPONENT 3  (HOUR) ---:3

* ENTER THE USEFUL LIFE OF COMPONENT 4  (HOUR) ---------:
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Figure 11. POPMP, modeling in MATLAB.

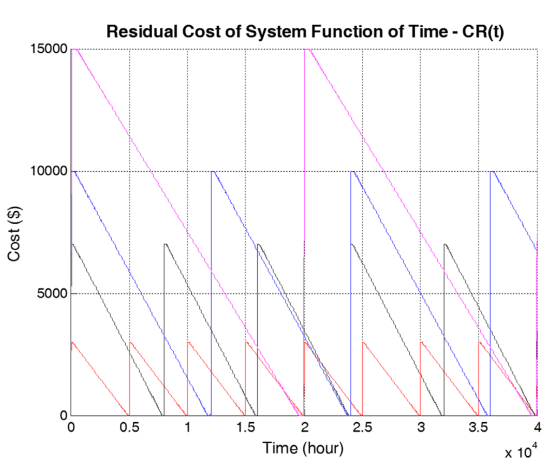


Figure 12. POPMP, residual cost in function of useful life.

In Figure 13 was obtained the total residual cost of the system, calculated by the Equation 3 and the cost of preventive repair calculated by the Equation 4, in function of time. It is observed that at this point of the program, there is only the cost of instantaneous maintenance in function of time and not according to the periodicity that is the main purpose of the POPMP.

From this point the program begins to simulate the periodicities, initially considering a quarter of minimum useful life of the system, in this example, 5.000 hours. The system initially will design a periodicity of 1.250 hours, as Figure 14.

From this point, the program begins to increment the periodicity in 0.5 times the value of minimum useful life as Equation 5 (see Figure 15).

For each periodicity simulated, the system calculate the total residual cost, taking in consideration that all the components will be replaced in the preventive event for a determined periodicity.

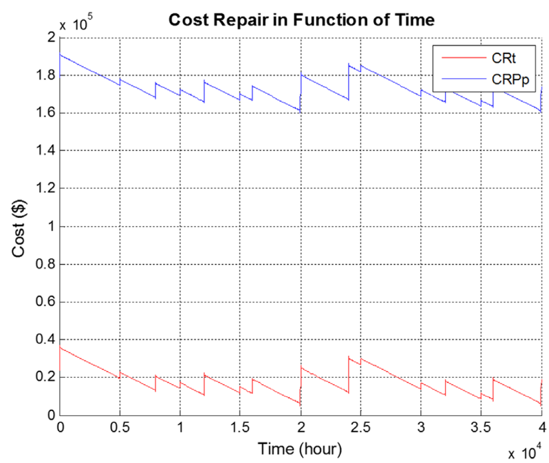


Figure 13. POPMP, cost of preventive repair x time.

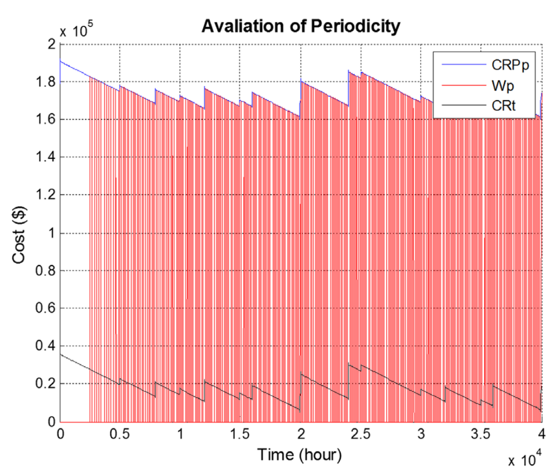


Figure 14. POPMP, evaluation of the periodicity.

In Figure 16, observes the behavior of the total residual cost in function of the periodicity. It is possible to observe that the smaller the preventive periodicity, greater are the residual costs. That means that the smaller the periodicity, the total life of the components will be used less, there is, therefore, a greater occurrence of premature exchanges.

For each periodicity simulated, being that the number of simulated periodicity depends on the difference between the minimum and the maximum useful life of the system, the system calculates the cost of preventive repair in function of periodicity, according to the Equation 6 (see Figure 17).

After calculating the cost of preventive repair in function of the periodicity, the system calculates the cost of corrective repair in function of the periodicity according to the Equation 7. It is worth remembering that, for each component that have a shorter life that the evaluated periodicity, the program considers a corrective event, calculating its cost from the cost of the component, MTTRc (medium time to repair corrective) of the respective component, and outgoing time of the process (see Figure 17), variable CRPc of the legend.

Finalizing with the Equation 8, comes the total cost of maintenance for the useful life of evaluation in function of the periodicity (see Figure 18).

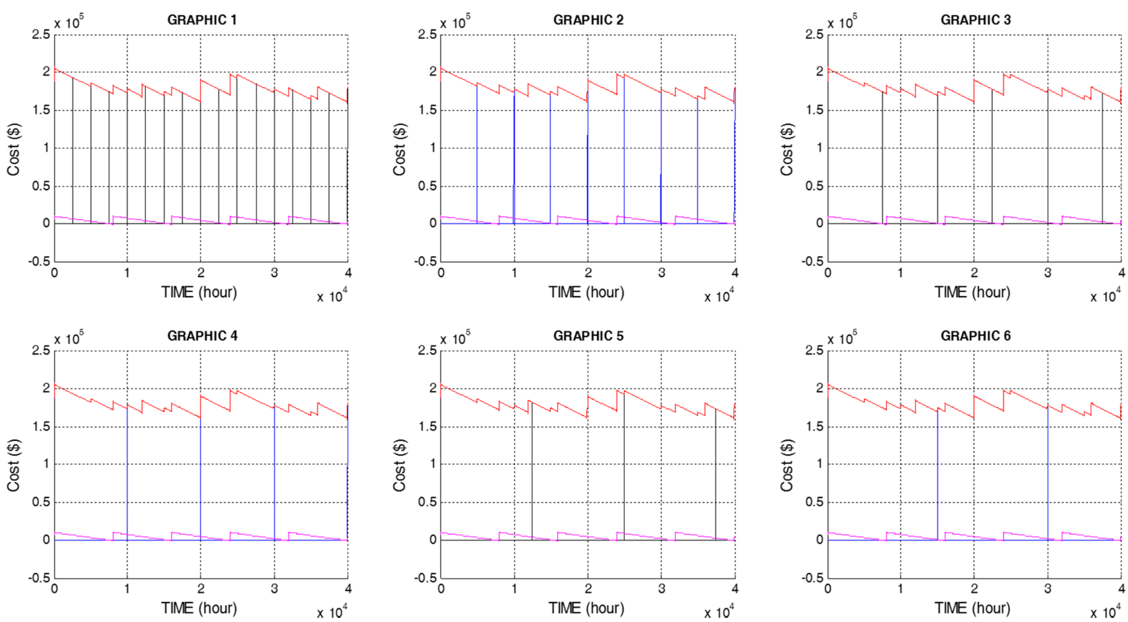


Figure 15. POPMP, interaction of the periodicities.

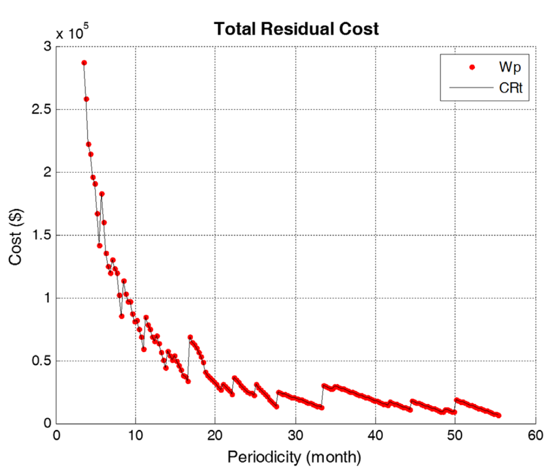


Figure 16. POPMP, total residual cost in function of the periodicity.

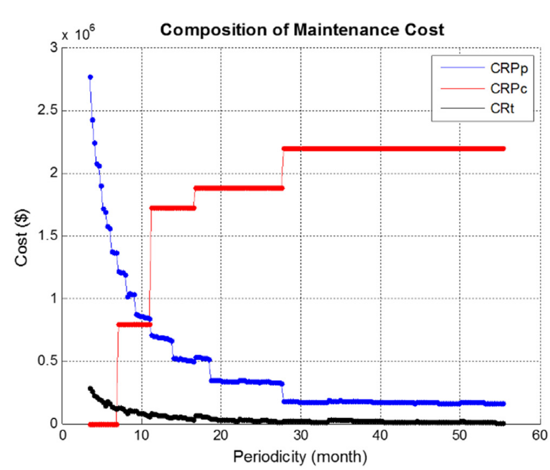


Figure 17. POPMP, cost of preventive repair in function of the periodicity.

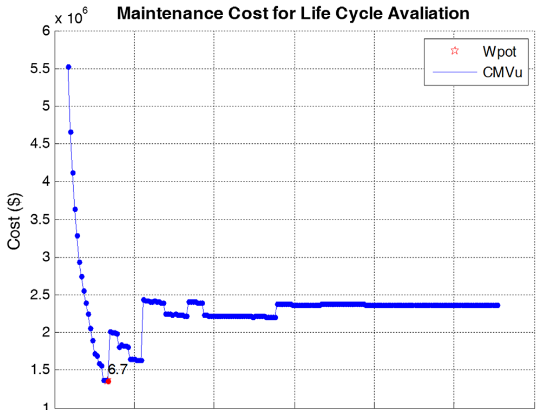
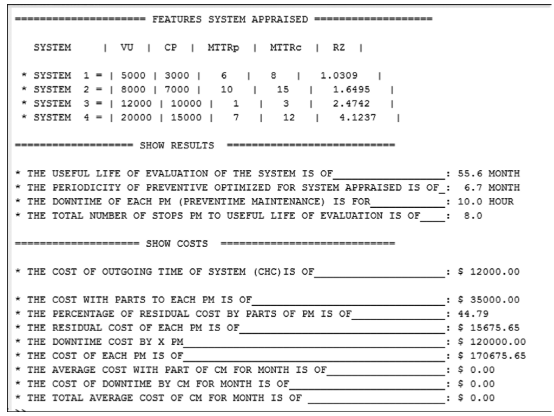


Figure 18. POPMP, maintenance cost of useful life evaluated in function of the periodicity.



```
==================== FEATURES SYSTEM APPRAISED ==================

   SYSTEM     |  VU  |  CP  |  MTTRp  |  MTTRc  |  RZ  |

 * SYSTEM  1 = | 5000 | 3000 |    6    |    8    |   1.0309   |
 * SYSTEM  2 = | 8000 | 7000 |    10   |    15   |   1.6495   |
 * SYSTEM  3 = | 12000 | 10000 |   1   |    3    |   2.4742   |
 * SYSTEM  4 = | 20000 | 15000 |   7   |    12   |   4.1237   |

=================== SHOW RESULTS  ===========================

* THE USEFUL LIFE OF EVALUATION OF THE SYSTEM IS OF__________________: 55.6 MONTH
* THE PERIODICITY OF PREVENTIVE OPTIMIZED FOR SYSTEM APPRAISED IS OF_:  6.7 MONTH
* THE DOWNTIME OF EACH PM (PREVENTIME MAINTENANCE) IS FOR____________: 10.0 HOUR
* THE TOTAL NUMBER OF STOPS PM TO USEFUL LIFE OF EVALUATION IS OF____:  8.0

=================== SHOW COSTS  ============================

* THE COST OF OUTGOING TIME OF SYSTEM (CHC)IS OF_____________________: $ 12000.00

* THE COST WITH PARTS TO EACH PM IS OF_______________________________: $ 35000.00
* THE PERCENTAGE OF RESIDUAL COST BY PARTS OF PM IS OF_______________: 44.79
* THE RESIDUAL COST OF EACH PM IS OF_________________________________: $ 15675.65
* THE DOWNTIME COST BY X PM__________________________________________: $ 120000.00
* THE COST OF EACH PM IS OF__________________________________________: $ 170675.65
* THE AVERAGE COST WITH PART OF CM FOR MONTH IS OF___________________: $ 0.00
* THE COST OF DOWNTIME BY CM FOR MONTH IS OF_________________________: $ 0.00
* THE TOTAL AVERAGE COST OF CM FOR MONTH IS OF ______________________: $ 0.00
```

Figure 19. POPMP, the output of the results.

It can be observed that for the system evaluated in the proposed modeling, the periodicity optimized is located in periodicity 6, 7 months (approximately 4.824 hours). Thus, if its elaborated a plan of maintenance planning a stop of 10 hours (see maximum MTTRp in Figure 10) to each 6,7 months, with the replacement of all four components of the system, it will be obtained the lowest cost of maintenance for the life of 55,6 months of the system.

According to the results presented in Figure 19, the total accumulated cost of maintenance for the periodicity of 6,7 months, is approximately R$ 1.359.000,00 for a life of 55,6 months. Each impairment will have a total maintenance cost of approximately R$ 170.675,65. It can be observed that the most representative is the cost of downtime of R$ 120.000,00 process and, for the simulation made, it has no occurrence of corrective for the evaluated systems, due to the great periodicity having an inferior time to the lowest useful life of component that, in the model, is 5.000 hours, approximately 6, 9 months.

4.3 CHC influences

Another important observation that can be obtained in the simulation is the influence of the CHC (outgoing time of the system).

To obtain this evaluation, the system will considerate the same model of Figure 10 by inserting a sensitivity analysis of the simulation. Starts the simulation with CHC equal to “zero” and increase its value of R$ 2.000,00 to each simulation, according to Figure 20.

It is observed that the values of CHC are represented by thousands, CHC x R$ 1.000.

As is incremented the value of CHC, it is observed a converged reduction of periodicity; that occurs due to the residual cost “CR” (due to the premature replacements) lose their significance in relation to the CHC. In this sense, the processes in which the cost of outgoing time has a ratio higher than the cost

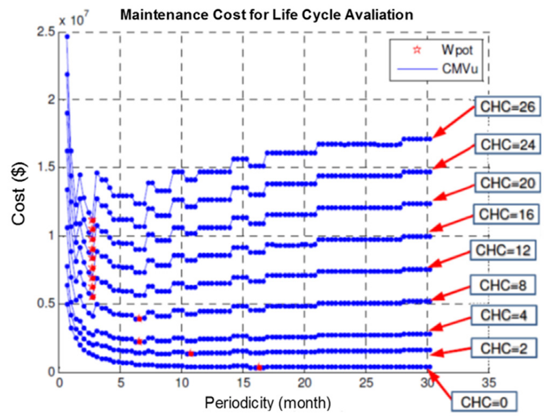


Figure 20. POPMP, CHC influence in the periodicity.

of the components, the premature replacement of components is advisable to increase the availability of the equipment, avoiding corrective events. It is emphasized that for the process in which CHC has a ratio higher than cost of the components, the corrective events have most significant costs, due to MTTRc be in most situations superior to MTTRp. For this reason, the greater the CHC the lesser will be the periodicity, to avoid corrective events.

Another observation refers to the occurrence of CHC very low: observe the curve where CHC = 0, the periodicity tends to increase, surpassing even the useful life of the component that has the most useful life. That means that for the cases in which the CHC is negligible, the tendency is to apply the strategy of corrective maintenance (MC), whose residual cost will be equal to “zero”, because there is no premature replacement.

5 Conclusions

This article shows a mathematical modeling to optimize periodicity of MP by means of the modeling of industrial systems. Its application enables observe the variation of financial impact in function of the periodicity and conclude that the premature exchange of component is necessary for a certain type of process. In Figure 16 is possible to observe the behavior of the residual cost in function of the periodicity, necessary information for the maintenance manager to make a decision.

In Figures 17 and 18 is possible to observe the behavior of the cost of corrective and preventive maintenance, and which periodicity provides lower maintenance cost over the life of the process.

It is also possible to observe the practical point of view of the modeling. It is known that most of the teams of maintenance of various segments develops knowledge of the behavior and the useful life of their systems; however, when there is need to rearrange all these systems to calculate the great periodicity, that would provide greater financial result to the process, these professionals have difficulty, because the modeling is laborious. In possession of a modeling, the maintenance manager have conditions to optimize the plan of preventive maintenance and the times of impairment.

It can be conclude that the mathematical modeling implemented as a computational program "POPMP" is of extreme importance to calculate the great periodicity of preventive maintenance of the industrial process and to provide a good visibility of the maintenance costs of the processes. This modeling ensures a periodicity of preventive maintenance that delivers the reliability suitable for each process, in function of the profitability of each business, without overloading the maintenance costs or the costs generated by low availability.

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