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Establishment of Satellite Formation with Initial Uncertainty by Control Lyapunov Function Approach

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Abstract: *In recent years, dynamics and control of satellite formation flying have been active areas of research. From the mission planning perspective, three main areas namely formation establishment, maintenance and reconfiguration have been discussed. In this paper, a study of formation establishment under initial uncertainty is presented. In this regard, dynamics of low Earth orbit satellite formation is discussed. Control Lyapunov function approach is adopted to bring a deputy satellite, with perturbed initial conditions into formation with a chief satellite. In order to take account of the initial orbit insertion error, uncertainty in initial conditions of the deputy satellite is considered. For a case study, a relatively small formation is adopted, with air-launched Pegasus as the launch vehicle. For several initial conditions, control function and required time to achieve a given mission accuracy are determined, and results are provided as illustration.*

Keywords: *Satellite Formation, Lyapunov Function, Dynamics, Control.*

INTRODUCTION

In recent years, there has been an increasing tendency to replace single large satellites with several small ones in formation flying (Baoyin *et al*., 2002; Kapila *et al*., 1999; Navabi *et al*., 2011; Orr *et al*., 2007; Carpenter *et al*., 2003; Schaub, 2004; Sabolet *et al*., 2001). For that purpose, dynamics of formation flying must be comprehensively studied, and effective control schemes must be devised. Formation control is carried out either by continuous or impulsive methods (Alfriend *et al*., 2009). Two of the most popular continuous methods are the Control Lyapunov Function (CLF) and the Linear Quadratic Regulator (LQR). Also, from the mission planning perspective, control methods can be studied in the context of formation establishment, maintenance, and reconfiguration (Alfriend *et al*., 2009).

Satellite formation missions with Projected Circular Orbit (PCO) in the local horizon plane are amongst promising missions and thus are considered here (No *et al*., 2009). Applications of these formations in missions, such as interferometric measurements, have been the theme study in papers such as in Peterson and Zee (2008). Yet, in the establishment phase, launch insertion errors frequently impose uncertainties on initial conditions of the formation and hence PCO conditions are violated. In this paper, applicability and effectiveness of CLF approach to bring a satellite formation into a specific PCO configuration are studied. In order to take account of the initial insertion errors, uncertainty in initial conditions of the deputy satellite is considered. From the previous launch logs, insertion error can be statistically computed. In most related literature, such as that of Pegasus XL from Orbital Sciences Corporation (2000), insertion error is given in terms of semi-major axis and inclination deltas from the intended values, and the same approach is taken here. Control laws to bring the satellites into a formation with a given position error budget are derived. Then, a case study of satellite formation with PCO is considered. It is assumed that satellites are placed in orbit by Pegasus XL. From the Pegasus launch log, three-sigma deltas of semi-major axis and inclination are derived. With initial conditions under uncertainty, the required time to bring the satellites into given bounded limits is determined and discussed.

The rest of the paper is as follows. In the next section, mathematical modeling of dynamics of satellite formation with PCO configuration is presented. Also, CLF to establish a formation with given bounded limits is discussed. Furthermore, a case study is developed and uncertainties in initial conditions due to insertion errors of Pegasus are considered. In the remaining chapters, results are presented and discussed, and possible future works are highlighted.

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MATHEMATICAL MODELING OF DYNAMICS AND CONTROL METHODOLOGY

Ordinary Differential Equations (ODE) can be employed to model dynamics of satellite formation flying (Hang *et al.*, 2008). Several ODE models were discussed in a previous work (Navabi *et al*., 2011), and their sensitivity to J_2 perturbation and eccentricity was investigated. It was found that the perturbed nonlinear model exhibits accurate results in all studied scenarios. Thus, the same model is used in this paper. Control methodology pursued here is the popular CLF approach. Mathematical foundations of dynamics and control employed in this paper are further discussed.

DYNAMICS OF SATELLITE FORMATION FLYING

The ODE approaches undertaken in dynamics modeling of satellite formation flying are often described in two main Cartesian coordinates. The first coordinate employed in this context is the earth-centered inertial (ECI) frame. In this frame, the fundamental plane is the equator, the unit vector $\hat{x}$ is directed from the Earth center towards the Vernal Equinox, $\hat{z}$ is normal to the fundamental plane and points in the Northward direction, $\hat{Y}$ completes the triad. This inertial coordinate frame is shown in Fig. 1

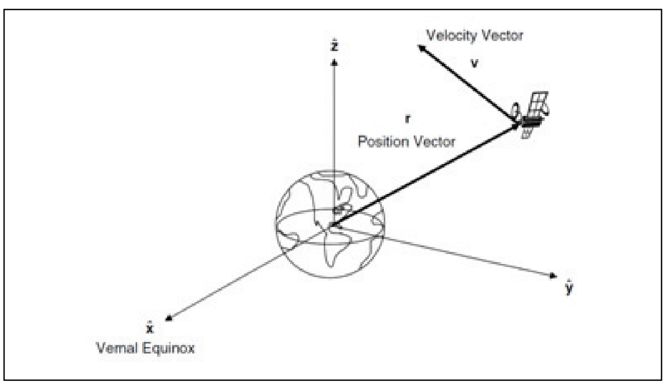

Figure 1. Earth-centered inertial frame.

It is assumed that the chief satellite is in free motion, and only the deputy satellite can be controlled to maintain a given desired relative orbit. If position vector of the chief satellite is denoted with $\mathbf{r_c}$ and that of the deputy with $\mathbf{r_d}$, one may obtain the perturbed nonlinear model as Eq. 1.1 and 1.2:

$$\ddot{\mathbf{r}}_c = \Gamma(\mathbf{r}_c) \tag{1.1}$$

$$\ddot{\mathbf{r}}_d = \Gamma(\mathbf{r}_d) + \mathbf{u} \tag{1.2}$$

where $\Gamma(\mathbf{r})$, taking account of J_2 effects, is defined as Eq. 1.3:

$$\Gamma(\mathbf{r}) = -\mu\frac{\mathbf{r}}{r^3} - \frac{3}{2}J2\left(\frac{\mu}{r^2}\right)\left(\frac{Re}{r}\right)\begin{pmatrix}\left(1-5\left(\frac{Z}{r}\right)^2\right)\frac{X}{r}\\ \left(1-5\left(\frac{Z}{r}\right)^2\right)\frac{Y}{r}\\ \left(3-5\left(\frac{Z}{r}\right)^2\right)\frac{Z}{r}\end{pmatrix} \tag{1.3}$$

In Eq. 1.3, μ is Earth gravitational parameter, J_2 is a coefficient of Earth gravitational harmonics, and R_e is equatorial radius of the Earth. Also, *X,Y,Z* are position coordinates in the ECI frame. If the desired relative orbit of deputy satellite is designated $\mathbf{r}_d$, position error of the deputy shown by $\delta\mathbf{r}$ is as Eq. 1.4. The control method for this purpose is discussed in the next subsection.

$$\delta\mathbf{r} = \mathbf{r}_d - \mathbf{r}_{dd} \tag{1.4}$$

It must be mentioned that the goal of CLF approach in this paper is to eliminate this position error. Yet, if one considers Eq. 1.4 as an error index, positive and negative errors may cancel out each other. Thus, l^2 norm of Eq. 1.4 is adopted as the error index, as shown by Eq. 1.5:

$$\partial_m = \left(\sqrt{(X_{d-}X_{d_d})^2 + (Y_{d-}Y_{d_d})^2 + (Z_{d-}Z_{d_d})^2}\right)_m \tag{1.5}$$

where, *m* is the index of simulation time steps.

In order to quantify the expression in Eq.1.4, orbits of the chief and deputy satellite and also the desired relative orbit must be determined. For that purpose, initial conditions for solving Eq. 1.1 can be either given by inertial states of the chief satellite or by its orbital elements. For the desired relative orbit, PCO conditions are considered. Therefore, desired relative orbit is given by chief satellite orbital elements added by certain delta values derived from PCO conditions, which are given by Eq. 1.6 (Alfriend *et al.*, 2009):

$$\begin{aligned}
&\delta\lambda(0) + \delta\Omega(0) * \cos(i_c) = \frac{\rho(0)e_c}{2a_c}\cos(\omega_c(0) + \alpha(0))\\
&e_d\sin(\delta M(0)) = \frac{\rho(0)}{2a_c}\cos(\omega_c(0) + \alpha(0))\\
&e_d\cos(\delta M(0)) = e_c - \frac{\rho(0)}{2a_c}\sin(\omega_c(0) + \alpha(0))\\
&\delta\Omega(0) = -\frac{\rho(0)}{a_c\sin i_c}\sin\alpha(0)\\
&\delta i = \frac{\rho(0)}{a_c}\cos(\alpha(0)
\end{aligned} \tag{1.6}$$

where indexes " c " and " d " correspond to chief and deputy satellite, respectively.

In Eq. 1.6, $a, e, i, \omega, \Omega, M$ are the classical orbital elements and $\lambda = M + \omega$. $\rho(0)$ and $\alpha(0)$ are initial PCO radius and phase angle in the magnitude-phase form. δa is obtained from no along-track d ⇄↗ ↗ ○ndition or the so-called J_2-invariant condition, given by Eq. 1.7 (Alfriend *et al*., 2009):

(1.7)

In Eq. 1.7, $\eta = \sqrt{1-e^2}$ Initial conditions of the deputy satellite are considered as those of the desired relative orbit with some initial uncertainty due to insertion error. In a compact form, one may write Eq. 1.8:

$$\Xi_d = \Xi_{d_d} + \delta\Xi_{insertion\ error} = \Xi_c + \delta\Xi_{PCO} + \delta\Xi_{insertion\ error} \quad (1.8)$$

where, Ξ_d and Ξ_{d_d} are the actual and desired orbital elements sets of the deputy satellite, and Ξ_c is the orbit al elements set of the chief satellite. $\delta\Xi_{PCO}$ are the expressions given by Eqs. 1.6 and 1.7. Also, the term $\delta\Xi_{insertion\ error}$ is considered to accommodate uncertainties in initial conditions of the deputy satellite orbit and it is quantified based on previous insertion error history of the launch vehicle of the formation. In the following sections, insertion error of Pegasus will be adopted as a case study.

With all the initial conditions given, dynamics modeling in ECI frame is accomplished. However, in most formation flying applications, it is desirable to visualize the formation in the rotating so-called Hill frame, shown in Fig. 2.

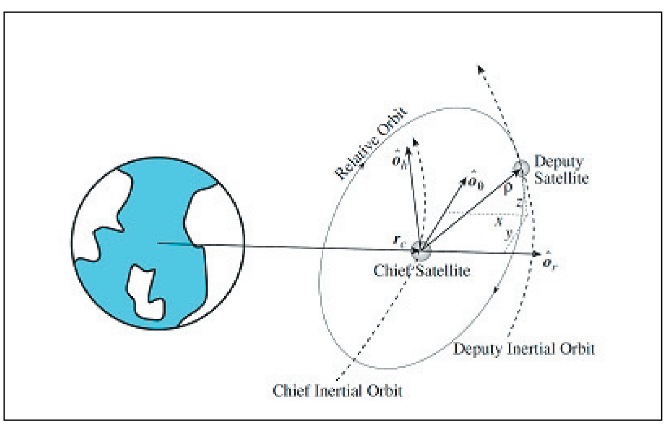

Figure 2. A general satellite formation in the Hill frame.

As it can be seen from Fig. 2, $\hat{O}_r, \hat{O}_\theta$ and $\hat{O}_h$ are Hill's unit vectors. These unit vectors are in radial, tangential, and orbit momentum direction, respectively. The transformation from ECI reference frame to Hill frame is given by Eq. 1.9.

$$T(\Omega, i, \theta) = \begin{bmatrix} c_\Omega c_\theta - s_\Omega s_\theta c_i & s_\Omega s_\theta + c_\Omega s_\theta c_i & s_\theta s_i \\ -c_\Omega s_\theta - s_\Omega c_\theta c_i & -s_\Omega s_\theta + c_\Omega c_\theta c_i & c_\theta s_i \\ s_\Omega s_i & -c_\Omega s_i & c_i \end{bmatrix} \quad (1.9)$$

where c_ξ and s_ξ denote cosine and sine of a given argument ξ. For illustration of our results regarding desired relative orbit and actual position of the deputy satellite, Hill frame will be used. As it was mentioned, in our context, it is desired to eliminate position error given by Eq. 1.4. The control method for this purpose is discussed in the next subsection.

CONTROL LYAPUNOV FUNCTION FOR SATELLITE FORMATION FLYING

To establish a formation within a bounded limit, in Schaub and Junkins (2002), the following form of Lyapunov function V was defined (eq. 1.10):

$$V = (\delta \boldsymbol{r}, \delta \dot{\boldsymbol{r}}) \frac{1}{2} \delta \dot{\boldsymbol{r}}^T \delta \dot{\boldsymbol{r}} + \frac{1}{2} \delta \boldsymbol{r}^T [K_1] \delta \boldsymbol{r} \quad (1.10)$$

where the 3x3 matrix $[K_1]$ is a positive definite position feedback gain and $(.)^T$ denotes transpose. It can be seen that Eq. 1.10 is positive definite. If one considers an equivalent dot product form of Eq. 1.10 as $V(\delta\boldsymbol{r}, \delta\dot{\boldsymbol{r}}) = \frac{1}{2}\delta\dot{\boldsymbol{r}}.\delta\dot{\boldsymbol{r}} + \frac{1}{2}\delta\boldsymbol{r}.([K_1]\delta\boldsymbol{r})$ and takes its derivative, one may find Eq. 1.11:

$$\dot{V} = \delta \dot{\boldsymbol{r}}^T (\ddot{\boldsymbol{r}}_d - \ddot{\boldsymbol{r}}_{d_d} + [K_1]\delta \boldsymbol{r}) \quad (1.11)$$

Substituting Eq. 1.2 for the 1.11 and for a J_2-invariant desired relative orbit, the following relationship is obtained:

$$\dot{V} = \delta \dot{\boldsymbol{r}}^T (\Gamma(\mathbf{r}_d) + \boldsymbol{u} - \Gamma(\mathbf{r}_{d_d}) + [K_1]\delta \boldsymbol{r}) \quad (1.12)$$

where $\Gamma(\mathbf{r})$ is given by Eq. 1.3.

To obtain a negative definite expression for $\dot{V}$, the following form is adopted for control law (Eq. 1.13):

$$u = -(\Gamma(\mathbf{r}_d) - \Gamma(\mathbf{r}_{d_d})) - [K_1]\delta \boldsymbol{r} - [K_2]\delta \dot{\boldsymbol{r}} \quad (1.13)$$

where the 3x3 $[K_2]$ matrix is a positive definite velocity feedback gain. From Eqs. 1.12 and 1.13, one can obtain the following relationship for $\dot{V}$ (Eq. 1.14):

$$\dot{V} = -\delta\dot{\boldsymbol{r}}^T[K_2]\delta\dot{\boldsymbol{r}} \qquad (1.14)$$

In order to ensure asymptotic stability, from Eq. 1.14 one may obtain $\delta\dot{\boldsymbol{r}}^T\ddot{V} = -2\delta\dot{\boldsymbol{r}}^T\delta\ddot{\boldsymbol{r}}.[K_2]\delta\dot{\boldsymbol{r}}$. Also, from Eq.1.11, $\delta\dot{\boldsymbol{r}}^T\delta\ddot{\boldsymbol{r}} = \dot{V} - \delta\dot{\boldsymbol{r}}^T[K_1]\delta\boldsymbol{r}$. After simplification, one obtains Eq. 1.15:

$$\ddot{V} = 2([K_2]\delta\dot{\boldsymbol{r}} + [K_1]\delta\boldsymbol{r})^T[K_2]\delta\dot{\boldsymbol{r}} \qquad (1.15)$$

One may note that $\ddot{V} = (\delta\dot{\boldsymbol{r}} = 0) = 0$ and:

$$\dddot{V} = 2(([K_2]\delta\ddot{\boldsymbol{r}} + [K_1]\delta\dot{\boldsymbol{r}})^T[K_2]\delta\dot{\boldsymbol{r}} + ([K_2]\delta\dot{\boldsymbol{r}} + [K_1]\delta\boldsymbol{r})^T[K_2]\delta\ddot{\boldsymbol{r}}) \qquad (1.16)$$

Evaluating Eq. 1.11 at $\delta\dot{\boldsymbol{r}} = 0$ and noting that $\dot{V} < 0$, one obtains $\delta\ddot{\boldsymbol{r}} = -[K_1]\delta\boldsymbol{r}$. Substituting the previous expression in Eq.1.16, it is easy to show that $\dddot{V}(\delta\dot{\boldsymbol{r}} = 0) = -2\delta\boldsymbol{r}^T[K_1]^T[K_2][K_1]\delta\boldsymbol{r} < 0$ and thus, asymptotic stability of CLF approach is ensured (Mukherjee and Chen, 1993). In the next section, a case study will be discussed for illustration.

CASE STUDY

In this section, a case study was developed to investigate effectiveness of the methodology already presented. In accordance with a previous work (Navabi *et al*., 2011), the following initial conditions are considered for the chief satellite, given in Table 1. Desired orbital elements of the deputy satellite are those of the chief satellite given in Table 1 added by delta values given by Eqs. 1.6 and 1.7. In Orr *et al*. (2007) and No *et al*., (2009), two case studies were discussed in which PCO radius had the value of 1 and 0.5km, respectively. Here, it is assumed that radius of PCO is 0.5km and also phase angle is 120º. Table 2 provides the desired orbital elements of deputy satellite.

To fully describe input data for this case study, initial uncertainty in deputy satellite initial conditions is discussed. As it was mentioned, this uncertainty is adopted as a mechanism to accommodate insertion error of the launch vehicle in formation establishment phase. To this date, few practical formation flying missions have been realized and thus there are not sufficient data regarding relative insertion error for such missions. As a result, relative insertion error is taken as a percentage of the absolute insertion error. For Pegasus XL, the absolute insertion errors are given in Table 3.

Table 1. Initial conditions of the chief satellite.

Orbit Element	Value	Dimension
a	7032.9998	Km
e	0.00998	-
i	97.9980	Deg
Ω	9.9964	Deg
ω	30.1762	Deg
f	-0.1803	Deg

Table 2. Desired initial conditions of the deputy satellite.

Orbit Element	Value	Dimension
a	7033	Km
e	0.0100	-
i	98	Deg
Ω	10	Deg
ω	30	Deg
f	0	Deg
Orbital period: 5869.8 seconds		

Table 3. Three-sigma absolute insertion errors of Pegasus XL.

	Delta in semi-major axis	Delta in inclination
Pegasus XL with HAPS configuration	±15 km	±0.08º

Relative insertion errors for formation establishment are assumed to be from 1 to 5% of the three-sigma absolute insertion errors. Assuming 1% step, five scenarios for relative insertion error of semi-major axis and inclination are obtained. For semi-major axis, the five insertion error scenarios are: $\delta a_{insertion\ error}$=150, 300, 450, 600, 700m.

And for inclination delta due to insertion error: $\delta a_{insertion\ error}$= 0.0008, 0.0016, 0.0024, 0.0032, 0.0040deg.

Thus, 25 scenarios must be studied to accommodate initial uncertainty in deputy orbital initial conditions, due to initial insertion error. A maximum acceptable error index of is considered so that response time of CLF approach for various initial conditions can be compared. A MATLAB® code was developed to fully study this case study. Results are given in the next section.

RESULTS AND DISCUSSION

For the case study presented in the previous section, simulations were carried out and results are given in this section. Initial conditions of the chief satellite orbit and those of desired relative orbit are constant in all scenarios, but initial conditions of the deputy satellite vary in each scenario. For the first scenario, initial errors in insertion are:

$$\delta\Xi_{insertion\ error} = \begin{cases} \delta a_{insertion\ error} = 150\text{m} \\ \delta i_{insertion\ error} = 0.0008\ \text{deg} \end{cases}$$

For illustration, in this scenario, Fig. 3 depicts the desired relative orbit along with the actual formation, without any control effort.

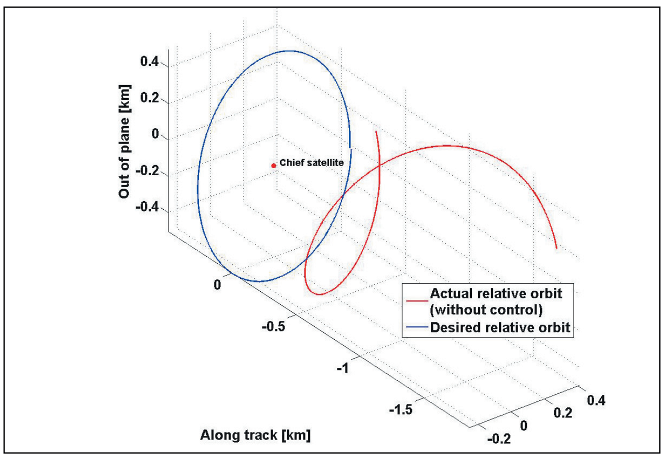

Figure 3. Actual (without control) and desired relative orbits in Hill frame.

As it can be seen from Fig. 3, actual and desired relative orbits do not initially coincide, due to the orbit insertion error. If not controlled, the deputy satellite will gradually drift away. In order to control the formation, control law given in Eq. 1.13 must be applied. In Fig. 4, control accelerations for this scenario are shown in the ECI frame.

With the above control accelerations, simulations resulted in a controlled orbit with prescribed accuracy, as shown in Fig. 5.

In Fig. 5, the desired relative orbit and its projections in three orthogonal, *xy*, *xz*, *yz*, planes are shown. As it can be seen, the *yz* projection is a circle of radius 0.5km. Also, in Fig.5, controlled actual orbit of the deputy satellite is given. In Fig. 6, time history of error index (Eq. 1.5) is presented.

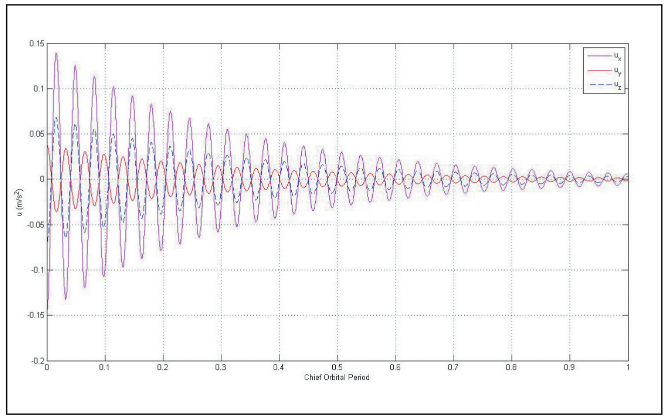

Figure 4. Control accelerations in the earth-centered inertial frame.

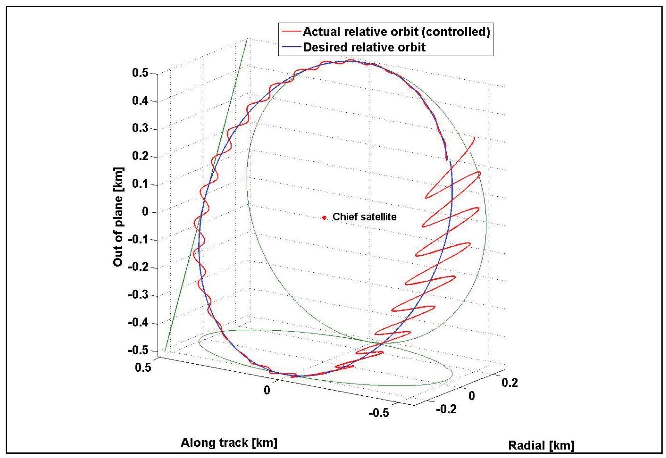

Figure 5. Controlled actual and desired relative orbits in the Hill frame.

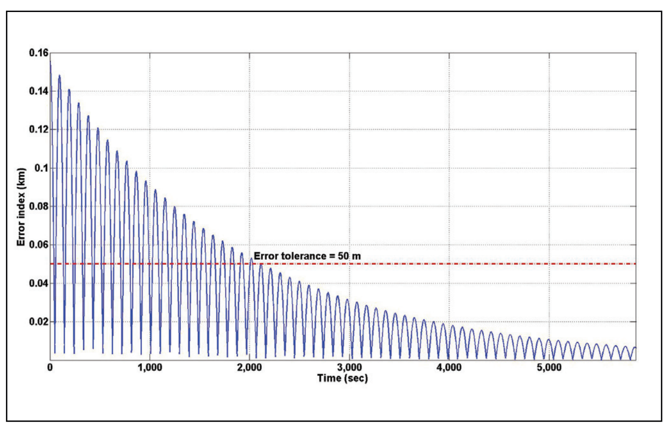

Figure 6. Error index time history.

From Fig. 6, initial error of the formation is 156.3m, which corresponds to $\delta a_{insertion\ error}$= 150m and $\delta a_{insertion\ error}$= 0.0008*deg*. At t = 2,114 seconds, i.e., in less than half a period of the chief satellite, formation is controlled in a manner that maximum acceptable error criterion is satisfied.

Including similar figures for the remaining, 24 scenarios would make this paper prohibitively long and, thus, results are summarized in Table 4.

Data presented in Table 4 are organized as follows. In each five successive scenarios, i.e., scenario numbers 1-5, 6-10, 11-15, 16-20, and 21-25, inclination error is propagated from 0.0008 to 0.0040 with steps of 0.0008. In the first five scenarios, δa = 150m and in the next five scenarios (scenario no. 6-10), δa = 300m. In the third, fourth and fifth five scenarios, i.e. scenarios numbers 11-15, 16-20 and 21-25, δa = 450, 600, 750m, respectively. The scenarios discussed in Table 4 have been chosen in a manner so that initial error follows a rising tendency with scenario number.

From Table 4, in the first scenario which has minimum insertion error (∂_0 = 156.3m), it takes 2,114 seconds for CLF approach to bring and keep the deputy and chief satellites to proximity of less than 50m (Fig. 6). In the fifth scenario, inclination error is largest and initial position error is approximately 285m. For this scenario, it takes 3,177 seconds (slightly more than half a period of chief satellite) to establish the formation, with maximum acceptable error of 50m. In the second five scenarios where δa = 300m, minimum and maximum insertion error is 301m (scenario number 6) and 384m (scenario number 10). For these scenarios, it takes CLF control law 3,272 and 3,753 seconds to establish the formation, respectively. Initial insertion error in scenario numbers 11 and 15 are 448m and 507m. For these scenarios, required time for formation establishment by CLF approach is 4,041 and 4,322 seconds, respectively. In the 16th and 20th scenarios, initial insertion error is 596m and 642m and required time for formation establishment with these initial errors is 4,611 and 4,711 seconds, respectively. Finally, for

Table 4. Response time of formation control based on Control Lyapunov Function approach for several initial conditions.

Scenario #	Initial error (m)	Time to maximum acceptable error (seconds)
1	$\delta a = 150,\ \delta i = 0.0008 \rightarrow \partial_0 = 156.3$m	2,114
2	$\delta a = 150,\ \delta i = 0.0016 \rightarrow \partial_0 = 177.5$m	2,311
3	$\delta a = 150,\ \delta i = 0.0024 \rightarrow \partial_0 = 208.1$m	2,600
4	$\delta a = 150,\ \delta i = 0.0032 \rightarrow \partial_0 = 244.6$m	2,889
5	$\delta a = 150,\ \delta i = 0.0040 \rightarrow \partial_0 = 284.8$m	3,177
6	$\delta a = 300,\ \delta i = 0.0008 \rightarrow \partial_0 = 301.0$m	3,272
7	$\delta a = 300,\ \delta i = 0.0016 \rightarrow \partial_0 = 312.5$m	3,366
8	$\delta a = 300,\ \delta i = 0.0024 \rightarrow \partial_0 = 330.9$m	3,463
9	$\delta a = 300,\ \delta i = 0.0032 \rightarrow \partial_0 = 355.0$m	3,653
10	$\delta a = 300,\ \delta i = 0.0040 \rightarrow \partial_0 = 383.8$m	3,753
11	$\delta a = 450,\ \delta i = 0.0008 \rightarrow \partial_0 = 448.2$m	4,041
12	$\delta a = 450,\ \delta i = 0.0016 \rightarrow \partial_0 = 456.0$m	4,041
13	$\delta a = 450,\ \delta i = 0.0024 \rightarrow \partial_0 = 468.8$m	4,134
14	$\delta a = 450,\ \delta i = 0.0032 \rightarrow \partial_0 = 486.1$m	4,229
15	$\delta a = 450,\ \delta i = 0.0040 \rightarrow \partial_0 = 507.5$m	4,322
16	$\delta a = 600,\ \delta i = 0.0008 \rightarrow \partial_0 = 596.0$m	4,611
17	$\delta a = 600,\ \delta i = 0.0016 \rightarrow \partial_0 = 601.9$m	4,614
18	$\delta a = 600,\ \delta i = 0.0024 \rightarrow \partial_0 = 611.6$m	4,616
19	$\delta a = 600,\ \delta i = 0.0032 \rightarrow \partial_0 = 625.0$m	4,708
20	$\delta a = 600,\ \delta i = 0.0040 \rightarrow \partial_0 = 641.8$m	4,711
21	$\delta a = 750,\ \delta i = 0.0008 \rightarrow \partial_0 = 744.1$m	4,999
22	$\delta a = 750,\ \delta i = 0.0016 \rightarrow \partial_0 = 748.8$m	5,001
23	$\delta a = 750,\ \delta i = 0.0024 \rightarrow \partial_0 = 756.7$m	5,001
24	$\delta a = 750,\ \delta i = 0.0032 \rightarrow \partial_0 = 767.5$m	5,091
25	$\delta a = 750,\ \delta i = 0.0040 \rightarrow \partial_0 = 781.2$m	5,095

scenario numbers 21 and 25, initial error is 744 and 781m and required time for formation establishment is 4,999 and 5,095 seconds.

It must be mentioned that even for the 25th scenario with worst-case insertion error, CLF can establish the formation in less than a chief satellite period. This finding proves effectiveness of CLF approach for initial formation establishment taking into account uncertainty due to initial insertion error. For any specific mission, one may adopt a similar approach, and considering various possible initial insertion errors, employ CLF for initial formation establishment

CONCLUSIONS

Dynamics of satellite formation flying was discussed. Initial conditions of a desired relative orbit, based on J_2-invariant and PCO configurations, were adopted. Uncertainty in initial condition was employed as a mechanism to take account of initial insertion error. CLF approach was adopted to establish satellite formation under these uncertainties. For a sunsynchronous chief satellite at 655km of altitude, it was shown that initial uncertainties in deputy satellite orbit is high 750m in semi-major axis and 0.004 degree in inclination can be corrected in less than a chief satellite period.

FUTURE WORK

In this study, required time for initial formation establishment by the means of CLF approach was discussed. In future works, this method will be extended by considering simultaneously required time and total control effort for formation establishment.

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