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Mode I+II Fatigue Crack Growth Delay by Stop-Holes

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ABSTRACT: The technique of retarding the growth of fatigue cracks by drilling holes on the crack tip is well known. Most of the research works on this subject are limited to fatigue cracks subjected to mode I loading conditions. In the present work the fatigue crack growth retardation by stop-holes of cracks under mode I+II loading is investigated. The proposed approximate solution is based on the implementation of a mixed-mode fatigue crack growth model and a multiaxial high cycle fatigue criterion. Numerical results for mode I+II fatigue crack growth retardation on Al-2024 thin plate are derived and the effect of the crack inclination angle, as well as the diameter of the stop-hole, are discussed and commented.

KEYWORDS: Structural integrity, Stop-holes, Mixed-mode loading, Multiaxial fatigue, Fatigue crack growth retardation.

INTRODUCTION

Since the drilling of a hole on the crack tip transforms the stress singularity of the crack tip to a notch, reducing thus the values of the stresses, it is a widely used technique for simple and economic repair method. Estimation of the delay (in terms of loading cycles) of the fatigue crack growth has been performed (e.g. Wu *et al.* 2010; Makabe *et al.* 2009; Ayatollahi *et al.* 2014; Fanni *et al.* 2015) to improve the structural integrity of cracked structures. However, most of the existing studies are focusing on cracks subjected to pure mode-I loading (Fig. 1), and few studies are analyzing cracks under pure mode-II loading (e.g. Ayatollahi *et al.* 2014). Since the structural integrity of aerospace structures is of vital importance for the aerospace industry (Mello Jr. and Mattos 2009; Mello Jr. *et al.* 2009; Mattos *et al.* 2009), the knowledge of the fatigue crack growth retardation due to stop-holes is important to improve the inspection plans.

Fatigue crack propagation is governed by the amplitude of the stress intensity factors (Fig. 1a). For any crack subjected to cyclic loading, fracture mechanics rules are used for fatigue crack propagation prediction. However, when a hole is drilled on the crack tip (Fig. 1b) the crack tip singularity is eliminated. The surface on the perimeter of the hole is smooth, crack initiation mechanisms take place, and the multiaxial stress state (instead of the stress intensity factor) is the driving force for fatigue damage accumulation (multiaxial fatigue). Then, crack initiation models (e.g. Pavlou 2018; Rege and Pavlou 2017) should be applied to estimate delay cycles.

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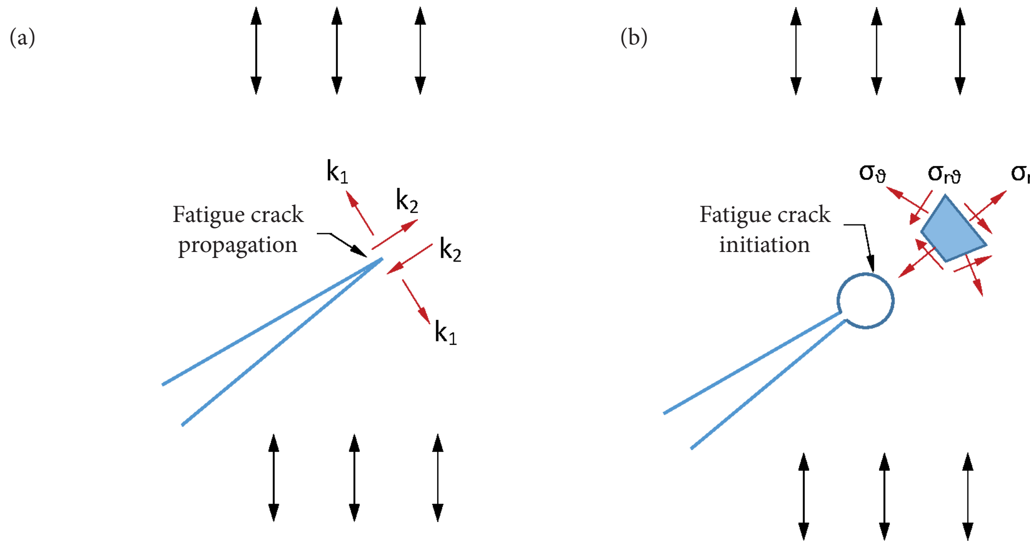

Figure 1. (a) Crack subjected to cyclic loading, (b) A hole drilled on the crack tip transforms the crack singularity to a smooth surface.

However, for cracks subjected to mode I+II loading (Fig. 2), the solution is more complicated due to the following reasons:

- The crack growth follows a zig-zag path. Therefore, the direction of crack deflection ϑ^* due to an inclined crack with inclination angle β should be initially calculated.
- Unlike the mode-I cracks, the stress state of a point A on the perimeter of the stop-hole is multiaxial (Fig. 2).
- The calculation of the k_1 and k_2 local stress intensity factors is not an easy task.

Therefore, the computation of the crack initiation period for the point A (Fig. 2) is based upon the computation of the stresses σ_{xx}, σ_{yy}, τ_{xy}. The values of these stresses depend on the angles β and ϑ^*. For multiaxial fatigue life prediction, several effective criteria have been published (Fatemi and Shamsaei 2011; Carpinteri *et al.* 2011; Mamiya *et al.* 2009; Liu and Mahadevan 2005; Lee *et al.* 2007). Among them, the criterion of the critical plane and the criterion of the equivalent stress are widely used. In the present work, the criterion of the equivalent stress is adopted due to its simplicity.

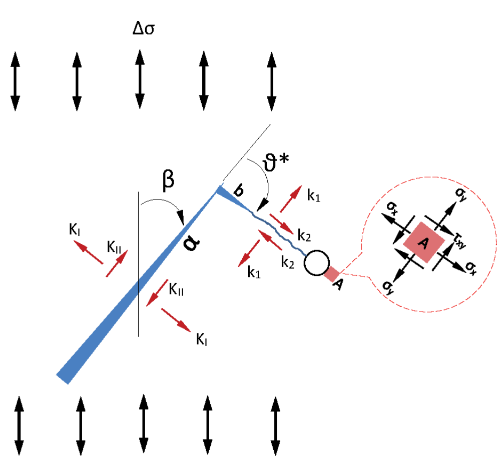

Figure 2. A mode I+II crack repaired by a stop-hole.

Regarding the crack propagation direction of cracks subjected to mode I+II loading, numerous criteria (models) have been published. The models can be classified in four categories: (a) stress-based criteria, (b) strain-based criteria, (c) crack tip displacement criteria, (d) energy-based criteria. The most known stress-based criterion is the Maximum Tangential Stress (MTS) criterion proposed by Erdogan and Sih (1963). This criterion assumes that crack propagates along the radial direction on which the stress σ_ϑ becomes maximum. Representative strain-based criterion is the maximum tangential strain criterion proposed by Chambers *et al.* (1991). According to this criterion, the maximum elastic tangential strain near the crack tip controls the mixed-mode crack propagation direction. A criterion proposed by Li (1989) assumes that mixed-mode crack propagates along the vector crack tip displacement (CTD). The main energy based criteria, namely the J-criterion, the T-criterion, and the S-criterion, have been proposed by Helen and Blackburn (1975), Theocaris and Andrianopoulos (1982), and Sih (1974), respectively. The J-criterion states that a mixed-mode crack propagates along the direction of the vector J. According to the T-criterion, the crack propagates along the maximum dilatational strain energy density direction. The S-criterion assumes that mixed mode crack propagates in a direction along with S-factor possesses a relative minimum. All the above criteria are based on energy density or stress or strain distribution before the event of crack propagation.

For the prediction of the direction ϑ^* of the propagation of a crack under mode I+II loading conditions, a model based on the principle of minimum potential energy, developed recently (Pavlou 2015a; Pavlou 2015b; Pavlou *et al.* 2004; Pavlou *et al.* 2003), is used in the present study. According to this model, an inclined crack (with inclination angle β) is going to propagate in a direction ϑ^* (Fig. 2), which yields minimum potential energy to the material (Fig. 3).

Even though the principle of the minimum potential energy was proposed many decades ago, its implementation on Fracture Mechanics was limited to take the derivative with respect to the crack length ($\partial\Pi / \partial\alpha$) in order to derive the formula for the fracture toughness. Pavlou 2015a, Pavlou 2015b, and Pavlou *et al.* 2004 proposed the condition of minimization of the potential energy with respect to the crack propagation angle $\partial\Pi / \partial\vartheta$ for the direction prediction of mode I+II fatigue crack growth. Verification on experimental results (Pavlou *et al.* 2004) has indicated that the predictions have been accurate.

It should be mentioned that there is no single model to provide accurate predictions for all materials, brittle or ductile, under all loading conditions (Qian and Fatemi 1996). Since most of the criteria are usually expressed in terms of the stress intensity factors, they are based on linear elastic fracture mechanics. All criteria are based on assumptions. Since the criterion of the minimum potential energy is not based on an assumption but on a physical principle, it has been selected in the present study as a tool for studying the effect of stop-holes on mode I+II fatigue loading conditions.

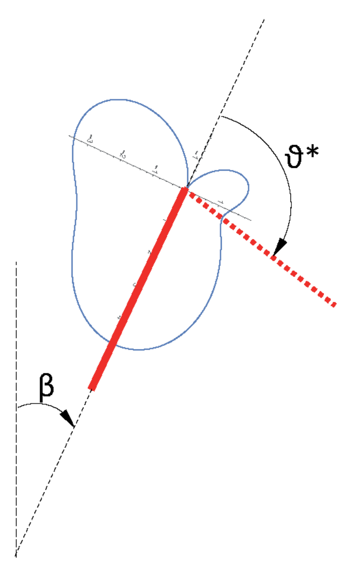

Figure 3. An inclined crack (with inclination angle β) propagates in a direction ϑ^* yielding minimum potential energy.

Due to the mode I+II conditions, the stresses in the vicinity of the perimeter of a stop-hole located on the branched crack (in direction ϑ*) are multiaxial. The analytic calculation of these stresses is a challenge, therefore, numerical analysis by using finite or boundary elements is the best practice. For the sake of simplicity, an analytic expression (Tada *et al.* 2000) valid for the points on the vicinity of the perimeter of the hole near the direction of the crack will be adopted. Then, the equivalent multiaxial fatigue criterion (Lee *et al.* 2012) will be used to calculate the number of cycles yielding fatigue crack initiation.

MIXED MODE FATIGUE CRACK GROWTH

As it has already been mentioned, the direction of the propagation of a fatigue crack under mode I+II conditions can be predicted by the Eq. 1:

$$\frac{\partial f(\vartheta)}{\partial \vartheta} = 0 \tag{1}$$

where: $f(\vartheta)$ is a function expressing the potential energy variation versus the angle ϑ of the direction of possible crack propagation (Fig. 3). Taking into account the references (Pavlou 2015a; Pavlou 2015b; Pavlou *et al.* 2004), the function to be minimized is Eq. 2:

$$f(\vartheta) = \lambda_1 k_1^2(\vartheta) + \lambda_2 k_1(\vartheta) k_2(\vartheta) + \lambda_3 k_2^2(\vartheta) \tag{2}$$

where::

$$\begin{Bmatrix} k_1(\vartheta) \\ k_2(\vartheta) \end{Bmatrix} = \begin{bmatrix} a_{11}(\vartheta) & a_{12}(\vartheta) \\ a_{21}(\vartheta) & a_{22}(\vartheta) \end{bmatrix} \begin{Bmatrix} K_I(\beta) \\ K_{II}(\beta) \end{Bmatrix} \tag{3}$$

$$a_{11}(\theta) = \frac{1}{4}\left[3\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right] \tag{4}$$

$$a_{12}(\theta) = -\frac{3}{4}\left[\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right] \tag{5}$$

$$a_{21}(\theta) = \frac{1}{4}\left[\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right] \tag{6}$$

$$a_{22}(\theta) = \frac{1}{4}\left[\cos\frac{\theta}{2} + 3\cos\frac{3\theta}{2}\right] \tag{7}$$

$$\begin{Bmatrix} K_I(\beta) \\ K_{II}(\beta) \end{Bmatrix} = \sqrt{\pi a}\,\sin\beta \begin{Bmatrix} \sin\beta \\ \cos\beta \end{Bmatrix} \tag{8}$$

$$\lambda_1 = 0.0000101419 \tag{9}$$

$$\lambda_2 \approx 0 \tag{10}$$

$$\lambda_3 = 0.0000300746 \tag{11}$$

Taking into account Eqs. 1 to 11, the crack propagation direction ϑ^* versus β (Fig. 3) can be obtained (Pavlou 2015a; Pavlou 2015b) by the Eq. 12:

$$\vartheta^* \text{ (deg)} = 114.348 - 0.155868\beta - 0.0129197\beta^2 \tag{12}$$

where the angle β should be placed in degrees.

STRESS STATE DUE TO A STOP-HOLE IN A BRANCHED CRACK

The stress state on a material element along the perimeter of a stop hole placed on the branch of a mode I+II fatigue crack (Fig. 2) is multiaxial. Analytic calculation of the stresses σ_{xx}, σ_{yy}, τ_{xy} is not an easy task, therefore, implementation of numerical methods (e.g. FEM or BEM) is the best practice. However, approximate formulae can be found in Tada *et al.* 2000, providing the stresses for small ϑ and $r \approx \rho$ (Fig. 4).

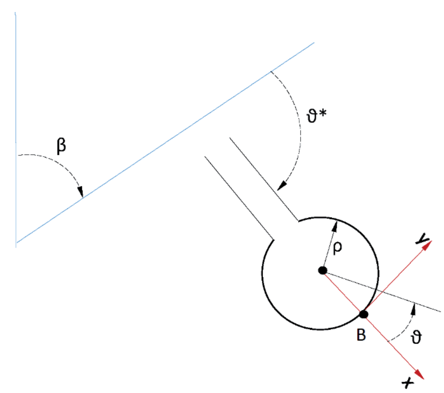

Figure 4. Topology of a hole placed on the branch of a mode I+II crack with inclination angle β.

According to Tada *et al.* (2000), the stresses on the point B in Fig. 4 can be obtained by the Eqs. 13 to 15:

$$\sigma_{xx} = -\frac{k_1}{\sqrt{\pi\rho}} \tag{13}$$

$$\sigma_{yy} = \frac{k_1}{\sqrt{\pi\rho}} \tag{14}$$

$$\tau_{xy} = -\frac{k_2}{\sqrt{\pi\rho}} \tag{15}$$

MULTIAXIAL FATIGUE MODEL

Taking into account the above stress state, a multiaxial fatigue crack initiation model should be used in order to estimate the delay (in terms of number of cycles) for further crack growth. Among the multiaxial fatigue crack initiation models, the equivalent stress model (Lee *et al.* 2012) is adopted due to its simplicity. According to this criterion, the equivalent stress is the driving force for the crack initiation. The Eq. 16 takes into account the equivalent stress amplitude (Eq. 17)

$$\sigma_{eq} = \left(\sigma_{VM,a} + a_{VM}\sigma_{VM,m}\right) \tag{16}$$

$$\sigma_{VM,a} = \frac{1}{\sqrt{2}}\sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a}\sigma_{y,a} + 3\tau_{xy,a}^2} \tag{17}$$

and the equivalent mean stress (Eq. 18)

$$\sigma_{VM,m} = \sigma_{x,m} + \sigma_{y,m} \tag{18}$$

The parameter a_{VM} reflects the effect of the mean stress (Eq. 19)

$$a_{VM} = \frac{\sigma_{E,R=-1} - \sigma_{E,R\neq-1}}{\sigma_{E,R\neq-1}} \tag{19}$$

where: $\sigma_{E,R} = -1$ is the fatigue endurance limit for fatigue loading with $R = \sigma_{min} / \sigma_{max} = -1$, i.e., $\sigma_m = 0$; and $\sigma_{E,R} \neq -1$ is the fatigue endurance limit for any loading with $R \neq -1$, i.e., $\sigma_m \neq 0$. The value of $\sigma_{E,R} = -1$ can be obtained by a standard S-N curve (Fig. 5).

For fatigue loading with $\sigma_m \neq 0$, the well-known Smith diagram (Fig. 6) should be used to calculate the value of $\sigma_{E,R} \neq -1$. In the present study, a fatigue loading with $\sigma_{\min} = 0$ (i.e. R = 0) or $\sigma_m = \sigma_a = \sigma_o / 2$ is considered (Fig. 7).

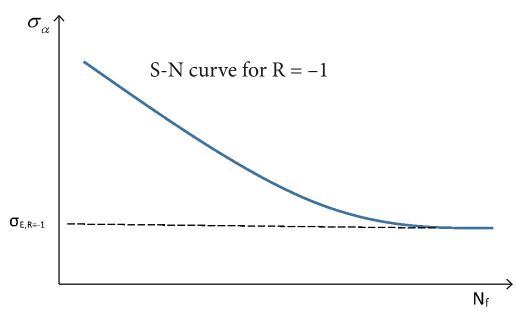

Figure 5. Schematic representation of an S-N curve.

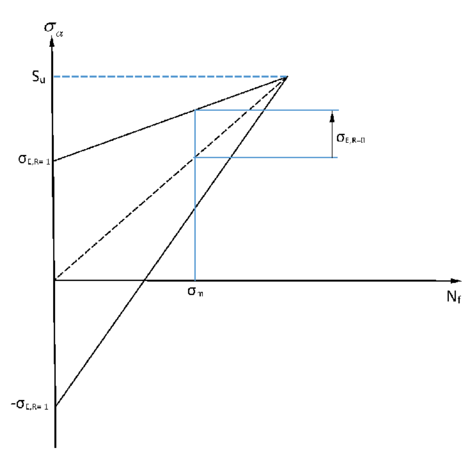

Figure 6. Smith diagram.

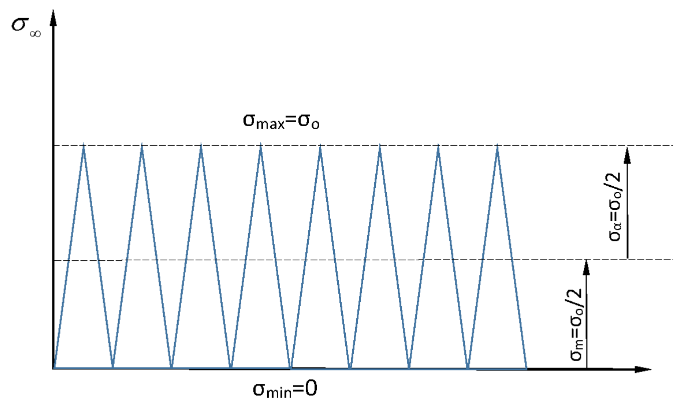

Figure 7. Fatigue loading type.

Therefore, the Smith diagram (Fig. 6) yields the Eq. 20:

$$\sigma_{E,R=0} = \left(\sigma_{E,R=-1} - \sigma_m\right) + \frac{\sigma_m}{S_u}\left(S_u - \sigma_{E,R=-1}\right) \tag{20}$$

The amplitude and the mean value of the stress components are given by the Eqs. 21 to 26:

$$\sigma_{x,a} = \sigma_x(\sigma_o/2) \tag{21}$$

$$\sigma_{y,a} = \sigma_y(\sigma_o/2) \tag{22}$$

$$\tau_{xy,a} = \tau_{xy}(\sigma_o/2) \tag{23}$$

$$\sigma_{x,m} = \sigma_x(\sigma_o/2) \tag{24}$$

$$\sigma_{y,m} = \sigma_y(\sigma_o/2) \tag{25}$$

$$\tau_{xy,m} = \tau_{xy}(\sigma_o/2) \tag{26}$$

RETARDATION OF THE FATIGUE CRACK GROWTH

Since the above stresses (Eqs. 21 to 26) are the consequence of the hole, the equivalent stress σ_{eq} given by Eq. (16) should be used to estimate the number of cycles causing crack initiation. To this end, a standard S-N curve can be used. For the well-known alloy Al-2024, an analytic expression (Eq. 27) (Siriwardane *et al.* 2010) can be adopted:

$$S = 118\left(\frac{N_f + 1023}{N_f + 1530000}\right)^{-0.1717} \tag{27}$$

If we replace the uniaxial stress amplitude *S* by the equivalent one, given by Eq. 16, the Eq. 28 for calculating the required cycles to cause fatigue crack initiation can be obtained:

$$N_f = \frac{-1023 + 1530000\left(\dfrac{\sigma_{VM,a} + a_{VM}\sigma_{VM,m}}{118}\right)^{-1/0.1717}}{1 - \left(\dfrac{\sigma_{VM,a} + a_{VM}\sigma_{VM,m}}{118}\right)^{-1/0.1717}} \tag{28}$$

NUMERICAL IMPLEMENTATION OF THE METHODOLOGY

A thin Al-2024 plate containing an inclined crack of 2.0 mm is subjected to fatigue loading with $\sigma_a = 90\ MPa$ and $\sigma_m = 90\ MPa$. A stop-hole is considered to be placed on the branch (oriented in an angle ϑ^*) closed to the crack tip of the crack α. The proposed methodology is applied to estimate the required number of cycles causing crack initiation on a point located on the perimeter of the stop-hole. In order to investigate the effect of the orientation angle β and the stop-hole diameter, the methodology is applied for seven values of β, namely β = 21.0, 30.4, 39.8, 49.1, 58.5, 64.9, 71.0 deg., and three values of the radius of the stop-hole, namely ρ = 2.5, 5.0, 7.5 mm. The results of the above example are demonstrated in Fig. 8.

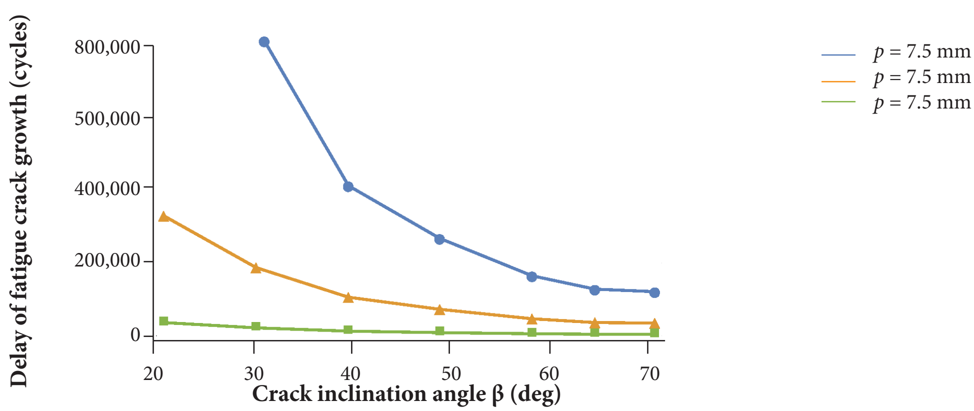

Figure 8. Delay of fatigue crack growth versus crack inclination angle for three values of the radius of the stop-hole.

It should be mentioned that the number of delay cycles depends on the angle ϑ^*, which is, in turn, related to β. Since the angle β is initially observed on an engineering structure (ϑ^* is a consequence of the angle β and the loading conditions), the Fig. 8 demonstrates the delay of fatigue crack growth versus β. This figure indicates that high β values tending to 90 deg. (mode I) yield a rapid decrease of the delay of the crack initiation. Therefore, the effectiveness of stop holes is better when we drill them in the early stages of the mixed mode fatigue crack propagation, before the transformation of the mode I+II crack to pure mode I crack. Furthermore, as it is expected, larger stop-hole diameter causes much more retardation to the crack initiation.

The fatigue crack length (along the path oriented in angle ϑ^*) versus the number of cycles for β = 49.1, 58.5, 64.9, 71.0 deg. is demonstrated in Figs. 9a, 9b, 9c, 9d for two values of stop-hole radius ρ = 5.0 mm and ρ = 7.5 mm. This figure indicates that small values of angle β yield slow crack growth rate, therefore longer fatigue life. On the other hand, larger stop-hole diameters are more effective than small ones.

The orientation of the crack plane in the crack growth based on Figs. 9a, 9b, 9c, 9d is $\gamma = \vartheta^* + \beta - 90$. Therefore, γ = 34.6º in Fig. 9a, γ = 29.6º in Fig. 9b, γ = 24.7º in Fig. 9c, and γ = 19.2º in Fig. 9d. Taking into account the results of Fig. 9, the number of delay cycles with respect to the crack growth curve without hole for a crack length α = 10 mm is shown in Table 1. From this table no large differences are observed. The number of delay cycles between γ = 19.2º and γ = 24.7º is very small. A bigger difference takes place between γ = 26.9º and γ = 34.6º. Since the differences in the orientation of the crack plane are not so large and the shear effect does not appear to change significantly, the results of the Table 1 seem to be reasonable.

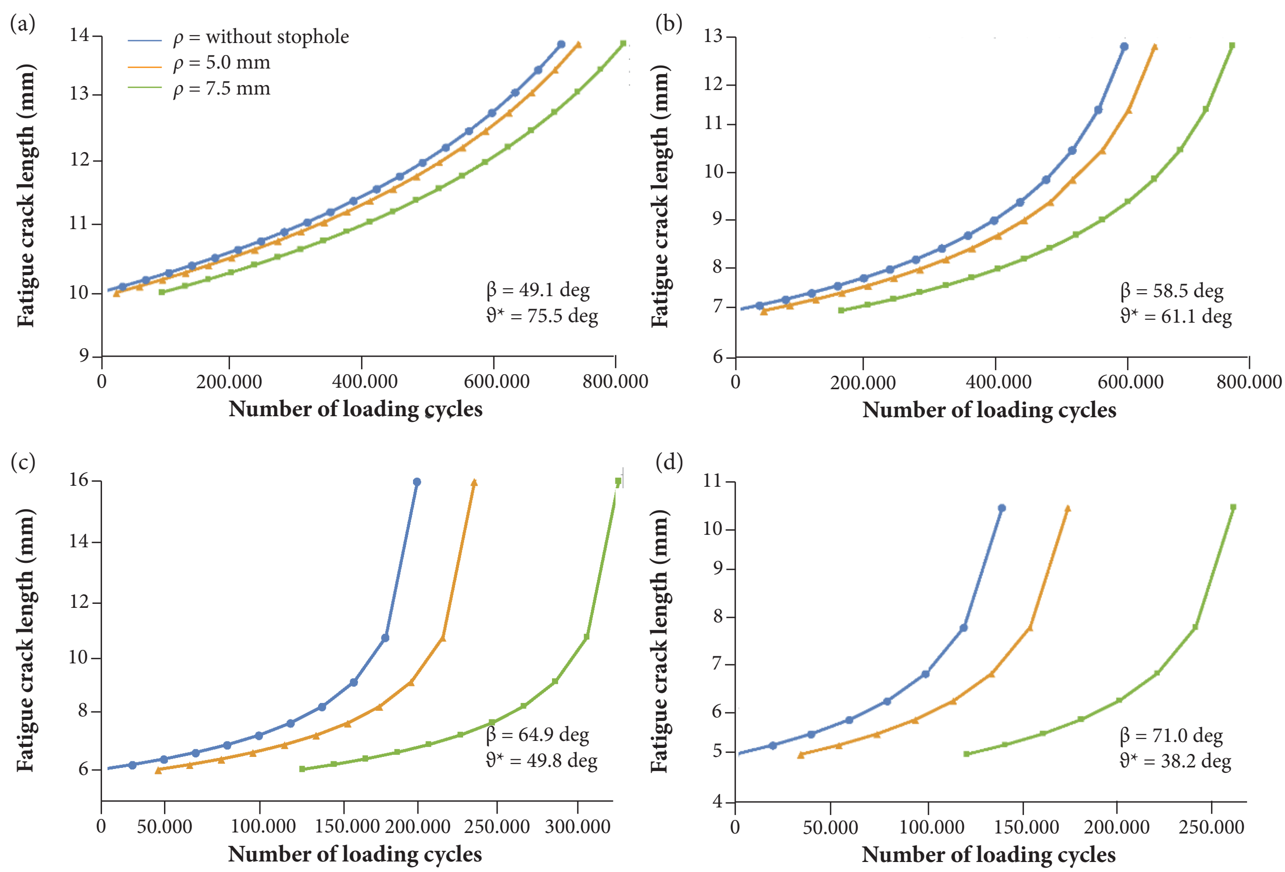

Figure 9. Crack length versus number of cycles for (a) β = 49.1 deg., (b) β = 58.5 deg., (c) β = 64.9 deg., and (d) β = 71.0 deg.

Table 1. Number of delay cycles versus γ.

Orientation of the crack plane γ (deg.)	Number of delay cycles for ρ = 7.5 mm	Number of delay cycles for ρ = 5.0 mm
34.6º	268,530	72,683
29.6º	165,222	40,462
24.7º	127,339	36,212
19.2º	121,427	34,588

CONCLUDING REMARKS

- The fatigue crack growth delay of mixed mode cracks due to stope-holes has been investigated.
- A methodology based on a mixed mode fatigue crack growth model and a multiaxial fatigue criterion is proposed.
- Implementation of the methodology on a thin Al-2024 plate containing a mode I+II crack has been performed.
- The results indicated that the fatigue crack growth delay is more effective for cracks with small inclination angle with respect to the loading direction. The mixed-mode fatigue crack growth rate of the deflected crack is smaller for cracks with small inclination angle. Large stop-holes cause much better extension of the fatigue life than small stop-holes.

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