Building 3D Frameworks of Accessory Aquatic Systems

C. B. P. Soares
Furnas Centrais Elétricas S.A.
Dep. Meio Ambiente
Real Grandeza 219, s801C
22283-900 Rio de Janeiro, RJ. Brazil
cassiobp@furnas.com.br

J. L. de Medeiros
Escola de Química
Federal University of Rio de Janeiro
Ilha do Fundão
21949-900 Rio de Janeiro RJ. Brazil
jlm@eq.ufrj.br

We consider a mathematical framework for spatially describing accessory aquatic systems that belong to a major hydric complex like a large reservoir created by damming a river. The central purpose is to provide a precise and complete mathematical covering of short-scale or localized water bodies, with typical lengths between 1 to 10 km, that are laterally attached to the main river. Such bodies may deserve a more detailed mathematical representation due, for instance, to their tendency to develop stagnant like hydric behaviors.
The proposed framework may work as an infrastructure for developing and/or installing dynamic water quality models. It can generate tetrahedrizations of the aquatic system in question by working according to a procedure which builds, successively, reliable candidates for the following entities: (a) Free Surface Triangular Mesh; (b) Submerged Terrain Triangular Mesh; (c) Three Dimensional Partition of the Domain; (d) Basic Tetrahedrization of 3D Partitions; and (e) Refinement of the Basic Tetrahedrization through a Multi-Layer Tetrahedrization Algorithm.
The required input for this procedure is only composed by terrain contour data and 3D located points. We present example applications including a real scenario belonging to a recent flooded system in Brazil.
***Keywords**: 3D Mesh, tetrahedrization, water reservoir, water quality*

Introduction

In certain parts of the globe, virgin land occupation processes by human populations are still on course. As human population spreads over wild continental zones, a stringent competition pattern is established between this trend and simultaneously certain acute needs of critical resources like hydraulic electricity and water offer. These last factors are heavily dependent on surface area, thus imposing strict preclusion of (or affecting) demographic distributions. This vision may seem somewhat epic, but it is not far from reality in some parts of the world. A recent and frequent situation belonging to this context, is the sudden transformation of a remote, agrarian and small human agglomeration into a lakeside population as a consequence of the filling of a reservoir created by a dam on a river basin. As a matter of fact, Brazil has, presently, several instances of this phenomenon taking place in its vast territory.

A new reservoir created by damming a river extends itself over a vast area of land, exhibiting a characteristic longitudinal shape with a length of hundreds or dozens of kilometers. This grand aquatic complex may be conceptually decomposed in at least three types of subsystems with structural and behavioral differences: (i) sections that behave like an enlarged river bed – which is responsible by the dominant water movement; (ii) few medium or large oblong sections that behave like lakes with very small stream velocity; and (iii) several minor aquatic subsystems with 1 to 10 km of typical length, that are laterally attached to the main enlarged river. Obviously, this view cannot be generalized, but serves to fix ideas about the central subject of this paper, which is the last and third class of mentioned aquatic objects. For sake of conciseness, they are herein referred as Small Lateral Basins or SLB.

Usually Water Quality (WQ) modeling faces a compromise between considered spatio-temporal scales, the degree of refinement of data and theory, and the accuracy of predictions that can be produced. When one models the main river in a reservoir system, frequently the approach is centered on macro scale descriptions characterized by low dimensionality geometry extending for hundreds of kilometers. This kind of model is supported by gross terrain description and long time series of climatic events. For practical reasons, the model is not fed with detailed topographic data, like a precise description of all meanders, islands, shore re-entrances and submerged topography of the reservoir bed. Such models are valuable for large scale predictions, but may be unable to accurately predict the effects of localized phenomena taking place in SLBs.

These bodies inherit stagnancy due to lateral, bottlenecked, attachment to the main water flow. Its characteristic abrupt changing of depth, as the main bed is approached, is a consequence of the flooding of the original canyon excavated by the main river in the system. The high degree of local tortuosity of the shoreline plays important role in their limnologic behavior, entailing that precise mathematical modeling of these systems can only be accessed if we complete the crude axial reservoir formula with accessory local models focusing on detailed spatial description of the site.

Water Quality (WQ) modeling of reservoirs is currently done within the scope of professional simulators of rivers, reservoirs and estuaries (Orlob, 1982; Stefan et al., 1989; Cerco and Cole, 1993; Cole and Buchak, 1995; Bowen, 1998). These systems are appropriate for large scale aquatic systems with hundreds of kilometers of characteristic lengths. In order to assure certain bounds on the utilization of human, computing and measurement resources, these models specialize on macro descriptions, where local topographic, demographic or micro-climatic aspects are usually not taken into account. In the majority of cases, only large scale factors are furnished to the model as input. Typical predictions are time series of spatial distributions of entities like temperature and chemical or biochemical concentrations. Nevertheless, large scale WQ models can generate valuable responses that are meaningful when focusing vast area scenarios and long time periods.

On the other hand, large scale WQ models do not seem be the most adequate tool for accessing precise predictions of trends associated to stagnant SLBs that show preoccupying characteristics like human agglomerations on the shoreline.

In these instances large scale WQ simulators play only a secondary role. They are still necessary in order to model the SLB boundary conditions at the contact zone with the main river, which is the central focus of the large scale model – i.e. the large scale model is responsible for modeling all background phenomena acting on the SLB.

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But the crucial object now needed is a dynamic and customized local WQ model constructed with transport equations superimposed on a precise mathematical description of all morphologic/topographic SLB aspects. The local model can then be solved by a numerical approach like the Finite Element Method (FEM) (Bathe, 1996).

This discussion corresponds to the following decomposition of tasks: (1) formulate a large scale fluvial model covering hundreds or thousands of squared kilometers, based on available (and probably coarse) climatic/atrophic/terrain data; (2) focus on a particular SLB of interest and construct a detailed and complete 3D grid of it; (3) on this grid, structure a mathematical model for transport phenomena, carefully locating the spatial distributions of pertinent influencing factors; (4) select a numerical solver for the local model; (5) start the resolution of the large scale model, generating time variant boundary conditions for the local model; (6) activate the local (or children) numerical processing, simulating the SLB dynamics as it interacts with the parent model.

In the present work we consider in detail a methodology to accomplish step (2) above via a tetrahedrization procedure.

The morphologic description of a SLB depends on its 3D partitioning. The most simple way of partitioning a highly irregular three dimensional domain is by means of a framework of irregular tetrahedrons, which are filtered in order to prevent bad geometric characteristics like shapes almost obeying sub-dimensional formats. This framework, if effective, – i.e. as a tetrahedrization – has also the advantage of offering a very suitable structure for the application of FEM schemes.

Current tetrahedrization techniques are very problem oriented (Si, 2001; Shewchuk, 1997), i.e. there is no universal tetrahedrization approach that can be used with high efficiency in all kind of problems and 3D scenarios, specially in geosciences applications. Also, as one can easily expect, effective tetrahedrization methodologies for a given 3D domain are not unique. In fact, tetrahedrization methodologies may differ on several aspects which are currently topics of active research, like the efficiency and completeness of the tetrahedrization process, the generation of meshes dominated by elements with good aspect ratio, and the degree of simplification of the necessary input data (Si, 2001).

In our approach, tetrahedrization is formulated for building a spatial framework for SLB like aquatic bodies, whose characteristics were discussed above. The required input information is a simple set of data corresponding to few contour data and a complementary set of located points on the submerged soil. The generated framework is completely tetrahedrized, offering resources for refinement, island representation and capabilities to generate grids according to multiple heights of water level as specified by the user. The final result is perfectly applicable as a mathematical infrastructure for 3D finite element solvers aiming local WQ modeling.

The paper covers the tetrahedrization methodology proposed here. We tried to make all presentation always accompanied by graphical exemplification through the use of two prototype SLBs. The first one, SLB1, is a quasi-elliptic sector of a reservoir (Fig. 1) with a terrain formation characteristic of potential islands. SLB2 corresponds to a compartment with two ramifications (Fig. 2).

Sections 2 to 5 are assigned to explain each phase of the tetrahedrization process, which are, respectively: Morphologic characterization of SLBs; triangular meshes; tetrahedrization via vertically aligned patches; and multilayer tetrahedrization algorithm. Finally, a real example is treated by our methodology in Section 6.

Morphologic Characterization of SLBs

To begin with, it is convenient to present firstly a short explanation about what we understand by the morphologic description of SLBs. Obviously we will have to focus on the main characteristic aspects of a typical SLB.

In some instances, SLBs exhibit morphologies coarsely resembling the shape of a hand, i.e. composed by a deep central bay and few long land penetrations of water. The central bay may be a compartment of the main river or be only laterally attached to it. In this last (and more common) case, the location of the *cutting line* between the central bay and the river defines the size of the problem and is usually a factor pertinent to the application in question. We call the “water wall” associated to this *cutting line* as the *aquatic boundary*.

A SLB is then a three dimensional domain defined (and enveloped) by three bi-dimensional loci: (i) the *submerged terrain*; (ii) the *free water surface*; and (iii) the *aquatic boundary*.

The *Submerged Terrain* is, indeed, the most important entity, being responsible for all morphologic issues of the SLB like the central bay shape, the existence of islands, the existence and lengths of land penetrations of water “fingers”, etc. The *free water surface* and the *aquatic boundary* only impose limits to the valid extension of the *submerged terrain*.

Other secondary entities are resultant from the intersection of pairs of objects. This is the case of the *shoreline* and the *cutting line* that are, respectively, the intersections between the *submerged terrain* and the *free water surface*, and between the *free water surface* and the *aquatic boundary*. It is easily recognized that the *shoreline* is naturally a terrain contour, while the *cutting line* is an aquatic contour.

The morphologic characterization of SLBs thus demands data addressing the *submerged terrain*, the *free water surface* and the *aquatic boundary*. These data constitutes what we call the basic set of 3d points for slb representation. In essence, this data must be sufficient for the construction of mathematical models for the *submerged terrain,* the *free water surface* and the *aquatic boundary.* These models are referred, respectively, as the Digital Terrain Model (DTM), the Free Surface Model (FSM) and the Aquatic Boundary Model (ABM). The last model will not be considered here. Indeed, it is not strictly necessary provided we have the other two models.

Basic Set of Points

The *basic set* of 3D points – i.e. points with *(x,y,z)* coordinates, where *z* points upward from the sea level – contains all primary usable information for construction of DTM and FSM. The information in the *basic set* is furnished by the user and is usually entered in a somewhat disordered manner, creating zones with varying degree of mathematical detail, which must be compensated by numerical processing.

Generally, uncertainties affecting data belonging to the Basic Set are inherent to the methods that gathered them. As a matter of fact, errors associated to charts with cartographic resolution are of the order of 0.05% with respect to the chart scale, while ordinary points located in field with GPS devices have typical accuracy superior to 5 meters (Clarke, 1995). Thus we consider that these two (and similar) sources of data are qualified to meet the requirements of accuracy for a reasonable morphologic representation of SLBs. In other words, the precision of the requested data (below) is compatible with the resolution of numerical and interpolation procedures that we will adopt.

Data in the *basic set* can be grouped in the following four classes of points:

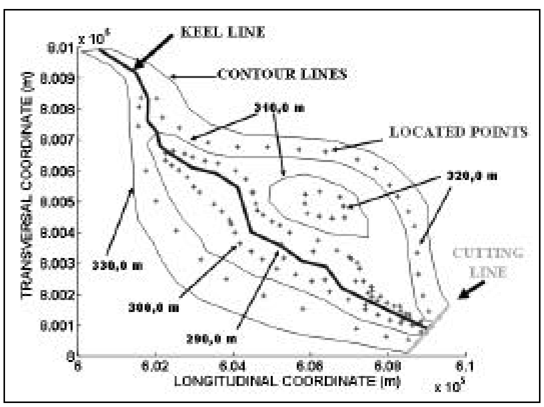
- terrain contours with several values of elevation
  Contours on the SLB terrain configure data of central importance. The higher the number of lines and/or the number of points in each line, the better will be the accuracy of the

models. In general, an acceptable prerequisite is three to five contour lines, being one of these the above mentioned *shoreline*, which must also include all islands contours whose existence can be inferred by the furnished data.

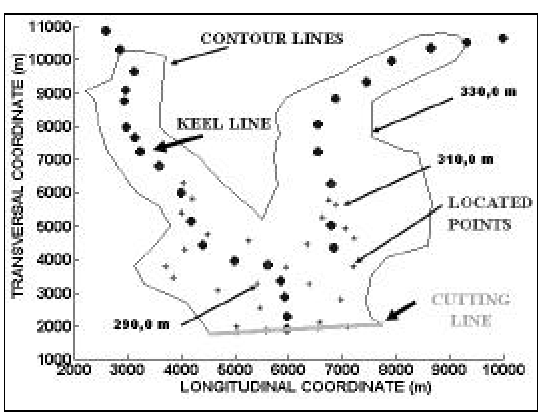
- located 3D points on submerged terrain
  These are points judiciously distributed on the terrain enabling that our algorithms could reckon critical morphologic aspects like island formations.
- located 3D points along channeling keels in the SLB soil
  These points belong to the same category of the points above. They are only necessary if the contour data is incapable of avoiding the appearance of incorrect flat bottom patterns in the representation of the SLB shell.
- cutting line for closure of the SLB domain
  This line was discussed above. It is constructed by drawing a line segment connecting points of same altitude belonging to different contour lines (i.e. on opposed SLB coasts).

The density of points in the contours is critical for balancing the resolution of the grid with the existent computing power. Additionally, this balance must be such that the application at hand could meet the proposed goals. As a first rule, it is proposed to start the methodology with lines as rarefied as possible, i.e. with the minimum set of points without significant losses of contour information (Douglas and Peucker, 1973). A “significant loss” of information can be perceived via detection of sensible alteration of perimeter or area values during exclusion of points (Hershberger and Snoeyink, 1992).

Figure 1 depicts prototypes SLB1 and SLB2 with their respective *basic sets* of points. SLB1 (Fig. 1a) has a quasi-elliptic form exhibiting a terrain formation characteristic of the appearance of islands. SLB2 (Fig. 1b) represents a bay with two land penetrations of water.



(a)



(b)

Figure 1. Basic sets of points for prototypes (a) SLB1 and (b) SLB2.

Triangular Meshes

The mathematical representation of SLBs demands firstly the preparation of mathematical models for the *submerged terrain* and for the *free water surface*. These models are, respectively, the Digital Terrain Model (DTM) and the Free Surface Mesh (FSM), which are triangular meshes extending over the terrain and over the water surface. Both meshes are constructed with the same data from the *basic set* of points. A difference between them is that the FSM is purely planar while the DTM extends over the submerged terrain. A strong connection between them is that FSM triangles are projections of DTM triangles onto water surface. This also means that the points involved in the discretization of the *shoreline* belong to the FSM and to the DTM at the same time, i.e. both meshes share all *shoreline* points.

Building FSM

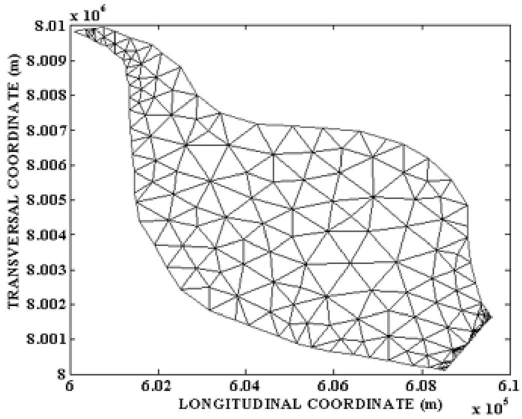
The FSM is built using part of the data in the *basic set*. It is created with contour points by means of Delaunay procedures of triangulation, which besides applying the habitual Delaunay criteria for avoiding bad triangle types (Shewchuk, 1997; Shewchuk, 2000; Shewchuk, 2002), operates imposing the following extra conditions:

- FSM is completely enclosed by the outer SLB contour, i.e. the *shoreline* contour.
- FSM may use all entered contours (Fig. 2b) or only a subset of it (Fig. 2a).
- A FSM triangle must not have all vertices projected on the same terrain contour line. This forbids FSM triangles with three vertices on the *shoreline* or on a contour in the *basic set*. This clause is necessary to avoid the collapsing of DTM triangles (i.e. 3D triangles on the terrain surface are not allowed to have the same value of vertex elevations).
- Gradual growing of FSM triangles. Normally algorithms generate uniform meshes. The technique used here, on the other hand, forces a gradual increase of triangle sizes, enhancing the capability to address irregularities on the peripheral contours; i.e. small and densely grouped triangles are posed near boundaries and gradually larger ones are distributed in the interior. The final product is a quasi-optimized mesh in terms of element collocation, being dense in the boundary regions and rarefied inside.
- Refinement clauses can be applied on a coarser FSM (Shewchuk, 2002; Edelsbrunner et al., 2000a; Edelsbrunner et al., 2000b; Edelsbrunner and Guoy, 2002). A simple procedure, also adopted here, is to insert midpoints on all edges of a given mesh to be refined.

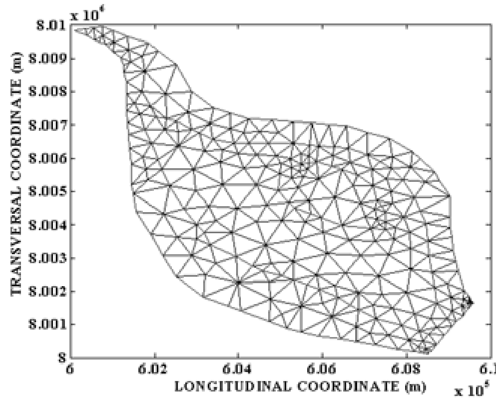
Figure 2 below shows a sequence of heterogeneous FSMs produced by this algorithm on prototype SLB1 (Figs 2a to 2d) and SLB2 (Fig. 2e). A coarser mesh for SLB1, which used only the *shoreline* contour, is shown in Fig. 2a. This mesh is one-step refined giving Fig. 2c. Figure 2b and 2d presents analogous FSMs if all entered contour data (including islands contours) are forced to belong to the mesh.

Information produced in the FSM creation is stored in the following matrices:

- Matrix $\mathbf{P}$ stores (x,y) planar coordinates of FSM nodes; it has size *2xnp*, where *np* is the number of nodes;
- Matrix $\mathbf{B}$ stores indexes of columns of matrix $\mathbf{P}$, corresponding to nodes that delimit rectilinear segments belonging to the *shoreline;* it has size *2xnb*, where *nb* is the number of such segments;
- Matrix $\mathbf{T}$ stores indexes of columns in matrix $\mathbf{P}$, corresponding to nodes belonging to each FSM triangle; it has size *3xnt*, where *nt* is the number of FSM triangles.



(a)



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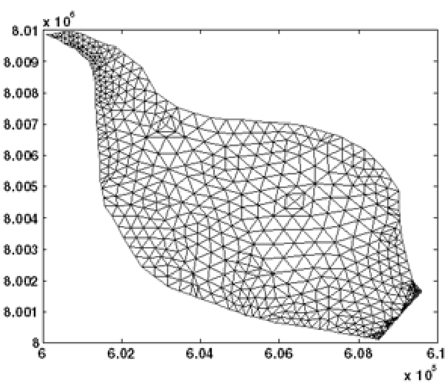
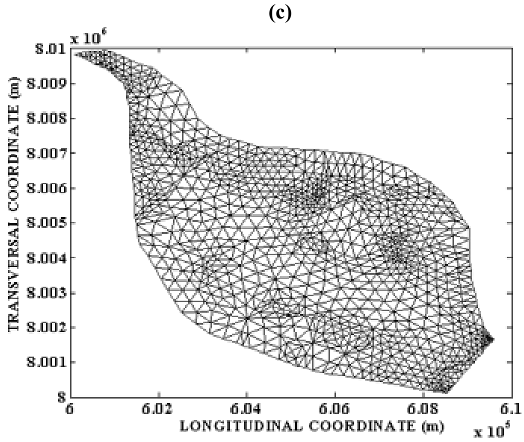
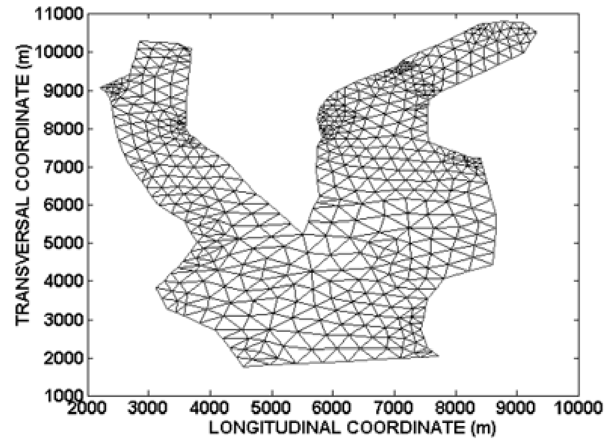


(c)



(d)

Figure 2. Heterogeneous FSMs for SLB1 (a,b,c,d) and SLB2 (e).



(e)

Figure 2. (Continued).

Building DTM

The proposition of DTM algorithms is still an active research topic. A search in the literature finds that there is no general method suitable to all cases, prevailing the situation of problem oriented applications. Wheatley (1994) focused DTMs for archeological applications. Wood (1996a,b) studied the automatic generation of drainage networks from terrain models. Tarboton (1997) studied digital elevation modeling and implications in determining contributing areas and flow directions. De Floriani et al. (2000) and Thibault and Gold (2000) cited recent techniques for upgrading terrain models. Wadzuk and Hodges (2001) addressed the description of Sinuous and Dendritic Reservoirs.

In the present work a simple technique was conceived for developing DTMs from the information gathered in the *basic set* of points and in the FSM.

Basically DTM creation consists in obtaining the three dimensional location of *all points* lying on the submerged terrain whose projection onto the free surface is a FSM point. Our procedure thus uses the *basic set* as a primary database from which the FSM points will be interpolated vertically. DTM 3D coordinates are stored in matrix $\mathbf{M}$ with size *3xnp*, where *np* is the number of DTM nodes. As can be seen by the sizes of $\mathbf{M}$ and $\mathbf{P}$, DTM and FSM nodes are univocally linked to each other.

Our procedure accesses directly matrix $\mathbf{P}$, finding the terrain elevation function $z(x, y)$ for all its (x, y) points that do not belong to the *basic set.* This is done through a bi-cubic interpolation scheme (Faux and Pratt, 1979) over the members of the *basic set*. The bi-cubic interpolation guarantees continuity of the elevation function and its gradient (Rogers and Adams, 1989). Given a (x, y) FSM point, the bi-cubic scheme first finds in the *basic set* its closest four neighbors, $\{(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), (x_4, y_4, z_4)\}$, whose 2D quadrilateral projection contains (x, y). The bi-cubic interpolated elevation for (x, y), $\tilde{z}(x, y)$, is generated from these points according to the following equations (Dierckx, 1981):

$$\tilde{z}(x, y) = Ø(x)^t ÙØ(y) \tag{1}$$

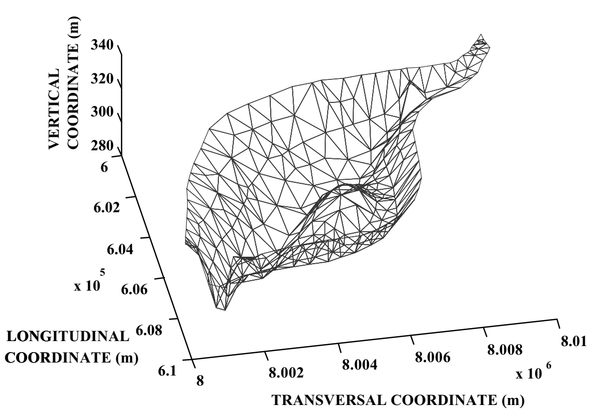
$$\text{Ø}(w)=\begin{bmatrix} 2w^3-3w^2+1 \\ -2w^3+3w^2 \\ w^3-2w^2+w \\ w^3-w^2 \end{bmatrix} \quad , \quad \text{Ù}=\begin{bmatrix} z_1 & z_3 & z_y(x_1,y_1) & z_y(x_3,y_3) \\ z_2 & z_4 & z_y(x_2,y_2) & z_y(x_4,y_4) \\ z_x(x_1,y_1) & z_x(x_3,y_3) & z_{xy}(x_1,y_1) & z_{xy}(x_3,y_3) \\ z_x(x_2,y_2) & z_x(x_4,y_4) & z_{xy}(x_2,y_2) & z_{xy}(x_4,y_4) \end{bmatrix} \tag{2}$$

where

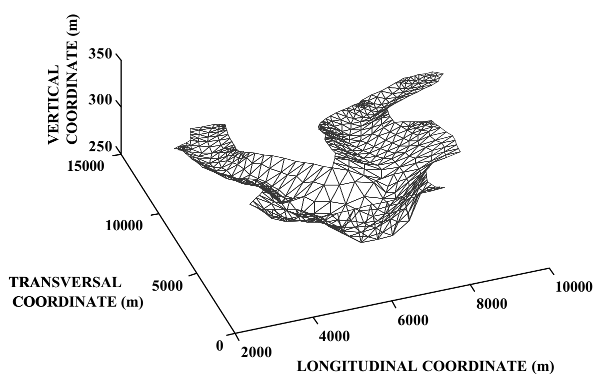
$$z_x(x_j,y_j)=\left(\frac{\partial z}{\partial x}\right)_{x_j,y_j} \ , \ z_y(x_j,y_j)=\left(\frac{\partial z}{\partial y}\right)_{x_j,y_j} \ , \ z_{xy}(x_j,y_j)=\left(\frac{\partial^2 z}{\partial x \partial y}\right)_{x_j,y_j}$$

are estimated from the *basic set*, using finite difference differentiation.

DTM creation is exemplified for SLB1 and SLB2 in Fig. 3. For SLB1, DTM is created from the FSM depicted in Fig. 2b, whereas in the DTM for SLB2, the FSM is shown in Fig. 2e.



(a)



(b)

Figure 3. DTM for (a) SLB1 (FSM in Fig. 2b) and (b) SLB2 (FSM in Fig. 2e).

Tetrahedrization Via Vertically Aligned Patches

DTM and FSM are 2D frameworks having strictly conforming topologies: They are both composed by triangular patches perfectly corresponding along the vertical direction. This feature can be used to create a rapid 3D partitioning of the SLB, which is important because it constitutes a preliminary step for tetrahedrization. The 3D slices for this partitioning are defined by polyhedrons containing, as faces, a FSM patch and a DTM patch. Since, by definition, partitions do not overlap, a *basic tetrahedrization* of SLBs can be easily created via union of simple schemes of tetrahedrization applied to these slices (or partitions). A viable *basic tetrahedrization* method for SLBs, obtained through this process, is the object that we examine in this Section.

To proceed further, we have to introduce the notion of *corresponding vertices* of the DTM with respect to a given FSM triangle. A *corresponding vertex* is a DTM point, *not belonging to the shoreline*, whose free surface projection is a vertex of the FSM triangle in question (for clarity, we kept all reference to *corresponding vertices* in italics).

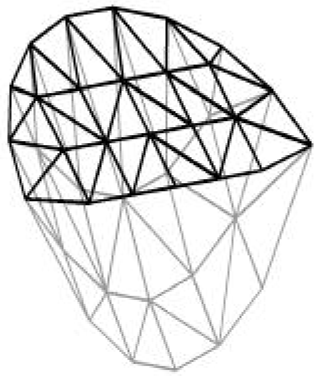
The 3D partitioning of SLBs can be understood with the help of Figs. 4a to 4d. Figure 4a depicts typical disposition of DTM (gray) and FSM (black) frameworks. The SLB is then perfectly visible in Fig. 4a, corresponding to the three dimensional domain enveloped by the two frameworks.

The SLB can be partitioned in slices generated by corresponding patches in the FSM and DTM. Due to the particularly simple proposition of DTM and FSM adopted here, only three kinds of slices appear. Figures 4b, 4c and 4d exemplify members of these three classes. We refer to the 3D slices as belonging to Class 1, Class 2 and Class 3 according to the types of patches that were joined to create them. Figure 4b depicts a typical Class 1 slice, whereas Figs. 4c and 4d do, respectively, the same for Class 2 and Class 3 Slices. This discrimination is important because the class of a slice will define how its tetrahedrization will take place.

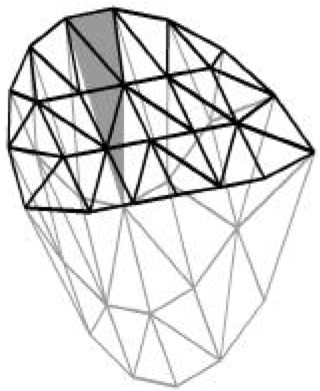
Class 1 slices are simple tetrahedrons born by DTM and FSM patches that have two vertices in common. There is then only a single *corresponding vertex* in the DTM patch. This occurs when a FSM triangle – and also the corresponding DTM triangle – has two vertices on the *shoreline* (Fig. 4b). For this reason, members of Class 1 may be encountered at the *shoreline*. Obviously Class 1 slices do not demand any further action for tetrahedrization.

Class 2 contains pyramidal objects with a quadrilateral face (Fig. 4c). These solids derived from FSM and DTM triangles with a single common vertex on the *shoreline*. They are created by connecting the other two FSM vertices to the two *corresponding DTM vertices*. Class 2 objects are simply tetrahedrized by sectioning in two tetrahedrons through the plane defined by a *corresponding vertex*, the single *shoreline* FSM point and the opposite interior FSM vertex.

Class 3 is populated by trunks of triangular prisms (with a planar face) generated by FSM and DTM patches without *shoreline* vertices (Fig. 4d). For this reason they are likely to be encountered in the interior of the SLB. Class 3 slices are created linking the three FSM vertices to *corresponding vertices* in the DTM. Children tetrahedrons can now be produced from the breakage of Class 3 parent objects. Since there are several ways of subdividing them in tetrahedrons, a criterion is needed. We formulated two strategies for breaking a Class3 slice. In the simpler Breakage Strategy 1, a slice is broken in three tetrahedrons via two sectioning planes that use, respectively, two and one FSM points connected to DTM *corresponding vertices* (Fig. 5). Breakage Strategy 2, on the other hand, creates more tetrahedrons than the previous method. It gives rise to eight tetrahedrons naturally defined by the centroid of the slice. Thus, this strategy inserts new interior points in the SLB model that do not exist in FSM and DTM frameworks. Another fact is that the new interior points (i.e. slices centroids) are also vertically aligned with the centroids of the involved FSM and DTM patches.



(a)



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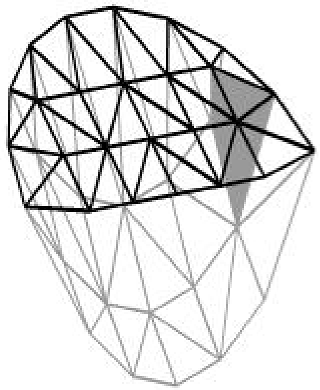

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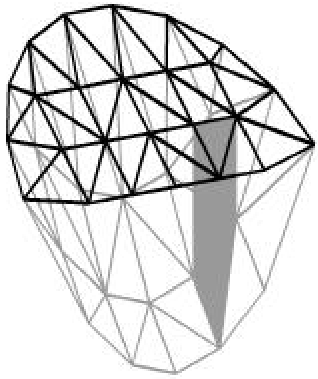


(d)

Figure 4. 3D Partitioning of SLBs via DTM (gray) and FSM (black) Meshes. Original situation; (b) Class 1 Slice; (c) Class 2 Slice; (d) Class 3 Slice.

Figure 6a and 6b depict tetrahedrizations for SLB1 (DTM in Fig. 3a, FSM in Fig. 2b) for two different water levels – 308m and 310m. In these figures, Class 3 slices were tetrahedrized through Breakage Strategy 1. The reader can notice that the DTM and FSM of this example were prepared for elevation of 330m. Modeled aquatic volumes, on the other hand, fill only portions of the original DTM. This shows that given DTM and FSM can be used with varying levels of water, which may be chosen according to the season of interest. In these cases, tetrahedrization is preceded by adapting (and contracting) the FSM and DTM to the required level through spatial interpolation and redefinition of the periphery of the FSM/DTM meshes. The example exposes the temporary character of the islands, configuring an interesting capability of the tetrahedrization resources developed here.

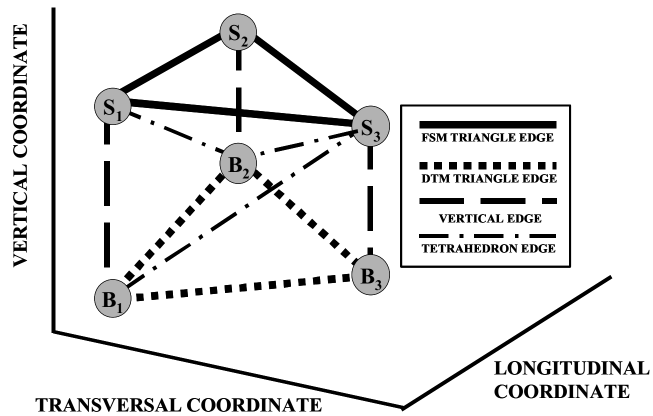
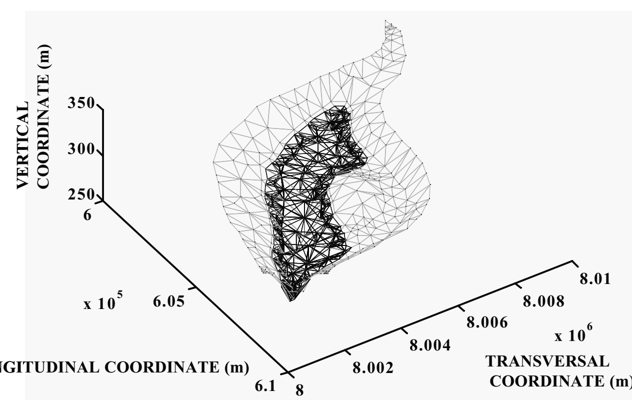
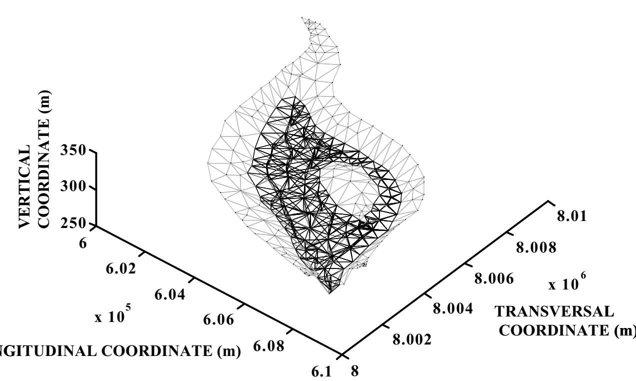


Figure 5. Breakage strategy 1 for class 3 objects. [Created tetrahedrons {S_1, S_2, S_3, B_2}, {S_1, S_3, B_1, B_2}, {B_1, B_2, B_3, S_3}].



(a)



(b)

Figure 6. Basic tetrahedrizations of SLB2 [DTM/FSM Built at 330m of elevation]. tetrahedrization for: (a) Water level at 308m; and (b) Water level at 310m.

Multi-Layer Tetrahedrization Algorithm

Accurate tetrahedrization of arbitrary SLBs may require a more refined framework than the basic mesh generated as described in Section 4.

The strategy adopted consists in introducing multiple horizontal layers of tetrahedrons – between the FSM and DTM – whose top heights are specified by the user. Tetrahedrization of SLBs can then be done with only one layer – via *basic tetrahedrization* – or with these multiple layers of tetrahedrons. In the last case, successive rounds of tetrahedrizations are performed until the algorithm achieves a satisfactory accuracy of representation.

Each multi-layer tetrahedrization demands only data existing in FSM and DTM frameworks, coupled with data from the insertion of new points necessary for multi-layering. Such new points belong to two groups: (i) **Group A** of points are necessarily imposed as intersections of the *basic tetrahedrization* with the height levels stipulated by user; and (ii) **Group B** of points which were inserted by Breakage Strategy 2 operating on Class 3 slices created in the new layers.

The multi-layer tetrahedrization algorithm is presented in Fig. 7. Table 1 presents basic definitions of terms, the objectives involved and initialization measures. The algorithm is a simplified one because the new tetrahedrizations perform mesh refinement only in the vertical direction, entailing that Group A of new points all lie on vertical lines connecting DTM points with FSM points that are not *shoreline* points (because, in our approach, FSM *shoreline* points are also DTM points). In other words, the FSM and DTM, which governed the *basic tetrahedrization*, will, again, orient the multi-layering. Another characteristic of the algorithm is that data associated with a tetrahedrization must be discarded if the user formulates a different set of Levels for multi-layer tetrahedrization; i.e. there is no recursivity.

The algorithm is quite similar to the strategy used for *basic tetrahedrization*, which proceeds first finding 3D slices defined by DTM and FSM triangles, and then breaks them – with different processes whether the slice is of Class 1, 2 or 3 – in tetrahedrons. Particularities of the multi-layer algorithm are the following:

- at a given stage of the search, we have *processed* and *non-processed* levels and layers; search stops when there are no more layers to be processed.
- the first (already) processed level is the FSM; the heights of all levels – excepting the FSM – are specified by user;
- A level has been processed when we found the projection of the DTM onto it, but excluding: (i) DTM points at a height greater than this Level, and (ii) DTM points that are also FSM points; this creates points of Group A at the present level. When these points are taken in conjunction with the excluded DTM points above, a new 3D triangular frame can be created; this frame quite resembles the DTM and, as well, is not in general completely planar; we call this frame the F frame associated with the level under processing. The first F frame is the FSM;
- a layer is the region between two adjacent F frames associated to two adjacent levels;
- a layer has been processed when we found a tetrahedrization within it; this is done by applying *basic tetrahedrization* using its two adjacent F frames passing at the top and bottom levels of the layer; the *basic tetrahedrization* operates as before except that top and bottom F frames replace FSM and DTM. Before a layer is put in processing, we must have its top and bottom levels in full processed condition, i.e. the respective F frames were already created.

Table 1. Definitions, objectives and initializing measures for multi-layer tetrahedrization.

Terms Defined	Meaning
$Zc_1 > Zc_2 > ... > Zc_n$	**Levels Specified by User for Defining Layers**
Zc_0 ($> Zc_1$)	**Original FSM Level**
M	*3* by *np* Matrix of *(x,y,z)* Coordinates of DTM Points
P	*2* by *np* Matrix of *(x,y)* Coordinates of FSM Points
Q	*3* by *np* Matrix of *(x,y,z)* Coordinates of FSM Points
T	*3* by *nt* Matrix of Indexes of FSM Points in each one of *nt* FSM Triangles
$\mathbf{F}\{k\}$	*3* by *np* Matrix of Coordinates of Points in the k^{th} F Frame, where *k=0, 1, ... n*
nt	Number of Tetrahedrons in the k^{th} F Frame
$ntet$	**Number of Tetrahedrons in the SLB Mesh**
TET	*4* by *ntet* Matrix of Indexes of Mesh Points
Initialization	
$\mathbf{F}\{0\} = \mathbf{Q}$	Creates the 0^{th} F Frame (i.e. the FSM itself)
$ntet = 0$	**Initializes the Number of Created Tetrahedrons**
$\mathbf{TET} = [\,]$	Initializes **TET** as an Empty Set
Objectives	
$\underline{\underline{F}}\{k\}, k = 1 \cdots n$	Creates all F Frames
TET	Creates the SLB Tetrahedrization with *n* Layers

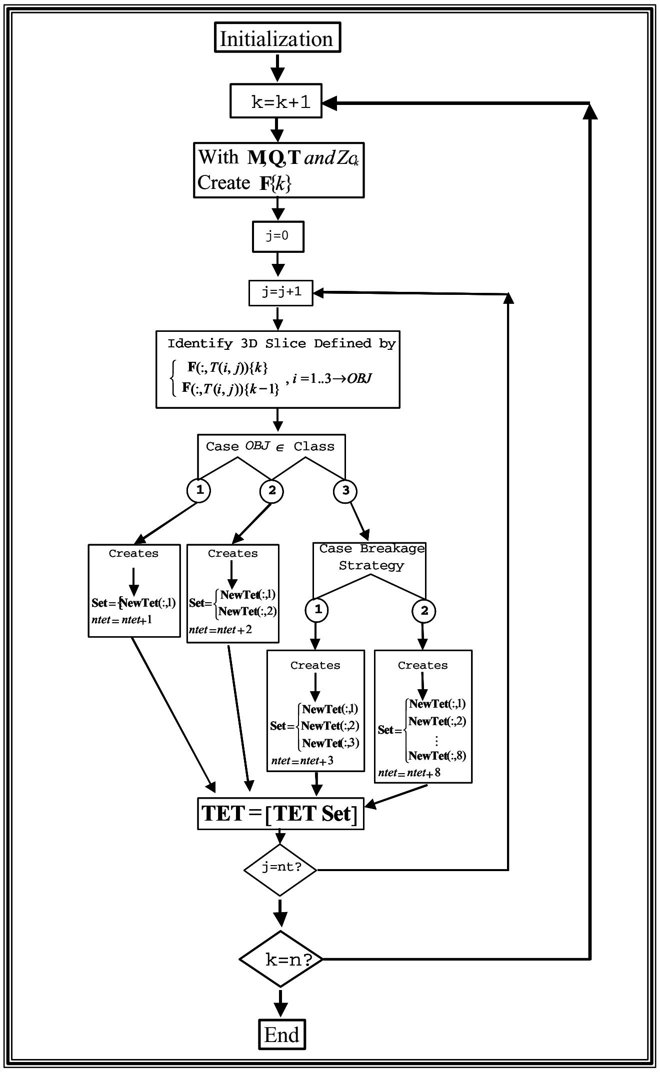
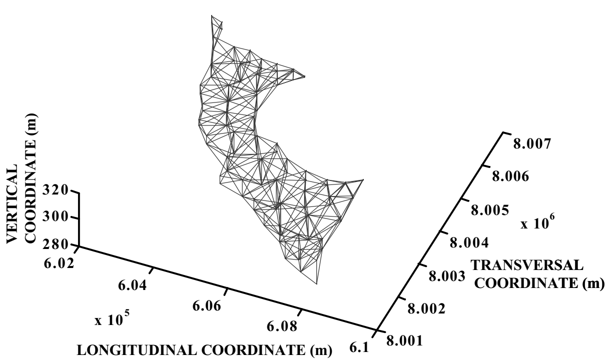


Figure 7. Multi-layer tetrahedrization algorithm.

Figure 8 illustrates the multi-layer tetrahedrization for SLB1. Tetrahedrization occurs in three layers adopting Breakage Strategy 1. Figures 8a to 8c show the tetrahedrization of each layer. The superposition of the layers – i.e. the complete tetrahedrization – is shown in Fig. 8d.



(a)

Figure 8. Tetrahedrization of SLB1 in 3 layers (Breakage Strategy 1).

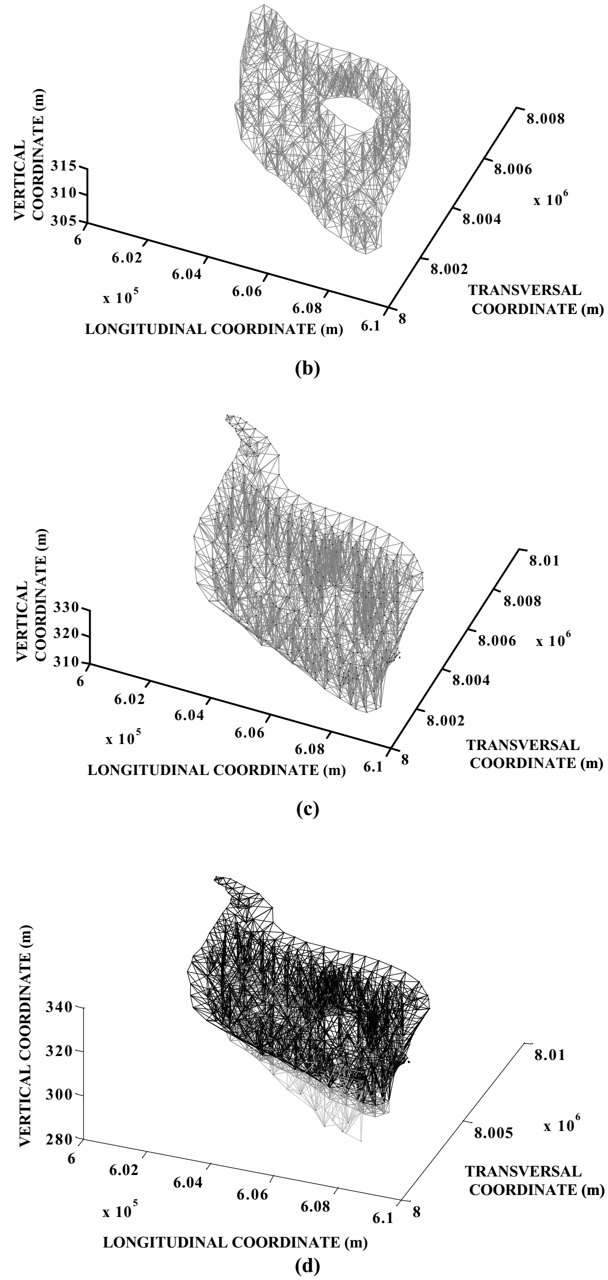


Figure 8. (Continued).

APM Manso Reservoir : SLB with Islands

The Manso and Casca Rivers form the APM Manso Reservoir, in the State of Mato Grosso, middle-west of Brazil. This reservoir has a V-shape, with two large branches, deriving, respectively, from the two main rivers that form it. At the north-eastern extremity of the branch created by the Manso river, there is a SLB with a rock formation in its interior. Changes on the free surface level of the reservoir may eventually isolate this formation from the neighboor terrain, entailing the appearance of an island. Figure 9 presents a Landsat 7 view of the APM Manso Reservoir. The SLB in question is shown at the north-eastern branch of the reservoir. We call it the NEMB-SLB.

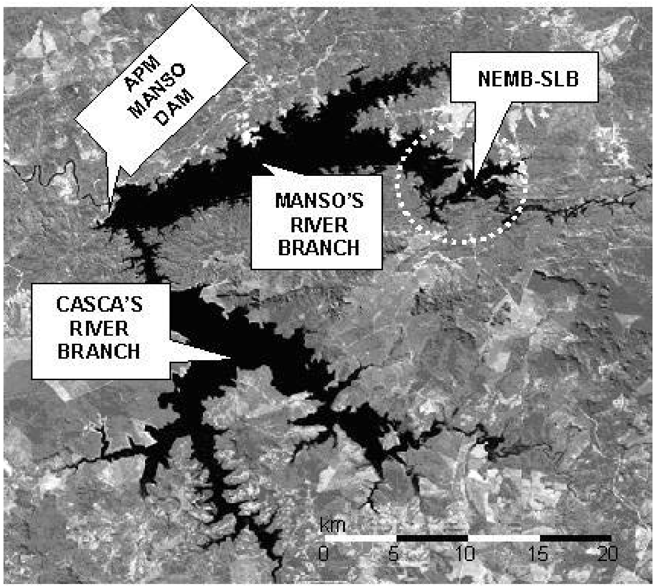


Figure 9. APM Manso reservoir with North-Eastern NEMB-SLB [Landsat 7 View].

A detailed view of the NEMB-SLB – at an occasion in which the level of the free surface was below 285m – is seen in Fig. 10. Due to the low water level, this figure shows the temporary island perfectly connected to the north margin of the Manso River. The model for NEMB-SLB was built from data gathered according to the scale of Fig.10.

The NEMB-SLB *basic set* consists of the contours at levels from 290.0m to 265.0m, and the keel tracing of river beds (Fig. 11). Spatial representation uses UTM coordinates. The *datum* is the SAD 69.

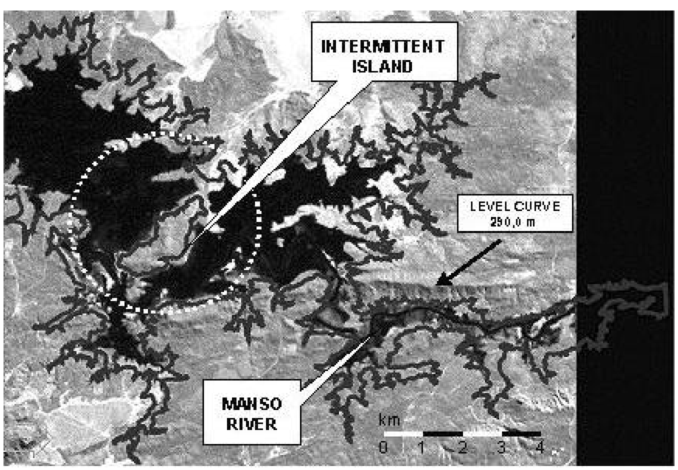


Figure 10. Detailed view of NEMB-SLB [Landsat 7 View].

The FSM is readily generated as shown in Fig. 12, while the DTM is shown in Fig. 13. Figure 14 depicts the *basic tetrahedrization* (i.e. Single Layer) of NEM-SLB adopting Breakage Strategy 1.

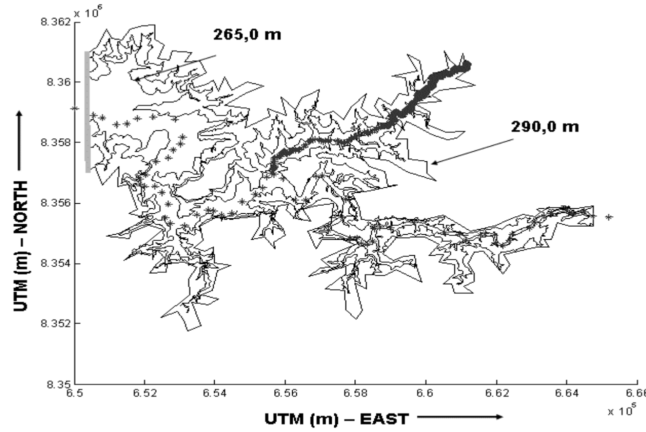


Figure 11. Basic set of data for tetrahedrization of NEMB-SLB.

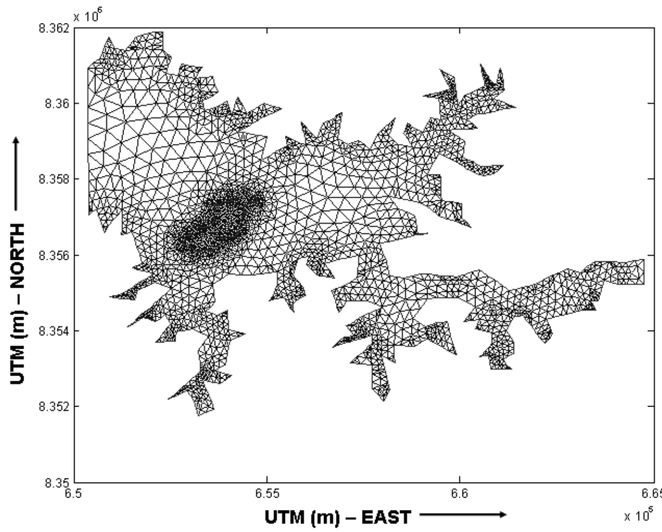


Figure 12. FSM for NEMB-SLB.

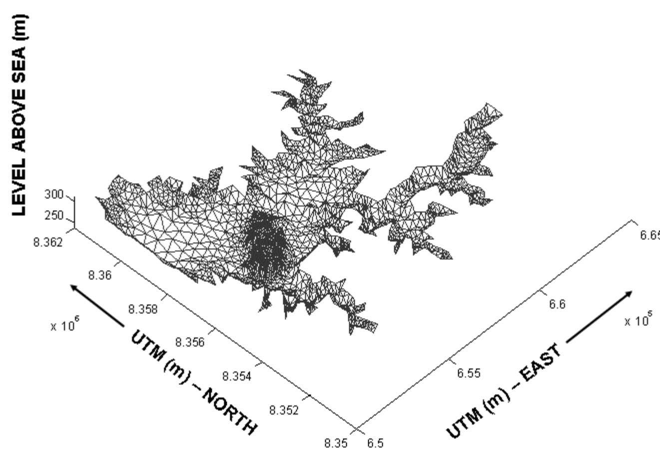


Figure 13. DTM for NEMB-SLB.

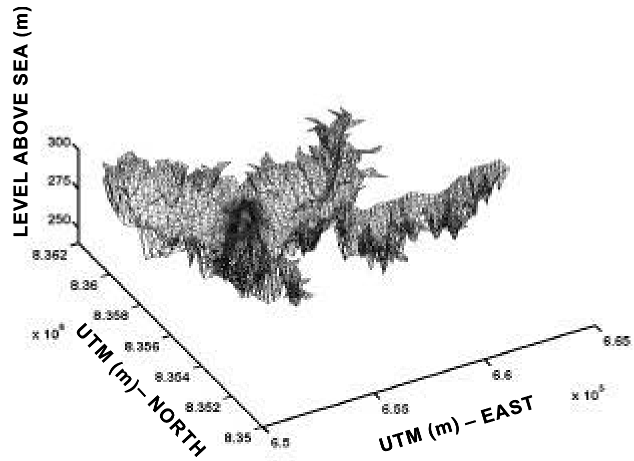


Figure 14. Basic tetrahedrization of NEMB-SLB with breakage strategy 1.

Concluding Remarks

The procedures of generation of bidimensional and tridimensional meshes presented in this work showed robustness and flexibility when faced with the task of mapping reservoir compartments – or SLBs – with highly irregular topographies (such as in the case of compartments with islands) being them real systems or ideal prototypes.

Accurate results were obtained from reasonably small *basic sets* of data, which are readily available on present reservoir applications. Small *basic sets* of data means low costs for processing and gathering input information. The volume of data manipulated and rendered in the numeric and geometric models here proposed, were stored within standard mathematical processing environments – like Matlab for Windows R12, The Mathworks ® –, through the use of optimized variables. This entails low requirements of machine power and allows access of generated data by other Windows based applications, like dynamic FEM solvers.

Most of the strategies adopted in the tetrahedrization procedure are quite flexible, and can be adjusted to cover ample spectrum of morphologic complexities of SLBs. This can be done regarding both what is needed for accurate description of SLBs as well the degree of detail required by users.

In all problems the mapping of volumes of SLBs can be refined, according to user demands, during the following stages of the methodology:

- the generation of the Free Surface Model (FSM);
- the choice of Breakage Strategy (if 1 or 2) for subdivision of 3D slices belonging to Class 3, that are created in tetrahedrizations, either in basic or multi-layer modes;
- the use of several layers in the multi-layer tetrahedrization procedure;
- the option of applying subdivision on selected tetrahedra after tetrahedrization.

The recognition of kinds of nodes, edges, triangles and tetrahedrons existing in the 3D mesh, is of utmost importance in many instances of the mathematical processing of SLB simulation problems, like the following ones:

- for determining appropriate surfaces for boundary conditions pertinent to the partial differential transport and flow equations applied in the SLB model;
- for node manipulation, which is essential in several stages of the numerical solver that is being used, such as in the case of the method of finite elements;
- for calculating several SLB statistics – after tetrahedrization was accomplished – like shoreline length, flooded area and water hold-up;
- for identifying bad shape tetrahedrons that deserve further refining or subdivision;

Acknoledgments

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