Modelling and Analysis of Cutting Force and Surface Roughness in Milling Operation Using TSK-Type Fuzzy Rules

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The present paper discusses on development of fuzzy rule based models (FRBMs) for predicting cutting force and surface roughness in milling operation. The models use Takagi-Sugeno-Kang-type (TSK-type) fuzzy rule to study the effect of four (input) cutting parameters (cutting speed, feed rate, radial depth of cut and axial depth of cut) on outputs (cutting force and surface roughness). The appropriate FRBM is arrived after a thorough investigation of different structures of rule-consequent function. A combined approach of genetic algorithm and multiple linear regression method is used to determine the rule-consequent parameters. Performance analysis of models by comparing with experimental data implies its potential towards practical application. Analysis of the influence of various input parameters on different outputs is carried out based on FRBMs and experimental data. It suggests that the cutting force becomes higher with increasing feed rate, axial depth of cut and radial depth of cut and lower with increase in cutting speed, whereas surface finish is improved with increase in cutting speed and gets poorer with increase in radial depth of cut.

Keywords: *fuzzy rule based model, TSK-type fuzzy rule, genetic linear regression, milling, surface roughness, cutting force*

Introduction

For a long time, manufacturing engineers and researchers have been realizing that in order to optimize the economic performance of metal cutting operations, efficient quantitative and predictive models are important. These models establishing the relationship between independent (input) parameters and output variable(s), are required for the wide spectrum of manufacturing processes, cutting tools and engineering materials (Armarego and Brown, 1969). Furthermore, it has been observed that the improvements in the output variables, such as tool life, cutting forces, surface roughness, etc., through the optimization of controllable/input parameters may result in a significant economic performance of machining operations (Armarego, 1994). The output variables that may have either direct or indirect indications on the performance of other variables such as tool wear rate, machining cost etc. are cutting forces and surface roughness.

Many researchers have conducted studies on predicting cutting forces produced in milling operations using theoretical and analytical approaches (Li et al., 1999; Li and Li, 2002; Yun and Cho, 2001; Yoon and Kim, 2004; Koenigsberger and Sabberwal, 1961; Sabberwal, 1960; Yun and Cho, 2000; Wang and Chiou, 2004), mechanistic model (Omar et al., 2007; Kang et al., 2007; DeVor et al., 1980; Sutherland and DeVor, 1986), etc. The problem with these approaches is that they are based on a big number of assumptions, which are not included in the analysis. This may reduce the reliability of the calculated cutting force values found by these methods. In addition, these approaches may be successfully applicable only for certain ranges of cutting condition. On the other hand, many other researchers have followed purely experimental approaches to study the relationship between cutting force and independent cutting conditions (Li et al. (2006)). It has reflected on the increased total cost of the study, as a large number of cutting experiments are required. Furthermore, with this purely experimental approach, researchers have investigated the effects of cutting parameters on output parameter(s) using machining experiments based on a one-factor-at-a-time design without having any idea about the behaviour of output parameter(s) when two or more cutting factors varied at the same time. So, some researchers had adopted the RSM (response surface methodology) technique, which is basically a group of mathematical and statistical techniques that are useful for numerical modelling the relationship between the input parameters (cutting conditions) and the output variable(s) (cutting force) (Montgomer, 2001). Although RSM saves cost and time, sometimes it becomes difficult to model the process having highly complex and non-linearity among input-output variables. For example, the 2nd order model (for cutting force in end milling operation) derived using RSM approach exhibits high mean square error value as observed during ANOVA analysis (Abou-El-Hossein et al. (2007)). There are many other approaches that have become of interest to researchers to adopt, for finding cutting force relationship in milling operation, namely, FEM analysis (Lee and Cho, 2007), Fuzzy logic (Zuperl et al., 2005), Evolutionary approach (Kovacic et al., 2004), etc.

Again, in case of analysis of surface roughness in end milling operation, many researchers have gone through experimental approach and mathematical relation(s) between output parameter (surface roughness) and cutting conditions allowing us to predict in general form (Dewes and Aspinwall, 1997; Alauddin et al., 1996; Chang, 1992; Kline et al., 1982; Chevrier et al., 2003; Vivancos et al., 2004). But it has been observed that such type of experimental and mathematical models result a great difference between real value(s) and theoretical value(s) due to consequence of movement error and building-ups edge as well as changes in the tool profile because of wear. Normally these causes are very difficult to maintain under precise control to obtain reproducible results. In order to overcome those difficulties, there were various approaches adopted concerning surface roughness in end milling operation, namely, Taguchi method in optimization of parameters (Ghani et al. (2004)), Computer-aided analysis for modelling (Alauddin et al. (1995)), ANN based modelling (Tsai et al. (1999)), etc.

From the above surveys, it has been observed that the prediction of surface roughness and cutting force in milling based on models which are constructed using conventional methods may not be accurate. This is so as milling process is a complex physical process, where the relationships of input-output variables are non-linear. In contrary, fuzzy logic concept is a well-established powerful tool to model physical processes, which are highly complex in nature and where the input-output relationships represent non-linearity,

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uncertainty and ambiguity. In the present study, cutting force and surface roughness produced during milling operation are investigated using FRBM (fuzzy rule based model) which are constructed using TSK-type fuzzy logic rule. A combined approach of multiple linear regression and genetic algorithm, so called genetic Linear Regression (GLR) approach is adopted to construct knowledge base (KB) of TSK-type FRBM. The models include four cutting (controllable) parameters: feed rate, cutting speed, axial depth of cut and radial depth of cut.

The rest of the paper is organized as follows: the second section describes FRBM using TSK-type fuzzy rule with construction of its KB based on GLR approach. Experimentation and experimental data analysis are discussed in the following section. Mathematical correlation models for cutting force and surface roughness with cutting parameters in milling which are determined based on the RSM are illustrated in the fourth section. The fifth section describes the training data and fitness evaluation procedure adopted in GLR approach. Details of TSK-type FRBMs for cutting force and surface roughness in milling process, as obtained based on GLR approach, are shown in the sixth section. Results and discussion on the prediction capabilities of FRBMs are discussed in the seventh section. Finally, concluding remarks are pointed out in eighth section.

Nomenclature

a	= *function coefficient*
$A_1, \ldots, A_n$	= *fuzzy subsets*
A_d	= *axial depth of cut, mm*
b, b_1, b_2, b_3, b_4	= *base-widths of membership function distributions*
C_p	= *crossover probability*
d, d_1, d_2, d_3, b_5	= *base-widths of overlapping between two fuzzy subsets*
F_c	= *cutting force, N*
F_d	= *feed rate, mm/rev*
FLR	= *fuzzy logic rule*
FRBM	= *fuzzy rule based model*
GA	= *genetic algorithm*
H	= *high*
KB	= *knowledge base*
L	= *low*
M_p	= *mutation probability*
MaxV	= *maximum value*
MFDs	= *membership function distributions*
MinV	= *minimum value*
N_g	= *number of generations*
P	= *population size*
R_d	= *radial depth of cut, min*
RB	= *rule base*
RCFs	= *rule consequent functions*
S_r	= *surface roughness, micron*
V_c	= *cutting velocity, m/min*

FRBM Using TSK-Type Fuzzy Rule

TSK-type fuzzy logic rules are widely used in developing rule-based systems. A fuzzy rule uses the fuzzy set theory proposed by Zadeh (1965). The syntax of a TSK-type fuzzy rule looks as follows (Sugeno and Kang, 1988; Takagi, and Sugeno, 1985):

If x_1 is A_1 and x_2 is A_2 and…and x_n is A_n, then $y = f(x_1,\ldots, x_n)$

where $A_1, \ldots, A_n$ are fuzzy subsets of the input variables $x_1, \ldots, x_n$, respectively. The consequent function of each rule is described as a (linear) function, in the form

$$y = \sum_{j=1}^{K} a_j \; f_j \; (x_1,\ldots, x_n),$$

where K is the number of parameters (coefficients) associated to a function and $f_j\ (x_1,\ldots,x_n)$ is a sub-function of the input variables $x_1, \ldots, x_n$. The overall output of the model can be obtained for the input tuple $(x_1, x_2, \ldots, x_n)$ using the following empirical expression.

$$Y = \frac{\sum_{r=1}^{R}\left(\prod_{v=1}^{n}\mu_v^r(x_v)\right)\sum_{j=1}^{K} a_j^r f_j^r(x_1,\ldots, x_n)}{\sum_{r=1}^{R}\left(\prod_{v=1}^{n}\mu_v^r(x_1,\ldots, x_n)\right)} \tag{1}$$

where n is the number of input variables that occur in the rule premise, R is the number of rules in the rule base. $\prod_{v=1}^{n}\mu_v^r(x_1,\ldots, x_n) = \eta_r$ is the firing degree of r^{th} rule. $\prod$ is the product representing a conjunction. $\sum_{j=1}^{K} a_j^r f_j^r(x_1,\ldots, x_n)$ is the rule consequent function (y) of the r^{th} rule and a_j^r are the function coefficients of the corresponding r^{th} rule consequent function. For a typical rule consequent function, say polynomial may be expressed by

$$y = a_1 x_1^{p_1} + a_2 x_2^{p_2} + a_3 x_3^{p_3} + a_4 x_4^{p_4} \tag{2}$$

The performance of this model mainly depends on the optimal values of the output function coefficients (a_1, a_2, a_3 and a_4) of the rules for a given values of the variable's exponential parameters (p_1, p_2, p_3 and p_4) and also on the choice of the type of MFDs considered for the input variables (x_1, x_2, x_3 and x_4). In addition to that the issue of having the optimized fuzzy sub-sets of each input variables is also an important concern for achieving the best performance of a model.

Model Construction

The main objective of constructing FRBM of a physical process is to design its optimum KB based on the measured example data. The KB of FRBM consists of rule base (RB) and fuzzy sub sets (or MFDs), also called database. Several methods had been suggested by various researchers for fuzzy rule generation. In this connection, work of Takagi, and Sugeno (1985), Abdelnour et al. (1991), Wang and Mendel (1992) are worth mentioning. Moreover, gradient descent method (Nomura et al., 1992), reinforcement learning technique (Fukuda et al., 1995), neural networks (Nauck et al., 1993), evolutionary algorithm (Hwang and Thompson, 1994), etc. are well employed to construct RB. In the present work, a combined approach of multiple linear regression and GA (Nandi, 2006), so called genetic linear regression approach is adopted to construct the KB of FRBM with TSK-type FLR, as illustrated in Fig. 1.

In this combined approach, the values of function coefficients are determined using linear regression method, while a GA is introduced to optimise the exponential parameters of input variables as well as optimisation of MFDs of input variables using the same GA. That means, once the values of exponential parameters of the RCFs and the parameters associated with the membership functions are obtained, the values of coefficients of the RCFs are evaluated by multiple linear regression method.

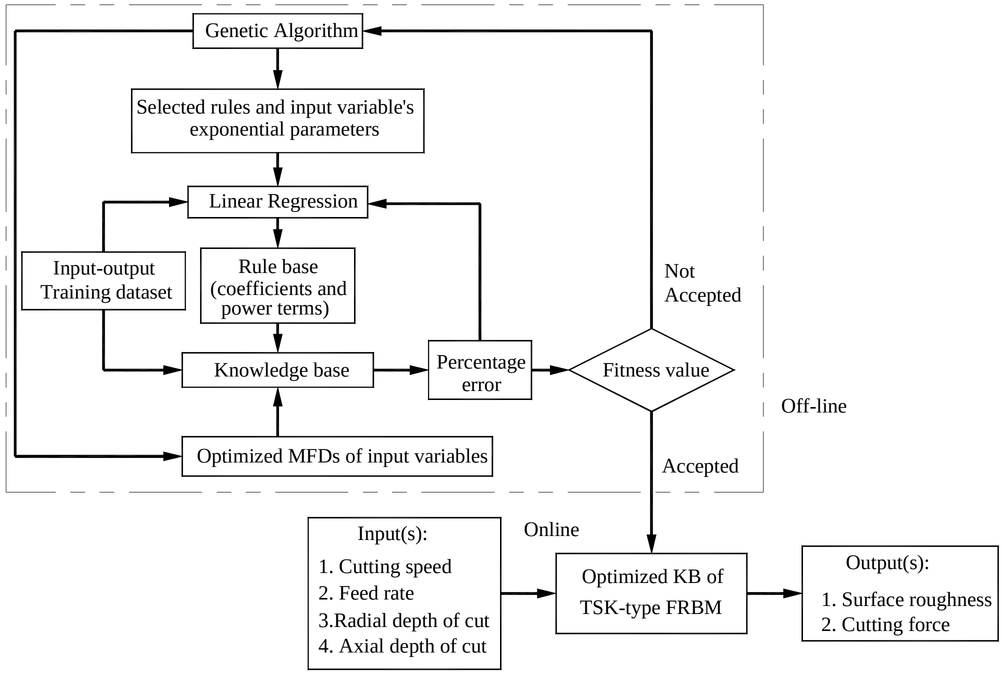

Figure 1. Flow chart of Genetic Linear Regression approach for construction of TSK-type FRBM.

The structure of trapezoidal MFDs as considered here for the input variables is represented in Fig. 2. Two parameters, b and d are needed to describe the (semi) trapezoidal MFDs. The scaling factors (MaxV – MinV) of all input variables are kept as same during optimization of MFDs in constructing each FRBMs for surface roughness and cutting force.

The optimal values of rule-consequent coefficients and power terms are obtained using genetic linear regression approach and simultaneous optimisation of input variable's MFDs using GA, as presented in Fig. 1. The optimum values of power terms of rule consequent functions (p_1, p_2, p_3 and p_4, according to Eq. (2)) and the parameters related to MFDs (b and d, according to Fig. 2) are determined using GA, while the rule-consequent coefficient (a_1, a_2, a_3 and a_4 according to Eq. (2)) are determined using multiple linear regression method in the framework of genetic linear regression approach. As the performance of a GA depends on the GA-parameters, the optimal choices of GA-parameters (namely population size, crossover probability and mutation probability) are fixed through a parametric study (Nandi, 2006) in order to achieve good results.

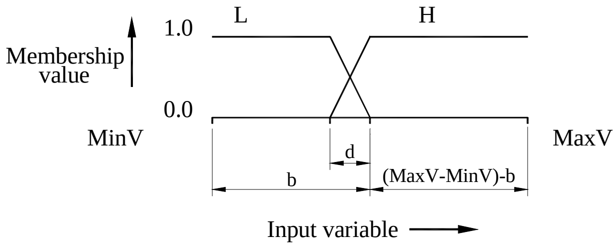

Figure 2. Structure of semi-trapezoidal MFDs with two fuzzy subsets.

Linear Regression Method with TSK-Type Fuzzy Model (Nandi and Klawonn, 2004)

A general expression of linear regression system with TSK-type fuzzy model is derived here to determine the coefficients of RCFs in GLR approach. Equation (1) may be rewritten by denoting $\prod_{v=1}^{n}\mu_v(x_{1,\ldots,n}) = \eta_r$ for simplicity, in the following form:

$$Y = F(x_1, \ldots\ldots x_n)$$

$$= \frac{\sum_{r=1}^{R^1}\eta_r\left(a_1^r f_1^r(x_{1,..,n}) + a_2^r f_2^r(x_{1,\ldots,n}) + \ldots.. + a_k^r f_k^r(x_{1,\ldots,n})\right)}{\sum_{r=1}^{R^1}\eta_r}$$

$$= \frac{\begin{array}{l}\eta_1\left(a_1^1 f_1^1(x_{1,..,n}) + \ldots\ldots + a_k^1 f_k^1(x_{1,.,n})\right) + \ldots\ldots\ldots\ldots \\ + \eta_{t_r}\left(a_1^{tr} f_1^{tr}(x_{1,..,n}) + \ldots\ldots.. + a_{tj}^{tr} f_{tj}^{tr}(x_{1,..,n}) + .. + a_k^2 f_k^2(x_{1,.,n})\right) + \\ \ldots\ldots.. + \eta_{R^1}\left(a_1^{R^1} f_1^{R^1}(x_{1,.,n}) + .. + a_k^{R^1} f_k^{R^1}(x_{1,.,n})\right)\end{array}}{\eta_1 + \eta_2 + \ldots\ldots.. + \eta_{R^1}}$$

Let us assume we have a set of input-output tuple (D) of S number of sample data where the output $y^{(i)}$ is assigned to the input $\left(x_1^{(i)}, x_2^{(i)}, \ldots\ldots\ldots, x_n^{(s)}\right)$.

$$D = \left\{\left(x_1^{(1)}, \ldots.., x_n^{(1)}, y^{(1)}\right), \left(x_1^{(2)}, \ldots, x_n^{(2)}, y^{(2)}\right) \ldots.. \left(x_1^{(s)}, \ldots.., x_n^{(s)}, y^{(s)}\right)\right\}$$

Now, the total quadratic error that is caused by the TSK-type FRBM with respect to the given data set is

$$E = \sum_{l=1}^{s}\left(f\left(x_1^{(1)}, x_2^{(1)}, \ldots. x_n^{(1)}\right) - y^{(1)}\right)^2 \tag{3}$$

In order to minimise E, we have to choose the following parameters appropriately:

$$\left\{\left(a_1^1,....,\ a_k^1\right),\left(a_1^2,.....\ a_k^2\right),......,\ \left(a_1^R,....,\ a_k^R\right)\right\},$$

where the parameter a_j^r indicates the j[th] coefficient of the output function of r[th] rule.

To determine the above parameters, we take the partial derivatives of *E* with respect to each parameter (a_j^r) and make them be zero, i.e.,

$$\frac{\partial E}{\partial a_j^r}=0\text{, where } j=\{1,2,....,k\} \text{ and } r=\{1,2,.....,R\}$$

Now, we obtain the partial derivation of E with respect to the parameter a_{tj}^{tr},

$$\frac{\partial E}{\partial a_{tj}^{tr}}=\sum_{l=1}^{S}2\cdot\left(f\left(x_1^{(l)},........,x_n^{(l)}\right)-y^{(l)}\right)\cdot\frac{\partial f\left(x_1^{(l)},........,x_n^{(l)}\right)}{\partial a_{tj}^{tr}}$$

$$=2\cdot\sum_{l=1}^{S}\left(\frac{\sum_{r=1}^{R^l}\eta_r\left(a_1^r f_1^r\left(x_{1,..,n}^l\right)+...+a_k^r f_k^r\left(x_{1,..,n}^l\right)\right)}{\sum_{r=1}^{R^l}\eta_r}-y^l\right)\times\frac{\eta_{tr}\cdot f_{tj}^{tr}\left(x_{1,..,n}^l\right)}{\sum_{r=1}^{R^l}\eta_r}.$$

$$=2\left(\left(\frac{\sum_{l=1}^{S}\sum_{r=1}^{R^l}\eta_r\left(a_1^r\right)f_1^r\left(x_{1,..,n}^l\right)\eta_{tr}f_{tj}^{tr}\left(x_{1,..,n}^l\right)}{\left(\sum_{r=1}^{R^l}\eta_r\right)^2}\right)+..............................+\left(\frac{\sum_{l=1}^{S}\sum_{r=1}^{R^l}\eta_r\left(a_k^r\right)f_k^r\left(x_{1,..,n}^l\right)\eta_{tr}f_{tj}^{tr}\left(x_{1,..,n}^l\right)}{\left(\sum_{r=1}^{R^l}\eta_r\right)^2}\right)\right)$$

$$-2\left(\frac{\sum_{l=1}^{S}y^l\cdot\eta_{tr}f_{tj}^{tr}\left(x_{1,..,n}^l\right)}{\sum_{r=1}^{R^l}\eta_r}\right)=0, \tag{4}$$

Thus, Eq. (3) provides the following system of linear equations from which we can compute the coefficients $\left\{\left(a_1^1,....,\ a_k^1\right),\left(a_1^2,.....\ a_k^2\right),......,\ \left(a_1^R,....,\ a_k^R\right)\right\}$:

$$\sum_{r=1}^{R}\sum_{j=1}^{K}a_j^r\sum_{l=1}^{S}\frac{\prod_{v=1}^{n}\mu_v^r\left(x_{1,..,n}^l\right)}{\left(\sum_{r=1}^{R^l}\prod_{v=1}^{n}\mu_v^r\left(x_{1,..,n}^l\right)\right)^2}f_j^r\left(x_{1,..,n}^l\right)\times f_{tj}^{tr}\left(x_{1,..,n}^l\right)\times\prod_{v=1}^{n}\mu_v^{tr}\left(x_{1,..,n}^l\right)$$

$$=\sum_{l=1}^{S}\frac{y^l\prod_{v=1}^{n}\mu_v^{tr}\left(x_{1,..,n}^l\right)}{\sum_{r=1}^{R^l}\prod_{v=1}^{n}\mu_v^r\left(x_{1,..,n}^l\right)}f_{tj}^{tr}\left(x_{1,..,n}^l\right) \tag{5}$$

In matrix form, Eq. (5) will be written as:

$$\begin{bmatrix}\alpha_{11}^r & \alpha_{12}^r & \cdot & \cdot & \cdot & \cdot & \cdot & \alpha_{1K}^r\\ \alpha_{21}^r & \alpha_{22}^r & \cdot & \cdot & \cdot & \cdot & \cdot & \alpha_{2K}^r\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \alpha_{K1}^r & \alpha_{K2}^r & \cdot & \cdot & \cdot & \cdot & \cdot & \alpha_{KK}^r\end{bmatrix}.\begin{bmatrix}a_1^r\\ a_2^r\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ a_K^r\end{bmatrix}=\begin{bmatrix}\beta_1^r\\ \beta_2^r\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \beta_K^r\end{bmatrix} \tag{6}$$

where $\alpha_{tj}^r=\sum_{l=1}^{S}f_j^r\left(x_1^l,x_2^l..,x_n^l\right)f_t^r\left(x_1^l,x_2^l..,x_n^l\right)$; $\beta_t^r=\sum_{l=1}^{S}y^l f_t^r$.

Thus Eq. (5) provides solutions of the function coefficients (a_j^r) of the TSK-type fuzzy rule consequents for given values of the input variable's exponential terms.

Experimentation

For modelling cutting force in milling, modified AISI P20 tool steel is considered as the work piece material (Abou-El-Hossein et al., 2007). It is a chromium-molybdenum alloyed which is considered as high speed steel. AISI P20 defers from normal P20 steel by containing 0.015% Sulphur, because of better machinability and more uniform hardness in all dimension. Its tensile strength is 1044 MPa and its hardness range is 280 HB to 320 HB. The cutting tool used in this study is a 00 lead-positive end milling cutter of 31.75 mm diameter and equipped with two square inserts whose all four edges can be used for cutting. Here, one insert per one experiment is mounted on the cutter. The inserts have the following specification: square shape, back rake angle of 00, clearance angle of 110, nose radius of 0.794 mm and without any chip breaker. These carbide inserts are KC735M which have a single layer of TiN. The coating is accomplished using PVD techniques to a maximum of 0.004 mm thickness. Experiments are performed in random with different cutting conditions and using a standard coolant to find the cutting force. Each experiment is stopped after 85 mm cutting length. F_c is measured with the aid of a piezoelectric cutting force dynamometer provided by Kistler. Each experiment is repeated three times using a new cutting edge every time and the average of these values is considered.

On the other hand, for surface roughness modelling, the material of workpiece used is W-Nr. 1.2344, hardened steel (50–54 HRC) (Vivancos et al., 2004). A cutting tool of KOBELCO series MIRACLE: (Al, Ti) N-coated micro grain carbide, two flute ball end mill VC2SBR0300, diameter 6 mm is used. Effective S_r is measured with a Taylor–Hobson form Taylsurf series 2 profile rugosimeter in every experiment conducted with different cutting conditions.

Now, the data collected based on experimentation are analyzed in the following sub-section to reveal the preliminary information underlying in the relationship between input-output variables. This information is used in the GLR approach to construct the KB of FRBMs.

Experimental Data Analysis

Surface roughness

In order to understand the relationship of surface roughness with cutting parameters (feed rate, radial depth of cut, axial depth of cut and cutting speed), it is essential to analyse the variation of surface roughness with respect to each of the individual cutting parameter as well as when more than one parameter are changing simultaneously. After analysing the experimental data, as shown in Figs. 3(i)-(iv) which describe the variation of surface roughness with feed rate, the following points are revealed:

i) Surface roughness is deteriorated with increasing feed rate at
  a) any value of A_d and V_c but lower value of R_d (0.1 mm)
  b) lower value of A_d (0.1 mm) but higher value of V_c and R_d (250 m/min and 0.1 mm, respectively), Fig. 3(iv)

ii) Surface roughness improves with increase in feed rate at
  a) any value of A_d, lower value of V_c (150 m/min) and higher value of R_d (0.3 mm), according to Fig. 3(iii)
  b) higher values of A_d (0.3 mm), V_c (250 m/min) and R_d (0.3 mm), according to Fig. 3(iv)

Figures 4(i)-(iv) describe the variation of surface roughness with respect to radial depth of cut. After analysing the data as shown in Figs. 4(i)-(iv), it has been revealed that surface roughness get worse by increasing the value of R_d at any values of axial depth of cut, feed rate and cutting speed, and the rate deterioration (considerably high) is almost the same for all values of A_d, F_d and Vc.

The variations of surface roughness with respect to axial depth of cut are illustrated in Figs. 5(i)-(iv). Analysis of data as presented in Figs. 5(i)-(iv) implies the following points:

i) Surface roughness is deteriorated (in different rates) with increasing axial depth of cut at
  a. lower value of R_d (0.1), any values of F_d and V_c, Figs. 5(i)-(ii)
  b. higher values of R_d (0.3) and V_c (250), and lower value of F_d (0.02), Fig. 5(iv)

ii) Surface roughness is improved with increasing axial depth of cut only at
  a. higher value of R_d (0.3), any value of F_d and lower value of V_c (150), Fig. 5(iii)

After analysing the data as shown in Figs. 6(i)-(iv), which describe the variation of surface roughness with cutting speed, the following points are revealed:

i) Surface roughness is deteriorated with increasing cutting speed at
  a. any value of A_d, higher value of R_d (0.3) and any value of F_d, Fig. 6(iv)
  b. higher value of A_d (0.3), lower value of R_d (0.1) and higher value of F_d (0.06), Fig. 6(ii)

ii) Surface roughness is improved with increasing cutting speed at
  a. lower value of A_d (0.1), lower value of R_d (0.1) and any value of F_d, Figs. 6(i), (ii) and (iii).

From the above analyses, it is stated that change in radial depth of cut influences much on surface roughness than other cutting parameters, namely axial depth of cut, cutting velocity and feed rate.

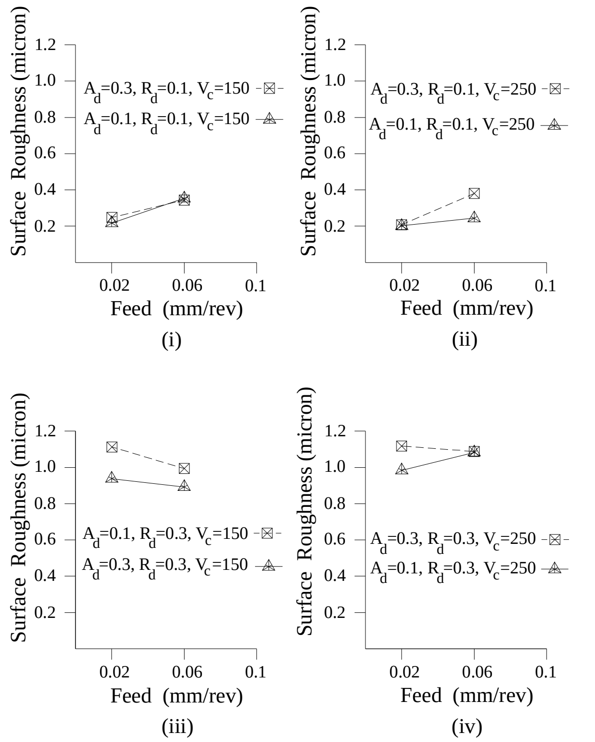

Figure 3. Variation of surface roughness with feed rate.

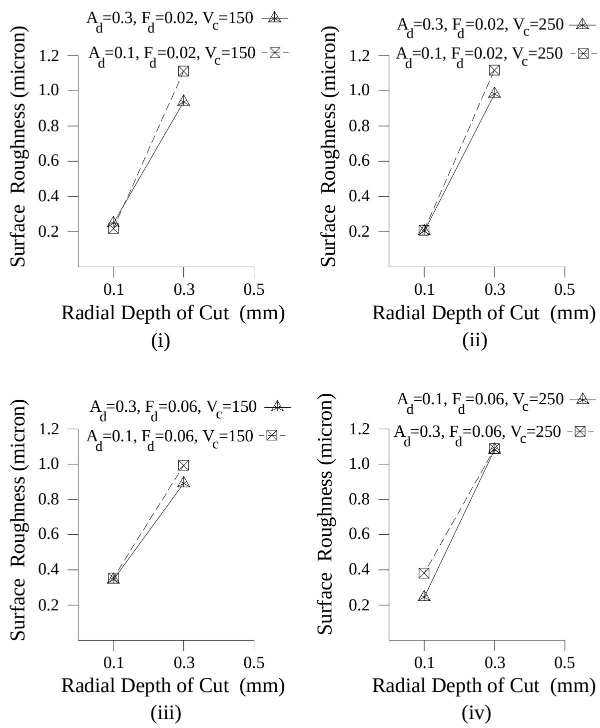

Figure 4. Variation of surface roughness with radial depth of cut.

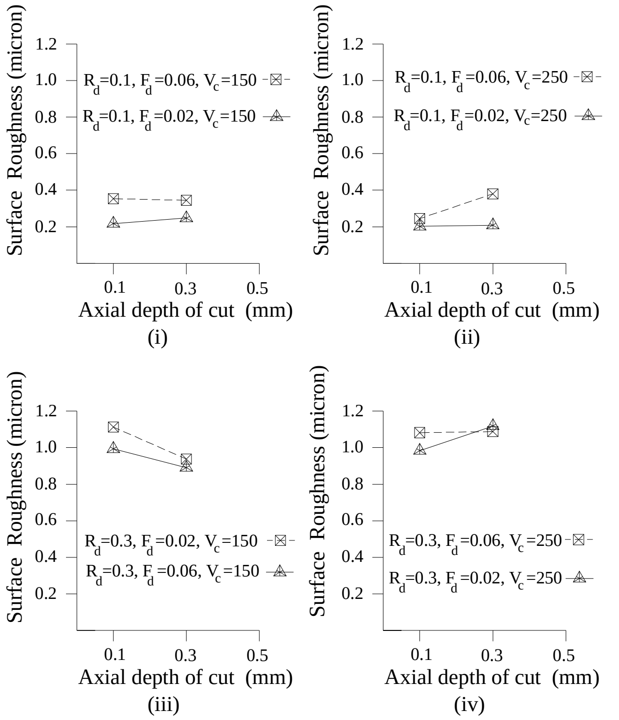

Figure 5. Variation of surface roughness with axial depth of cut.

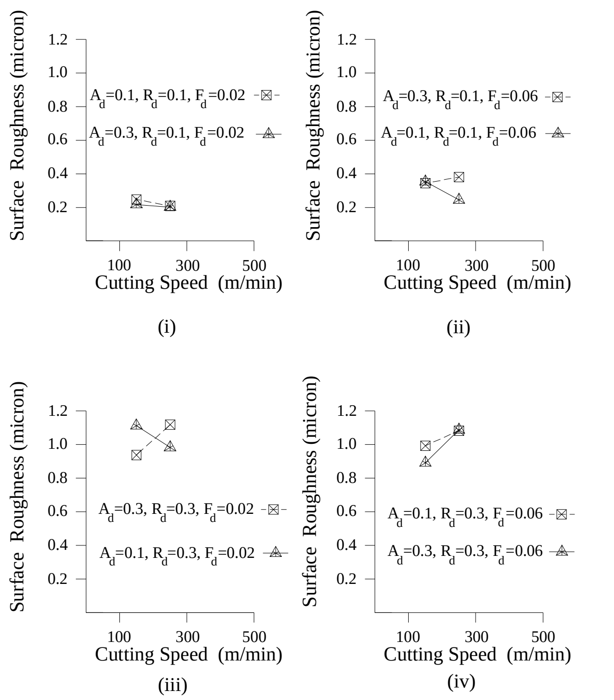

Figure 6. Variation of surface roughness with cutting speed.

Cutting Force

Like surface roughness, the influences of different cutting parameters (F_d, A_d, R_d and V_c) on cutting force generated during milling operation are illustrated in graphical manner based on experimental data. This underlying information in the cutting force relation with cutting parameters extracted from experimental data is later utilized during learning of FRBM for constructing cutting force model.

Figures 7(i)-(iii) show the graphs representing the variation of cutting force with axial depth of cut. It is observed that cutting force increases with increasing axial depth of cut at almost equal rate at any values of V_c, F_d and R_d. Again it is observed in Fig. 7(iii) that, when cutting velocity is decreased, the amount of cutting force value is comparatively higher for the constant values of F_d and R_d.

Figures 8(i)-(iii) represent the variation cutting force with feed rate. It is found that cutting force increases with increase in feed rate for any values of V_c, A_d and R_d, but the increasing rate varies in different cases. Again in Fig. 8(i), when A_d changes the value from 1 mm to 2 mm, with increase in feed rate, the cutting force increases but it starts from a high value as well as with higher rate.

In Figs. 9(i)-(ii), the graphs are drawn showing the variation of cutting force with radial depth of cut. It is observed that cutting force increases with increase in radial depth of cut. It is observed that, if the value of A_d changes from 1 mm to 2 mm (Fig. 9(i)) and V_c changes value from 180 m/min to 100 m/min (Fig. 9(ii)), with increase in R_d, the cutting force value becomes high and it increases with almost equal rate.

In Figs. 10(i)-(iii), the curves are drawn representing the variation of cutting force with cutting speed. Here it is observed that with increase in cutting speed, the cutting force decreases for any values of F_d, A_d and R_d, i.e. proportionally inverse. For a given cutting speed, the cutting force value becomes high if R_d changes from 2 mm to 5 mm and A_d changes from 1 mm to 2 mm, as shown in Fig. 10(i) and Fig. 10(ii), respectively.

From the above analysis of experimental data, it is clearly observed that the outputs (surface roughness and cutting force) in milling are not linearly related with the cutting parameters and ambiguity is involved when more than one cutting parameters vary simultaneously.

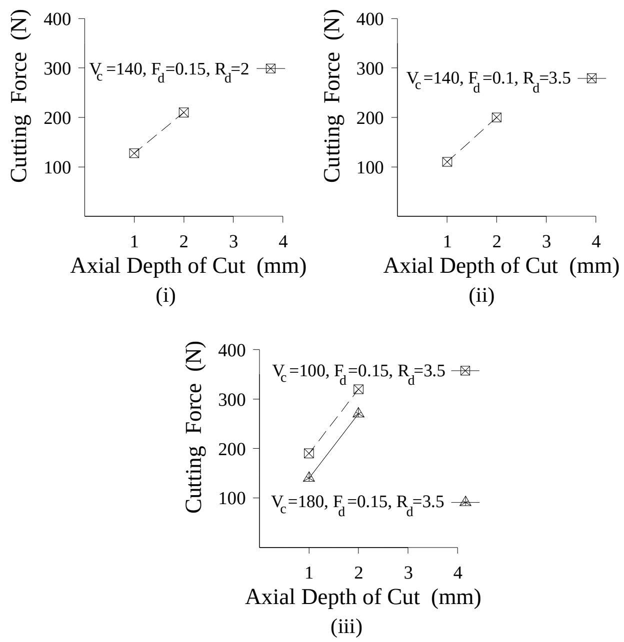

Figure 7. Variation of cutting force with axial depth of cut.

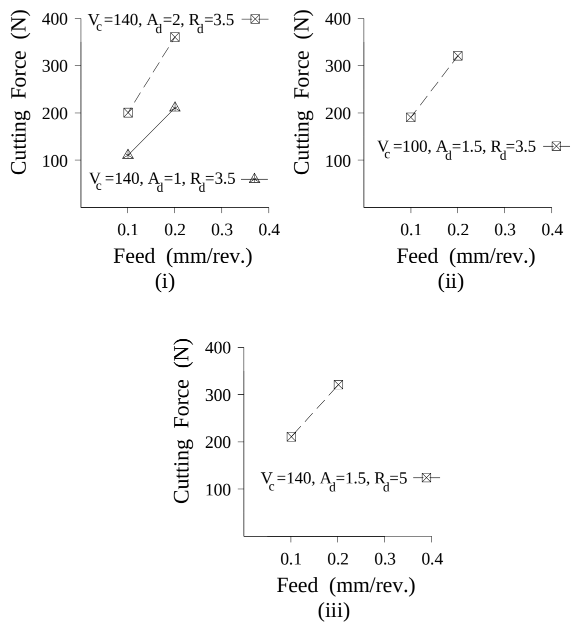

Figure 8. Variation of cutting force with feed rate.

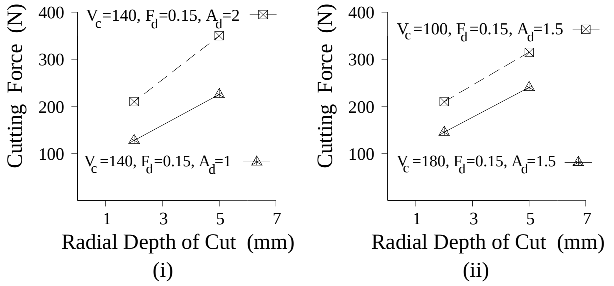

Figure 9. Variation of cutting force with radial depth of cut.

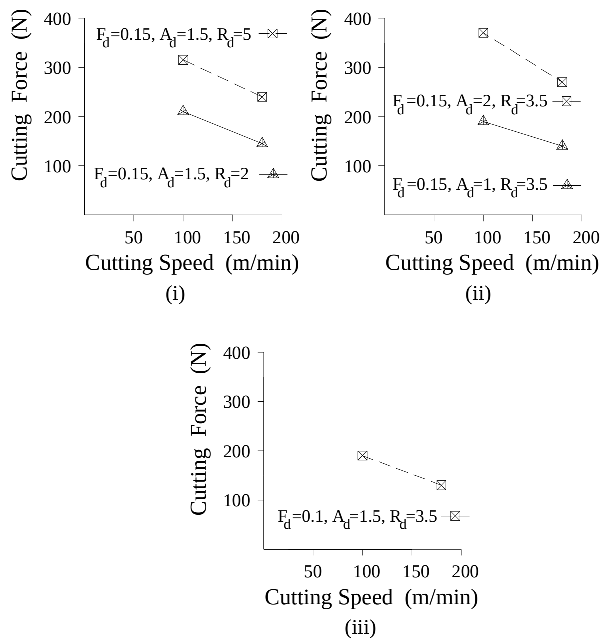

Figure 10. Variation of cutting force with cutting speed.

Mathematical Model

The mathematical model between cutting parameters (cutting velocity, feed rate, axial depth of cut and radial depth of cut) and the cutting force in milling operation (with workpiece material of AISI P20) was derived by using Box-Behnken design (one type of RSM) and it is defined by:

$$F_c = 330.66 - 3.09V_c + 292.30F_d - 166.07A_d + 2.57R_d + 0.0089V_c^2 - 307.67F_d^2 + 48.15A_d^2 + 2.02R_d^2 - 0.0417V_cR_d + 600F_dA_d + 33.33F_dR_d + 14.05A_dR_d \quad (7)$$

The regression model of surface roughness with cutting parameters for (climb) milling (with workpiece material of W-Nr) is derived by Vivancos et al. (2004), as follows:

$$S_r = 0.683042 - 1.34515A_d - 2.49037R_d + 3.4081F_d - 0.00250345\,V_c + 0.00672575\,A_dV_c + 14.6044R_d^2 - 17.0406R_dF_d + 0.0057915R_dV_c \quad (8)$$

Training Data and Fitness Evaluation of GA

Training data

In order to determine the rule consequent function coefficients and power terms of a TSK-type FRBM, a huge number of example data are required. In the present study, 81 numbers of data (Fig. 11 and Fig. 12 related to cutting force and surface roughness, respectively) are considered for constructing KB of FRBMs. These data are obtained through real experimentation as well as based on empirical correlation models (as stated in the section "Mathematical Model"). However, those empirical models are not accurate. Hence, the results obtained using the empirical models do not follow the real characteristics of the relationships among input-output variables in milling process. For this reason, it is required to modify the data obtained using mathematical models to suit the process input-output relationship as discussed in experimental data analysis (in sub-section "Experimental Data Analysis").

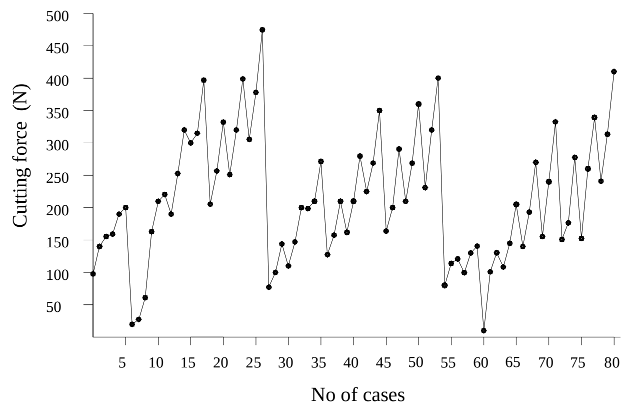

Figure 11. Training data: Cutting force.

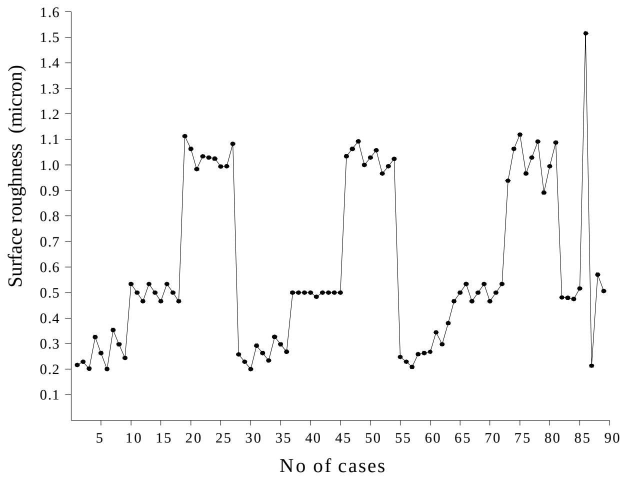

Figure 12. Training data: Surface roughness.

Fitness Evaluation of GA

During the iteration process of genetic algorithm, the GA population (individuals/chromosomes) having lower fitness value (for error minimization) is chosen in order to reproduce the child chromosomes in the next iteration using the three GA- operators, namely selection, cross-over and mutation. On the other hand, to have a better reliability of FRBM, the performance of FRBM is to be uniform throughout the entire input space. To achieve such consistent result of an FRBM, in every region of the input space the errors of all training data samples that are considered to be uniformly distributed over the whole range of the input variable's space should be equally important for minimization in finding a lower fitness value. Thus, the fitness value of a GA solution is estimated based on the percentage error (instead of simple error) of each training data sample. The error of each set of training data is the deviation of the result (surface roughness) of the FRBM from that of the desired one. Since the error may be positive or negative, absolute value of the error is considered in determining average percentage error as a fitness value of GA-solution.

For cutting force, the fitness value of GA-solution during model construction is calculated in the same way as discussed above for surface roughness.

TSK-Type FRBM for Milling Process

In order to develop a suitable model for milling operation in the present work, four input process variables (cutting speed, feed rate, axial depth of cut and radial depth of cut) are considered. For each of the output variables (cutting force and surface roughness), the model is constructed based on the training data as depicted in Fig. 11, and Fig. 12, respectively. Each of the four input variables are considered to have semi-trapezoidal MFDs with two different linguistic values (L and H) (as shown in Fig. 2) and the corresponding scaling factors are 80, 0.1, 1.0 and 3.0, respectively, for all the TSK-type FRBMs corresponding to different outputs. Since each input variable has two linguistic terms within its range, there could be a maximum of $2\times2\times2\times2=16$ rules in the RB of FRBM.

Model of Cutting Force

In order to develop a FRBM for cutting force in milling process, the structure of rule consequent function (as shown in Eq. (9)) considered here has four coefficients and four power terms. Thus the RB, with a maximum of 16 rules in the rule premise, would have a total of 64 (16x4) coefficients and 64 power terms.

A GA-string of 720-bits long is considered for finding the RCFs parameters using GLR approach as well as optimization of MFDs of input variables. First 80 bits (10 bits for each variable) of the GA-string carry information of the eight continuous variables (two variables related to MFDs, b and d for each of the four inputs). The remaining 640 bits (10 bits for each variable) are used to obtain the values of 64 power terms. It is noted that during optimization of MFDs of input variables, the scaling factors (length of input range) of all input variables are not changed.

During GA-based optimization, the parameters related to MFDs – b_1 and d_1 (for cutting speed); b_2 and d_2 (for feed rate); b_3 and d_3 (axial depth of cut) and b_4 and d_4 (radial depth of cut), as shown in Fig. 2, are varied in the range of **{**(20, 60) and (0, 20)**}**; **{**(0.02, 0.05) and (0, 0.02)**}**; **{**(0.2, 0.8) and (0, 0.2)**}** and **{**(1, 2) and (0, 1)**},** respectively. The values of power terms lie in the range of 0.0 to 3.0. The fitness values of GA solution are calculated using the procedure as discussed in sub-section "Fitness Evaluation of GA". The optimal choices of GA-parameters (namely population size, crossover probability and mutation probability) are fixed through a parametric study in order to achieve good results.

After a parametric study of GA, the following GA parameters are selected for the best optimization during training of FRBM for cutting force prediction:

$P = 100$; $C_p = 0.87$; $M_p = 0.011$; $N_g = 125$.

$$F_c = c_1 V_c^{p_1} + c_2 F_d^{p_2} + c_3 A_d^{p_3} + c_4 R_d^{p_4} \tag{9}$$

The optimized data base and rule base of FRBM for cutting force in milling obtained using Eq. (9) are shown in Fig. 13 and Table 1, respectively.

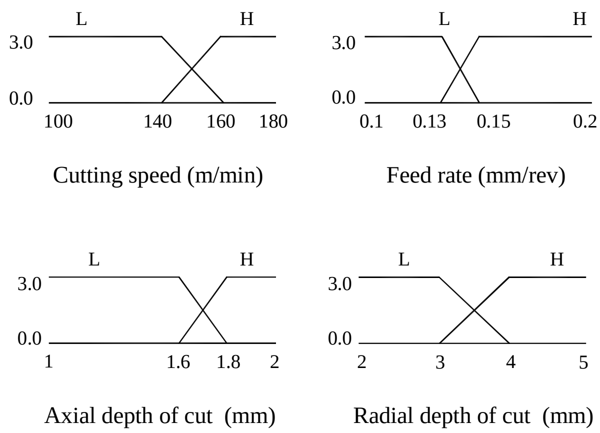

Figure 13. Optimized semi-trapezoidal MFDs of TSK-type FRBM for cutting force.

Model of Surface Roughness

Like cutting force, the structure of rule consequent function for surface roughness (as shown in Eq. (10)) has four coefficients and four power terms. Since the rule base consists of a maximum 16 rule in the rule premise, there would be a total of 64 (16x4) coefficients and 64 power terms in the RB. A GA-string of 720-bits long is considered here for the GLR technique as well as the optimization of MFDs of input variables. The first 80 bits (10 bits for each variable) are used to carry information of the eight continuous variables related MFDs of input variables. The remaining 640 bits

(10 bits for each variable) are used to obtain the values of 64 power terms. It is noted that during optimization of MFDs of input variables, the scaling factors are not changed.

During GA-based optimization, the parameters related to MFDs – b_1 and d_1 (for cutting speed), b_2 and d_2 (for feed rate), b_3 and d_3 (for axial depth of cut), and b_4 and d_4 (for radial depth of cut), as shown in Fig. 2, are varied in the range of {(55.359, 105.359) and (0, 55.359)}, {(0.012, 0.052) and (0, 0.012)}, {(0.111, 0.211) and (0, 0.111)}, and {(0.111, 0.211) and (0, 0.111)}, respectively. In this case, the values of power terms are kept in the range of 0.0 to 2.0. The fitness values of GA solution are calculated using the same procedure as used in case of cutting force. After a parametric study of GA, the following GA parameters are selected for best optimization during tuning of FRBM used for power prediction in milling:

P = 50; Cp = 0.98; Mp = 0.011; Ng = 125.

$$S_r = c_1 V_c^{P_1} + c_2 f_d^{P_2} + c_3 A_d^{P_3} + c_4 R_d^{p_4} \quad (10)$$

The optimized data base and rule base of the TSK-type FRBM for surface roughness obtained using Eq. (10) are shown in Fig. 14 and Table 2, respectively.

Table 1. Values of coefficients and power terms of TSK-type rules in optimized rule base of FRBM cutting force [(a) coefficient, (b) power terms].

(a) Coefficient

Rule No.	Rule Antecedent				Cutting Force			
	V_c	F_d	A_d	R_d	C_1	C_2	C_3	C_4
1	L	L	L	L	-0.258446	-454.6900	-20.94110	567.5480
2	L	L	L	H	0.502033	-4586.570	21.87050	36.92860
3	L	L	H	L	-0.000921	565305000	-420648.0	7.543430
4	L	L	H	H	0.042156	-16225.20	385.0280	-48.26390
5	L	H	L	L	-0.322918	4358.690	195.8770	1.738540
6	L	H	L	H	-11.65450	1961.360	13059.10	2262.630
7	L	H	H	L	0.005987	1565.370	-113.280	11.31310
8	L	H	H	H	-10.20730	2541.820	1671170	-341836.0
9	H	L	L	L	-0.001152	964.6000	45.1810	81.62920
10	H	L	L	H	0.000432	-1100.050	61.04240	-2.226270
11	H	L	H	L	-5.284520	-26908300	15328.60	20.63840
12	H	L	H	H	279.5600	-1001790	568.4150	475.0060
13	H	H	L	L	-1.440340	2516.390	134.7630	209.5880
14	H	H	L	H	-2.696420	2835.470	22.03080	38286.10
15	H	H	H	L	0.043724	1538.670	-702.4380	560.9770
16	H	H	H	H	-0.067508	3940.300	-59498.00	19939.90

(b) Power terms

Rule No.	Rule Antecedent				Cutting Force			
	V_c	F_d	A_d	R_d	P_1	P_2	P_3	P_4
1	L	L	L	L	1.23460	0.03225	0.55718	0.13489
2	L	L	L	H	1.39883	0.60410	0.76832	1.92962
3	L	L	H	L	2.74194	2.98240	0.48387	1.87097
4	L	L	H	H	1.90909	2.38710	2.23460	2.30205
5	L	H	L	L	1.23754	2.34604	0.50146	2.81232
6	L	H	L	H	1.91789	2.10264	0.17008	2.10557
7	L	H	H	L	2.07038	0.38709	2.49853	1.87390
8	L	H	H	H	1.24633	1.69795	1.48387	1.62463
9	H	L	L	L	2.31672	0.87096	1.71261	0.28739
10	H	L	L	H	2.14370	1.42815	2.24633	1.78299
11	H	L	H	L	1.61584	2.89443	1.91202	0.79472
12	H	L	H	H	0.53958	2.19062	1.16716	0.24633
13	H	H	L	L	1.04985	2.11144	0.82991	0.27272
14	H	H	L	H	2.66276	1.60411	1.85337	2.65103
15	H	H	H	L	2.24047	1.10850	2.98240	0.17008
16	H	H	H	H	2.63930	1.91202	1.94428	1.66276

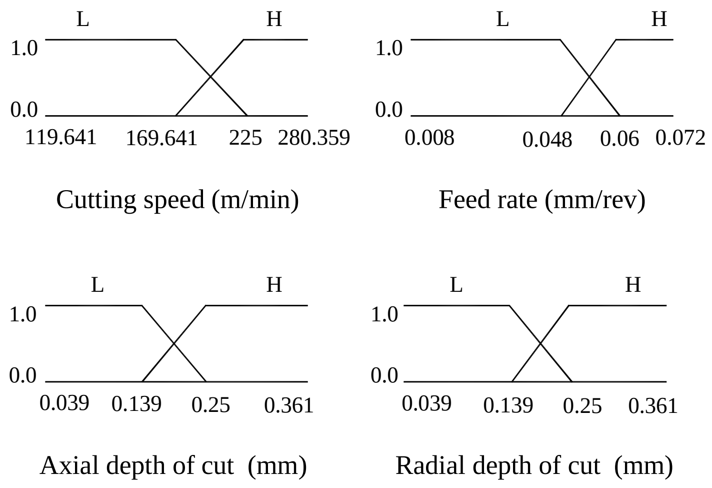

Figure 14. Optimized semi-trapezoidal MFDs of TSK-type FRBM for surface roughness.

Table 2. Values of coefficients and power terms of TSK-type rules in optimized rule base of FRBM: Surface roughness [(a) coefficient, (b) power terms].

(a) Coefficient

Rule No.	Rule Antecedent				Surface Roughness			
	V_c	F_d	A_d	R_d	C_1	C_2	C_3	C_4
1	L	L	L	L	0.027802	51.07870	0.054873	-1.363000
2	L	L	L	H	0.000003	-0.369016	0.052852	10.89830
3	L	L	H	L	0.056442	6.056330	1.163300	-80.00000
4	L	L	H	H	-0.118721	0.353802	-0.926731	4.708700
5	L	H	L	L	0.011655	0.697784	-0.221793	2.202150
6	L	H	L	H	-0.000188	83.26960	-0.311935	7.627010
7	L	H	H	L	-0.002009	-116.8980	4.830190	0.000000
8	L	H	H	H	-2.695120	-155.1670	360.7270	6.013470
9	H	L	L	L	-0.018567	0.763311	0.333107	0.748771
10	H	L	L	H	-0.000006	-1.843820	0.154931	9.455500
11	H	L	H	L	-0.000018	1.928620	0.427665	0.082161
12	H	L	H	H	-0.000003	-15.44880	-0.035930	8.419270
13	H	H	L	L	0.020635	-8.261550	0.635194	27.01110
14	H	H	L	H	-0.000007	-2.869020	-0.101510	5.972600
15	H	H	H	L	0.687483	-27.87730	3.052320	-8.210860
16	H	H	H	H	-0.000158	-7.426170	-88.00250	17.59480

(b) Power terms

Rule No.	Rule Antecedent				Surface Roughness			
	V_c	F_d	A_d	R_d	P_1	P_2	P_3	P_4
1	L	L	L	L	0.47702	1.94526	1.91398	1.16520
2	L	L	L	H	2.00000	0.70576	0.25219	2.00000
3	L	L	H	L	0.44379	1.35679	0.18377	1.81036
4	L	L	H	H	0.38123	0.37536	0.45747	0.62952
5	L	H	L	L	0.67644	0.94428	0.48875	1.83773
6	L	H	L	H	1.46628	1.74976	0.18377	1.79277
7	L	H	H	L	0.99120	1.83187	1.08504	0.59628
8	L	H	H	H	0.37145	0.42619	1.43109	1.91007
9	H	L	L	L	0.36950	0.93841	1.99413	0.37536
10	H	L	L	H	1.62463	1.32356	0.48289	1.83773
11	H	L	H	L	1.17693	1.43109	0.84262	0.18181
12	H	L	H	H	1.61877	1.97654	0.26197	1.63832
13	H	H	L	L	0.09775	1.43891	1.99218	1.88856
14	H	H	L	H	1.75171	0.81524	1.70674	1.18084
15	H	H	H	L	0.17008	1.30010	0.75268	0.63734
16	H	H	H	H	1.71261	0.32258	1.82991	0.08797

Results and Discussions

Cutting force

The developed FRBM will be used for prediction cutting force and parameter optimization to achieve a desired objective in milling operation. In order to demonstrate the prediction capability of FRBM, both the results of FRBM and mathematical correlation model (available in the literature) are compared with the experimental data. For this comparative study, 22 numbers of cases are considered at random and the results of FRBM, mathematical model and experimentation for the 22 cases are enlisted in Table 3. In Table 3, Error I is the deviation (in percentage) of the result obtained using FRBM from that of the experimental value. Whereas, Error II is the percentage deviation of the result obtained using mathematical correlation model (Eq. (7), as shown in the section “Mathematical model”) from that of the experimental value.

In Table 3, it is observed that for almost all the cases, FRBM outperforms over the mathematical correlation model. For 11 cases (case no 1, 2, 7, 9, 10, 12, 14, 16, 17, 20 and 22), it is found that the results obtained by the FRBM are much better than the corresponding mathematical correlation results. Moreover, it is observed that RMS (root mean square) value (4.097) of Error I (evaluated in TSK-type FRBM model) is less than the RMS value (4.248) of Error-II (evaluated in mathematical model).

Thus, the developed FRBM may be adopted for prediction of cutting force to achieve a desired objective in drilling. The performance of FRBM may be improved by considering the interaction effect(s) of the four cutting parameters in the rule consequent functions. But, in such cases, the computational complexity during model construction will be higher. For this reason, it is important to investigate the level of contribution(s) of the independent parameter’s interactions toward cutting force, which may be achieved using statistical approach such as analysis of variance (ANOVA).

Surface roughness

The developed FRBM for surface roughness will be used for prediction and parameter optimization to achieve a desired surface roughness in milling operation. The prediction capability of FRBM is verified by comparing the results of FRBM and mathematical correlation model with the experimental results. For this comparative study, 25 cases are considered at random and the results of FRBM, mathematical model and that of experimentation for the 25 cases are enlisted in Table 4. In Table 4, Error I is the deviation (in percentage) of the result obtained using FRBM from that of the experimental value. Whereas, Error II is the percentage deviation of the result obtained using mathematical model (Eq. (8), as shown in the section “Mathematical model”) from that of the experimental value.

In Table 4, it can be seen that in most of the cases, FRBM gives better results than mathematical correlation model, except in cases no. 5 and 9. Moreover, it is observed that RMS value (3.410) exhibited by the TSK-type FRBM model is less than that found by mathematical correlation model (RMS value = 11.65456173).

Table 3. Comparative results of FRBM and mathematical model: Cutting force.

Test Case	V_c	F_d	A_d	R_d	Experimental Value	GLR Based FRBM	Error I	Mathematical Correlation Model	Error II
1	100	0.1	1.5	3.5	190	189.478	0.274736	191.1438	0.602000
2	100	0.15	1.5	2	210	206.895	1.478571	199.1439	5.169559
3	100	0.15	2	3.5	320	312.049	2.484687	323.7356	2.167398
4	100	0.15	1.5	5	315	328.413	4.258095	315.4374	0.138865
5	100	0.2	1.5	3.5	320	302.672	5.415000	312.8092	2.247125
6	140	0.1	1.5	2	100	99.9121	0.087900	98.54580	1.454200
7	140	0.1	1	3.5	110	117.244	6.585454	115.2308	4.755272
8	140	0.1	2	3.5	200	203.076	1.538000	203.1358	1.567900
9	140	0.1	1.5	5	210	209.779	0.105238	204.8358	2.459142
10	140	0.15	1	2	127.46	115.983	9.004393	121.3454	4.797250
11	140	0.15	2	2	210	203.237	3.220476	218.0254	3.821630
12	140	0.15	1.5	3.5	210	208.009	0.948095	208.7476	0.596345
13	140	0.15	1	5	225	205.522	8.656888	211.4099	10.040033
14	140	0.15	2	5	350	357.804	2.229714	350.5399	0.154264
15	140	0.2	1.5	2	200	210.038	5.019000	215.2117	7.605850
16	140	0.2	1	3.5	210	210.914	0.435238	206.8962	1.478000
17	140	0.2	2	3.5	360	366.947	1.929722	354.8012	1.444111
18	140	0.2	1.5	5	320	302.295	5.532812	331.5007	3.593968
19	180	0.1	1.5	3.5	130	130.565	0.434615	131.6278	1.252153
20	180	0.15	1.5	2	145	153.624	5.947586	144.6319	0.253844
21	180	0.15	1	3.5	140	139.679	0.229285	146.3146	8.510482
22	180	0.15	2	3.5	270	269.233	0.284074	264.2196	2.140861

Table 4. Comparative results of FRBM and mathematical model: Surface roughness.

Test Case	V_c	F_d	A_d	R_d	Experimental Value	GLR Based FRBM	Error I	Mathematical Model	Error II
1	150	0.02	0.1	0.1	0.21673	0.236293	9.02643	0.29185	34.6637
2	250	0.02	0.1	0.1	0.20214	0.195465	3.30216	0.16668	17.5402
3	150	0.06	0.1	0.1	0.35369	0.354553	0.24399	0.36001	1.78954
4	250	0.06	0.1	0.1	0.24439	0.246806	0.98858	0.23484	3.90484
5	150	0.02	0.1	0.3	1.11283	1.053010	5.37548	1.06771	4.05387
6	250	0.02	0.1	0.3	0.98330	1.030370	4.78694	1.05837	7.63497
7	150	0.06	0.1	0.3	0.99341	0.989709	0.37255	0.99955	0.61860
8	250	0.06	0.1	0.3	1.08217	1.044580	3.47357	0.99021	8.49748
9	200	0.04	0.2	0.2	0.48307	0.517010	7.02478	0.50011	3.52727
10	150	0.02	0.3	0.1	0.24766	0.245965	0.68440	0.22459	9.31149
11	250	0.02	0.3	0.1	0.20844	0.203716	2.26635	0.23394	12.2345
12	150	0.06	0.3	0.1	0.34434	0.344356	0.00464	0.29276	14.9788
13	250	0.06	0.3	0.1	0.38016	0.380166	0.00157	0.30210	20.5322
14	150	0.02	0.3	0.3	0.93807	0.951987	1.48357	1.00045	6.65086
15	250	0.02	0.3	0.3	1.11840	1.113520	0.43633	1.12563	0.64666
16	150	0.06	0.3	0.3	0.89143	0.890864	0.06349	0.93229	4.58451
17	250	0.06	0.3	0.3	1.08790	1.088140	0.02206	1.05747	2.79710
18	200	0.04	0.361	0.2	0.48070	0.480699	0.00020	0.50011	4.03877
19	200	0.04	0.039	0.2	0.47929	0.485235	1.24037	0.50011	4.34484
20	119.641	0.04	0.2	0.2	0.47523	0.475034	0.04124	0.50011	5.23628
21	280.359	0.04	0.2	0.2	0.51620	0.482322	6.56296	0.50011	3.11615
22	200	0.04	0.2	0.361	1.51547	1.485410	1.98354	1.49499	1.35115
23	200	0.04	0.2	0.039	0.21347	0.213480	0.00468	0.26235	22.9008
24	200	0.008	0.2	0.2	0.57057	0.540709	5.23353	0.50011	12.3483
25	200	0.072	0.2	0.2	0.50609	0.506086	0.00079	0.50011	1.18067

Likewise cutting force model, the performance of FRBM of surface roughness may be improved by considering the interaction effect(s) of the independent input parameters in the rule consequent functions. However, investigation on the level of contribution(s) of the independent parameter's interactions is important.

Conclusion

In this work an attempt has been made to develop suitable TSK-type FRBMs for modelling of surface roughness and cutting force in milling operation.

In order to carry out these objectives, the present research work is carried out in three successive stages:

1. Experimentation and data analysis
2. Use of suitable techniques for constructing FRBM based on example data
3. Validation of FRBM

From experimental study, it is found that change in radial depth of cut influences much on surface roughness than other cutting parameters such as axial depth of cut, cutting velocity and feed rate. On the other hand, surface roughness and cutting force in milling are not linearly related to the cutting parameters and ambiguity happens by varying multiple cutting parameters simultaneously. For constructing the TSK-type FRBM, a combined approach of multiple linear regression method and genetic algorithm is utilized. The function coefficients are determined by linear regression whereas the optimized values of the exponential parameters are obtained by using GA. In addition to that, the MFDs of input variables (cutting speed, feed rate, axial depth of cut and radial depth of cut) are simultaneously optimized in order to improve the performances of the FRBMs. After validation of each of the models corresponding to different outputs (surface roughness and cutting force) with the experimental data, it is suggested that both the FRBMs give satisfactory results showing excellent trade-off and practical implementation.

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