Hazem A. Attia
Department of Mathematics, College of Science
Al-Qasseem University
P.O. Box 237, Buraidah 81999
Kingdom of Saudi Arabia

Unsteady MHD Flow of a Dusty Non-Newtonian Bingham Fluid Through a Circular Pipe

In this paper, the transient magnetohydrodynamic (MHD) flow of a dusty incompressible electrically conducting non-Newtonian Bingham fluid through a circular pipe is studied taking the Hall effect into consideration. A constant pressure gradient in the axial direction and an uniform magnetic field directed perpendicular to the flow direction are applied. The particle-phase is assumed to behave as a viscous fluid. A numerical solution is obtained for the governing nonlinear equations using finite differences.
***Keywords**: Fluid mechanics, magneto-fluid mechanics, circular pipe flow, non-Newtonian fluid, Bingham fluid*

Introduction

The flow of a dusty and electrically conducting fluid through a circular pipe in the presence of a transverse magnetic field has important applications such as MHD generators, pumps, accelerators, and flowmeters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so-called particle-phase viscosity. There have been many articles dealing with theoretical modelling and experimental measurements of the particle-phase viscosity in a dusty fluid (Soo 1969, Gidaspow *et al*. 1986, Grace 1982, and Sinclair *et al*. 1989).

The flow of a conducting fluid in a circular pipe has been investigated by many authors (Gadiraju *et al*. 1992, Dube et al. 1975, Ritter *et al*. 1977, and Chamkha 1994). Gadiraju *et al*. (1992) investigated steady two-phase vertical flow in a pipe. Dube et al. (1975) and Ritter *et al*. (1977) reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stresses. Chamkha (1994) obtained exact solutions which generalize the results reported in Dube et al. 1975 and Ritter *et al*. 1977 by the inclusion of the magnetic and particle-phase viscous effects. It should be noted that in the above studies the Hall effect is ignored.

A number of industrially important fluids such as molten plastics, polymers, pulps and foods exhibit non-Newtonian fluid behavior (Nakayama *et al*. 1988). Due to the growing use of these non-Newtonian materials, in various manufacturing and processing industries, considerable efforts have been directed towards understanding their flow characteristics. Many of the inelastic non-Newtonian fluids, encountered in chemical engineering processes, are known to follow the so-called "power-law model" in which the shear stress varies according to a power function of the strain rate (Metzner *et al*. 1965). It is of interest in this paper to study the influence of the magnetic field as well as the non-Newtonian fluid characteristics on the dusty fluid flow properties in situations where the particle-phase is considered dense enough to include the particulate viscous stresses.

In the present study, a new element is added to the problem studied by Attia (2003) by taking the Hall effect into consideration. Therefore, the unsteady flow of a dusty non-Newtonian Bingham fluid through a circular pipe is investigated considering the Hall effect. The carrier fluid is assumed viscous, incompressible and electrically conducting. The particle phase is assumed to be incompressible pressureless and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing nonlinear momentum equations for both the fluid and particle-phases are solved numerically using the finite difference approximations. The effect of the Hall current, the non-Newtonian fluid characteristics and the particle-phase viscosity on the velocity of the fluid and particle-phases are reported.

Paper accepted May 2006. Technical Editor: Aristeu da Silveira Neto.

Governing Equations

Consider the unsteady, laminar, and axisymmetric horizontal flow of a dusty conducting non-Newtonian Bingham fluid through an infinitely long pipe of radius "*d*" driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The Hall current is taken into consideration and the magnetic Reynolds number is assumed to be very small, consequently the induced magnetic field is neglected (Sutton *et al*. 1965). We assume that both phases behave as viscous fluids and that the volume fraction of suspended particles is finite and constant (Chamkha 1994). Taking into account these and the previously mentioned assumptions, the governing momentum equations can be written as

$$\rho\frac{\partial V}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial V}{\partial r}\right) + \frac{\rho_p\phi}{1-\phi}N(V_p - V) - \frac{\sigma B_o^2 V}{1+m^2} \quad (1)$$

$$\rho_p\frac{\partial V_p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu_p r\frac{\partial V_p}{\partial r}\right) + \rho_p N(V - V_p) \quad (2)$$

where t is the time, r is the distance in the radial direction, V is the fluid-phase velocity, V_p is the particle-phase velocity, ρ is the fluid-phase density, ρ_p is the particle-phase density, $\partial P/\partial z$ is the fluid pressure gradient, φ is the particle-phase volume fraction, N is a momentum transfer coefficient (the reciprocal of the relaxation time, the time needed for the relative velocity between the phases to reduce e^{-1} of its original value (Chamkha 1994), σ is the fluid electrical conductivity, $m=\sigma\gamma B_o$ is the Hall parameter, γ is the Hall factor (Sutton *et al*. 1965), B_o is the magnetic induction, μ_p is the particle-phase viscosity which is assumed constant, and μ is the apparent viscosity of the fluid which is given by,

$$\mu = \mu_o + \frac{\tau_o}{\left|\frac{\partial V}{\partial r}\right|}$$

where μ_o is the plastic viscosity of a Bingham fluid and τ_o is the yield stress. In this work, ρ, ρ_p, μ_p, φ, and B_o are all constant. It should be pointed out that the particle-phase pressure is assumed negligible and that the particles are being dragged along with the fluid-phase.

The initial and boundary conditions of the problem are given as

$$V(r,0)=0, V_p(r,0)=0, \tag{3a}$$

$$\frac{\partial V(0,t)}{\partial r}=0, \frac{\partial V_p(0,t)}{\partial r}=0, V(d,t)=0, V_p(d,t)=0 \tag{3b}$$

where "d" is the pipe radius.

Equations (1)-(3) constitute a nonlinear initial-value problem which can be made dimensionless by introducing the following dimensionless variables and parameters

$$\bar{r}=\frac{r}{d}, \bar{t}=\frac{t\mu_o}{\rho d^2}, G_o=-\frac{\partial P}{\partial z}, k=\frac{\rho_p\phi}{\rho(1-\phi)}, \bar{\mu}=\frac{\mu}{\mu_o}$$

$$\bar{V}(r,t)=\frac{\mu_o V(r,t)}{G_o d^2}, \bar{V}_p(r,t)=\frac{\mu_o V_p(r,t)}{G_o d^2},$$

$\alpha = Nd^2\rho/\mu_o$ is the inverse Stokes' number,

$\beta=\mu_p/\mu_o$ is the viscosity ratio,

$\tau_D=\tau_o/G_o d$ is the Bingham number (dimensionless yield stress),

$H_a=B_o d\sqrt{\sigma/\mu_o}$ is the Hartmann number (Sutton *et al*. 1965).

By introducing the above dimensionless variables and parameters as well as the expression of the fluid viscosity defined above, Eqs. (1)-(3) can be written as (the bars are dropped),

$$\frac{\partial V}{\partial t}=1+\frac{\partial^2 V}{\partial r^2}+\left(1+\frac{\tau_D}{\left|\frac{\partial V}{\partial r}\right|}\right)\frac{1}{r}\frac{\partial V}{\partial r}+k\alpha(V_p-V)-\frac{H_a^2 V}{1+m^2} \tag{4}$$

$$\frac{\partial V_p}{\partial t}=\beta\left(\frac{\partial^2 V_p}{\partial r^2}+\frac{1}{r}\frac{\partial V_p}{\partial r}\right)+\alpha(V-V_p) \tag{5}$$

$$V(r,0)=0, V_p(r,0)=0, \tag{6a}$$

$$\frac{\partial V(0,t)}{\partial r}=0, \frac{\partial V_p(0,t)}{\partial r}=0, V(1,t)=0, V_p(1,t)=0 \tag{6b}$$

The volumetric flow rates and skin-friction coefficients for both the fluid and particle phases are defined, respectively, as (Chamkha 1994)

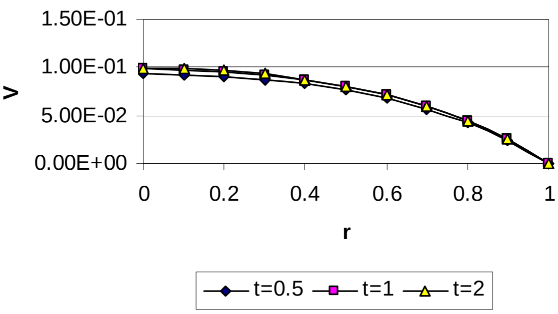
$$Q=2\pi\int_0^1 rV(r,t)dr, Q_p=2\pi\int_0^1 rV_p(r,t)dr, C=-\frac{\partial V(1,t)}{\partial r}, C_p=-\beta k\frac{\partial V_p(1,t)}{\partial r} \tag{7}$$

Results and Discussion

Equations (6) and (7) represent coupled system of non-linear partial differential equations which are solved numerically under the initial and boundary conditions (8) using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. The computational domain is divided into meshes each of dimension Δt and Δr in time and space, respectively. Then the Crank-Nicolson implicit method is used at two successive time levels (Mitchell *et al*., 1980; Evans *et al*., 2000). An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas-algorithm (Mitchell *et al*., 1980; Evans *et al*., 2000). Computations have been made for α=1 and k=10. Grid-independence studies show that the computational domain $0<t<\infty$ and $0<r<1$ can be divided into intervals with step sizes Δt=0.0001 and Δr=0.005 for time and space respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns V, V_p, $\partial V/\partial r$, and $\partial V_p/\partial r$ for the last two approximations differ from unity by less than 10^{-6} for all values of r in $0<r<1$ at every time step. It should be mentioned that the results obtained herein reduce to those reported by Dube *et al*. (1975) and Chamkha (1994) for the cases of non-magnetic, inviscid particle-phase (B=0), and Newtonian fluid. These comparisons lend confidence in the accuracy and correctness of the solutions.

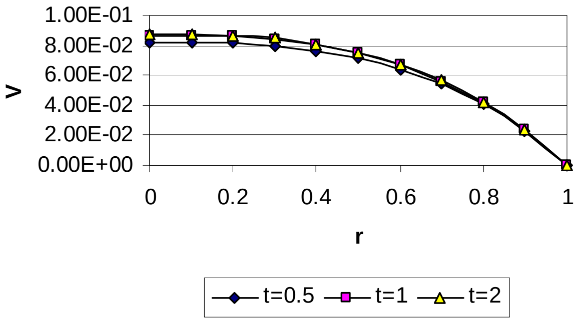
Imposing of a magnetic field normal to the flow direction gives rise to a drag-like or resistive force and it has the tendency to slow down or suppress the movement of the fluid in the pipe, which in turn, reduces the motion of the suspended particle-phase. This is translated into reductions in the average velocities of both the fluid- and the particle-phases and, consequently, in their flow rates. In addition, the reduced motion of the particulate suspension in the pipe as a result of increasing the strength of the magnetic field causes lower velocity gradients at the wall. This has the direct effect of reducing the skin-friction coefficients of both phases. Including the Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity. Therefore, the Hall parameter leads to an increase in the average velocities of both the fluid- and the particle-phases and, consequently, in their flow rates and the velocity gradients at the wall.

Figures 1 and 2 present the time evolution of the profiles of the velocity of the fluid V and dust particles V_p, respectively, for various values of the Bingham number τ_D and for m=0, Ha=0.5 and B=0.5. Both V and V_p increase with time and V reaches the steady-state faster than V_p for all values of τ_D. It is clear from Figs. 1 and 2 that increasing τ_D, which increases the driving force for V, increases V and, consequently, increases V_p while its effect on their steady-state times can be neglected.

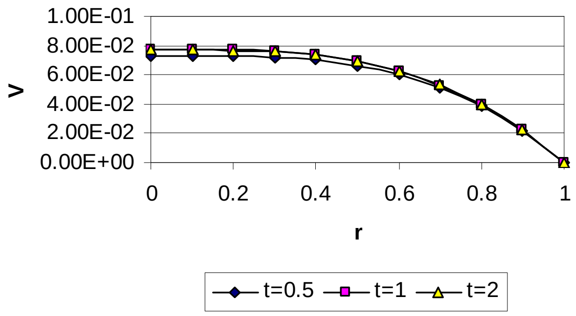


(a) $\tau_{D=0.0}$

Figure 1. Time development of V for various values of τ_D (m=0).

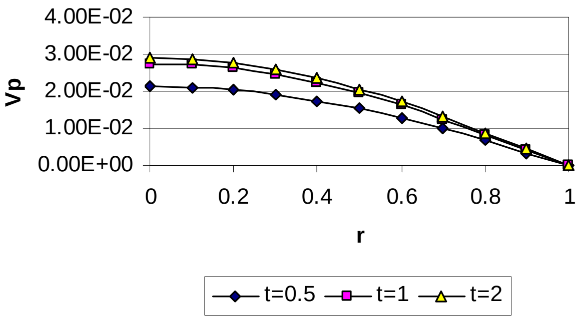


(b) $\tau_{D=0.025}$

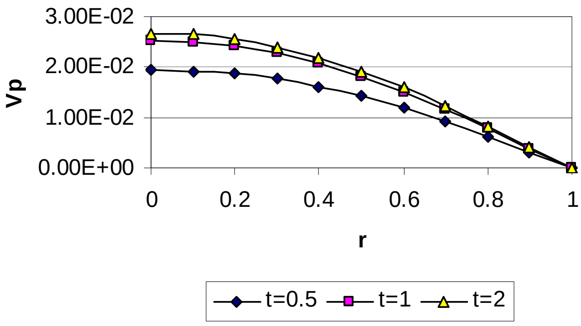


(c) $\tau_{D=0.05}$

Figure 1. (Continued).

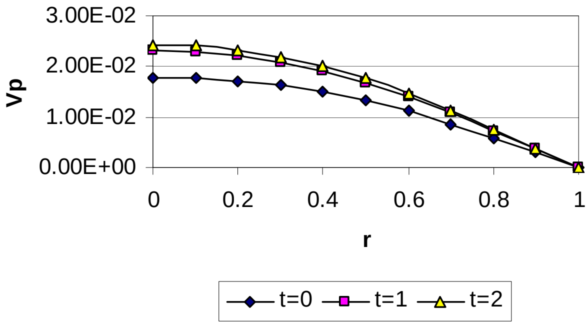


(a) $\tau_{D=0.0}$



(b) $\tau_{D=0.025}$

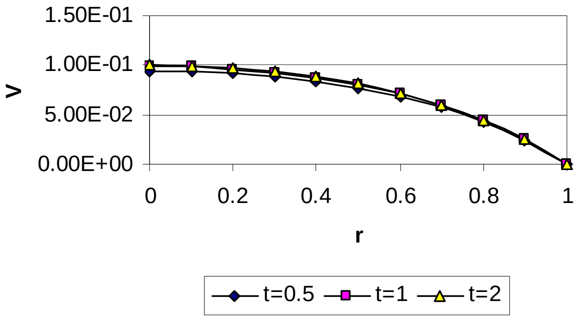
Figure 2. Time development of V_p for various values of τ_D (m=0).



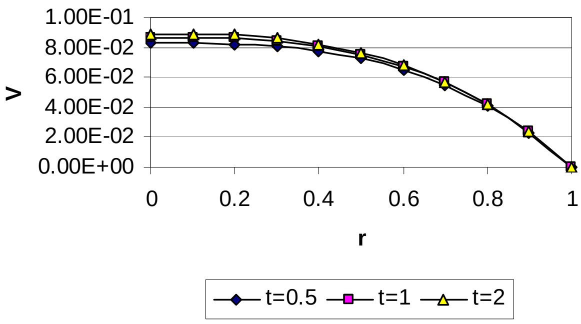
(c) $\tau_{D=0.05}$

Figure 2. (Continued).

Figures 3 and 4 present the time evolution of the profiles of the velocity of the fluid *V* and dust particles V_p, respectively, for various values of the Bingham number τ_D and for *m*=1, *Ha*=0.5 and *B*=0.5. It is indicated in the figures that increasing m increases *V* and, in turn, V_p due to the decrease in the effective conductivity ($\sigma/(1+m^2)$) which reduces the damping magnetic force on *V*. It is shown that the influence of the Hall parameter *m* on *V* is more apparent for higher values of τ_D.

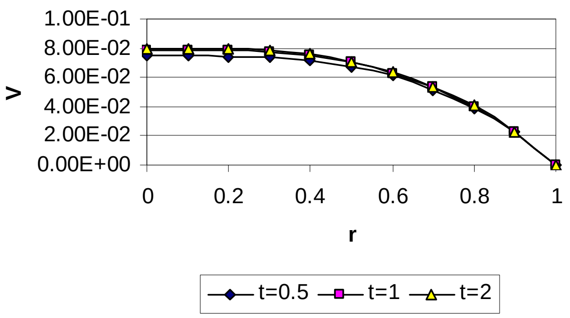


(a) $\tau_{D=0.0}$



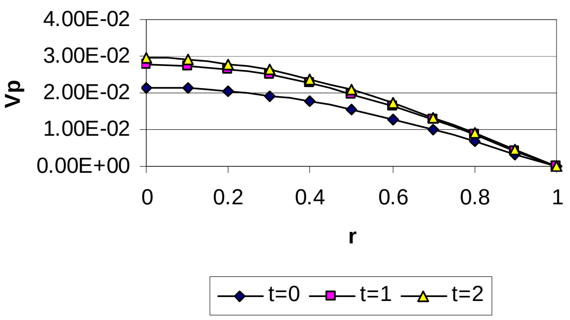
(b) $\tau_{D=0.025}$

Figure 3. Time development of V for various values of τ_D (m=1).

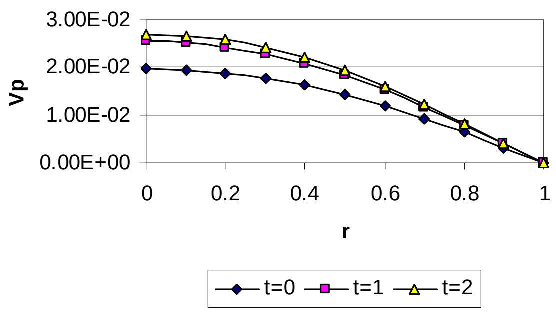


(c) $\tau_{D=0.05}$

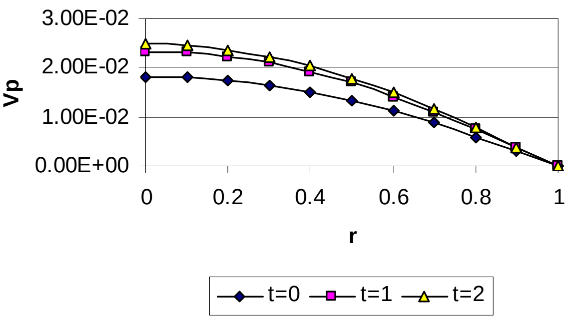
Figure 3. (Continued).



(a) $\tau_{D=0.0}$



(b) $\tau_{D=0.025}$



(c) $\tau_{D=0.05}$

Figure 4. Time development of V_p for various values of τ_D (m=1).

Table 1 presents the steady state values of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of the parameters τ_D and m and for Ha=0.5 and B=0.5. It is clear that increasing the parameter m increases Q, Q_p, C, and C_p for all values of τ_D. This comes from the fact that increasing m increases the velocities and their gradients which increases the average velocities of both the fluid- and the particle-phases and, consequently, increases their flow rates and skin-friction coefficients of both phases. It is also shown that increasing τ_D increases Q, Q_p, C, and C_p for all values of m as a result of increasing the velocities of both phases.

Table 1. The steady state values of Q, Q_p, C, C_p for various values of m and τ_D.

τ_D=0	m=0	m=1	m=2
Q	0.1764	0.1779	0.1789
Q_p	0.0426	0.0430	0.0433
C	0.2818	0.2834	0.2844
C_p	0.2111	0.2129	0.2140
τ_D=0.025	m=0	m=1	m=2
Q	0.1649	0.1663	0.1673
Q_p	0.0396	0.0400	0.0402
C	0.2704	0.2719	0.2729
C_p	0.1975	0.1995	0.2005
τ_D=0.05	m=0	m=1	m=2
Q	0.1525	0.1535	0.1564
Q_p	0.0364	0.0369	0.0372
C	0.2583	0.2598	0.2612
C_p	0.1834	0.1859	0.1868

Table 2 presents the steady state values of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of the parameters m and

B and for Ha=0.5 and τ_D=0. It is clear that, increasing m increases Q, Q_p, C, and C_p for all values of B and its effect becomes more pronounced for smaller values of B. Increasing the parameter B decreases the quantities Q, Q_p, and C, but increases C_p for all values of m. This can be attributed to the fact that increasing B increases viscosity and therefore the flow rates of both phases as well as the fluid-phase wall friction decreases considerably. However, since C_p in defined as directly proportional to B, it increases as B increases at all times.

Table 2. The steady state values of Q, Q_p, C, C_p for various values of m and β.

β=0	m=0	m=1	m=2
Q	0.3032	0.3075	0.3101
Q_p	0.2582	0.2615	0.2635
C	0.4125	0.4167	0.4193
C_p	0	0	0
β=0.5	m=0	m=1	m=2
Q	0.1764	0.1779	0.1789
Q_p	0.0426	0.0430	0.0433
C	0.2818	0.2834	0.2844
C_p	0.2111	0.2129	0.2140
β=1	m=0	m=1	m=2
Q	0.1640	0.1654	0.1662
Q_p	0.0226	0.0228	0.0229
C	0.2702	0.2716	0.2724
C_p	0.2231	0.2249	0.2260

Conclusion

The unsteady MHD flow of a particulate suspension in an electrically conducting non-Newtonian Bingham fluid in a circular pipe is studied considering the Hall effect. The governing nonlinear partial differential equations are solved numerically using finite differences. The effect of the magnetic field parameter Ha, the Hall parameter, the non-Newtonian fluid characteristics (Bingham number τ_D), and the particle-phase viscosity β on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle-phases is studied. It is shown that increasing the magnetic field decreases the fluid and particle velocities, while increasing the Hall parameter increases both velocities. It is found that increasing the parameter m increases Q, Q_p, C, and C_p for all values of τ_D. The effect of the Hall parameter on the quantities Q, Q_p, C, and C_p becomes more pronounced for smaller values of β.

References

Attia, H.A., 2003, "Unsteady flow of a dusty conducting non-Newtonian fluid through a pipe", *Can. J. Phys*., Vol. 81(3), pp. 789-795.

Chamkha, A.J., 1994, "Unsteady flow of a dusty conducting fluid through a pipe", *Mechanics Research Communications*, Vol. 21(3), pp. 281-286.

Dube, S.N. and Sharma, C.L., 1975, "A note on unsteady flow of a dusty viscous liquid in a circular pipe", *J. Phys. Soc. Japan*, Vol. 38(1), pp. 298-310.

Evans, G.A., Blackledge, J.M. and Yardley, P.D., 2000, "Numerical methods for partial differential equations", *Springer Verlag*, New York.

Gadiraju, M., Peddieson, J. and Munukutla, S., 1992, "Exact solutions for two-phase vertical pipe flow", *Mechanics Research Communications*, Vol. 19(1), pp. 7-13.

Gidaspow, D., 1986, "Hydrodynamics of fluidization and heat transfer: super computer modeling", *Appl. Mech. Rev*., Vol. 39, pp. 1-23.

Grace, J.R., 1982, "Fluidized-Bed Hydrodynamic, Handbook of Multiphase Systems", *McGraw-Hill*, New York.

Metzner, A.B., 1965, "Heat Transfer in non-Newtonian fluid", *Adv. Heat Transfer*, Vol. 2, pp. 357-397.

Mitchell, A.R. and Griffiths, D.F., 1980, "The finite difference method in partial differential equations", *John Wiley & Sons*, New York.

Nakayama, A. and Koyama, H., 1988, "An analysis for friction and heat transfer characteristics of power-law non-Newtonian fluid flows past bodies of arbitrary geometrical configuration", *Warme-und Stoffubertragung*, Vol. 22, pp. 29-37.

Ritter, J.M. and Peddieson, J., 1977, "Transient two-phase flows in channels and circular pipes", *Proceedings of the 6th Canadian Congress of Applied Mechanics*, 1977.

Sinclair, J.L. and Jackson, R., 1989, "Gas-particle flow in a vertical pipe with particle-particle interactions", AICHE J., Vol. 35, pp. 1473-1486.

Soo, S.L., 1969, "Pipe flow of suspensions", *Appl. Sci. Res*., Vol. 21, pp. 68-84.

Sutton, G.W. and Sherman, A., 1965, "Engineering Magnetohydrodynamics", *McGraw-Hill*, New York.