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Optimal Configurations of Composite Multiple Mass Dampers in Tall Buildings

The effectiveness of multiple tuned mass dampers (MTMD) for suppressing harmonically forced oscillations is studied in the paper. In particular the influence of possible connections between the masses of the damper on the main system performance is investigated using four different configurations of a double mass damper. For this, a minimax procedure, which considers all dampers parameters and variables, is used to optimize each configuration and compare their influence on the minimum value of the maximum magnification factor of the main system. A parametric study shows that small variations in the MTMD parameters and the way in which the masses are connected have a marked influence on the main system response. This sensitivity gives the designer more freedom in choosing the proper damper configuration in a practical situation.
Keywords*: tuned mass damper, multiple tuned mass damper, vibration control, structural dynamics, damper optimization*

Introduction

A tuned mass damper (TMD) is a passive vibration control device that has been used in some engineering structures and machines. Usually it consists of a single mass-spring-dashpot system connected to the main structure. In particular, TMDs have been used in recent years in several tall buildings and towers to reduce the energy dissipation demand of these structures under the action of wind loads (Holmes, 1995). In these applications the TMD is placed on the top of the building to maximize its efficiency. The basic concepts for the design of a damped TMD for an undamped structure were presented by Den Hartog (1956). He showed that under a simple harmonic load the main structure could be kept completely stationary when the attached absorber is chosen to be tuned to the excitation frequency. As a result, the vibrational structural energy of the building is transferred to the TMD.

One of the drawbacks when a single TMD is used is its sensitivity to small variations in system parameters, in particular the natural frequency of the structure and/or the TMD damping considered in the design. Uncertainties in the damper and particularly in the system are inherent to engineering constructions. To improve the reliability and effectiveness of the damper, experimental measurements have to be made to determine the dynamic properties of one structure. Alternatively the use of more than one damper has been proposed (Xu & Igusa, 1992; Park & Reed, 2001; Abé & Fujino, 1994; Igusa & Xu, 1994; Jangid, 1999; Jangid & Datta, 1997; Yamaguchi & Harnpornchai, 1993; Kareem & Kline, 1995; Gu *et al.,* 2001; Poovarodom, 2003; Yau & Yang, 2004). In particular, previous studies by Abé & Fujino (1994), Igusa & Xu (1994) and Jangid (1999), among others, have shown that multiple tuned mass damper (MTMDs) can be more effective and robust than a single TMD and that, in this case, the response of the main system is not much influenced by relatively small changes or errors in the values of system parameters used in the TMD design. According to Janjid & Datta (1997) there is a region around the optimum frequency of the damper where the optimized MTMD exhibit an almost constant effectiveness.

Some practical restraints, however, must be observed in the design of a TMD (Abé & Fujino, 1994; Soong & Dargush, 1997). The amount of added mass placed on the top story of a building, the TMD excursion travel relative to the floor, the friction between the sliding mass and the bearing surface, the amount of spring force on the building and also the enclosure of space occupied by the TMD are some of the issues to be addressed in its design. Now the use of a conception of a system of MTMD can give the designer more freedom in choosing the properties and the best configuration of the multiple dampers.

There are several propositions for the selection of the dampers parameters: one can vary the mass or stiffness of the damper (or both) to cover a certain frequency range encompassing the main tuning frequency. According to Xu & Igusa (1992), it is easier in practice to fix the stiffness of each spring and vary the mass of each damper to cover the desired frequency range.

To improve the MTMD effectiveness, several optimization procedures are found in literature (Jangid, 1999; Tsai & Lin, 1993; Hoang & Warnitchai, 2005; Magluta *et al.*, 2003; Carneiro, 2004; Li & Qu, 2006). According to Jangid (1999), the optimal parameters of a MTMD cannot be obtained by a procedure similar to that employed by Warburton (1982) for a single damped TMD. He determines the optimal parameters by the *minimax* procedure proposed by Tsai & Lin (1993) for a single damped TMD. An optimization procedure for a MTMD was proposed by Li (2000) for a structure submitted to a base excitation. Hoang & Warnitchai (2005) developed a new method to design MTMD using a numerical optimizer.

In some practical applications the masses of the damper are connected in different ways (Soong & Dargush, 1997), on the other hand the majority of the investigations on MTMDs consider no connection between the dampers. In this paper the influence of possible connections between the masses of the dampers is studied in detail. For this, four different configurations of a double mass damper are compared. In each case the damper parameters are optimized using a minimax procedure that considers mass, tuning frequency and damping ratio of each damper as free variables.

Nomenclature

$\boldsymbol{C}$ = *damping matrix*
c = *TMD damping*
C_1^* = *modal damping*
d_i = *total displacement*
$\boldsymbol{D}$ = *location vector*
$\boldsymbol{K}$ = *stiffness matrix*
k = *TMD stiffness*
K_1^* = *modal stiffness*
$\boldsymbol{F}$ = *external load vector*

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$f(t)$ = *modal dynamic excitation force*
$g(t)$ = *external load acting on the structure*
$\boldsymbol{M}$ = *mass matrix*
m = *TMD mass*
M_1^* = *modal mass*
$p(t)$ = *interaction force between the TMD and the structure*
q_i = *relative displacement*
R_d = *magnitude of the first element of the complex-frequency response transfer matrix*
$\boldsymbol{y}$ = *displacement vector*
$y_i\ (t)$ = *the absolute lateral displacement of the* i=*th floor where the TMD is installed relative to the building base.*
$\boldsymbol{Y}(\omega)$ = *transfer function matrix*
$z(t)$ = *relative displacement*

Greek Symbols

α = natural frequency ratio
β = forced frequency ratio
ϕ_1 = first mode shape
ω_s = structure natural frequency
ω_e = excitation force frequency
ω_{TMD} = TMD frequency
ξ = damping ratio
μ = mass ratio

System Description and Equations of Motion

System with TMD

The equation of motion of a building-TMD system can be expressed in matrix form as

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}\dot{\boldsymbol{y}}(t) + \boldsymbol{K}\boldsymbol{y}(t) = \boldsymbol{F}(t) + \boldsymbol{D}p(t) \tag{1}$$

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = -m\ddot{y}_N(t) + g(t) \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are, respectively, the mass, damping and stiffness matrixes of the NDOF structural system while m, c and k are the mass damper parameters; F(t) and g(t) are the external loads acting on the structure and TMD, respectively; $p(t) = c\dot{z}(t)+kz(t)$; $y_i(t)$ is the absolute lateral displacement of the i-th floor of the building relative to its base; $z(t)$ is the relative displacement of the TMD with respect to the floor where it is installed; $\boldsymbol{D}$ is a localization vector whose components d_j are given by

$$d_j = \begin{cases} 0, j \neq k \\ 1, j = k \end{cases} \tag{3}$$

where k is the floor where the damper is installed.

Let us consider a tall building where the natural frequencies are well spaced so that the structure oscillates around a predominant mode. In this case, response vector of the structure can be approximately represented by a single coordinate y_N and a mode shape $\square_1$

$$y = \phi_1 \boldsymbol{y_N}. \tag{4}$$

Substituting Eq. (4) into Eq. (2) and pre-multiplying Eq. (2) by ϕ_1^T, one obtains the following reduced system

$$M_1^*\ddot{y}_N + C_1^*\dot{y}_N + K_1^* y_N = \left(c\dot{z} + kz\right)\boldsymbol{\varphi}_1^T + f(t) \tag{5}$$

where $M_1^* = \boldsymbol{\varphi}_1^T \boldsymbol{M} \boldsymbol{\varphi}_1$ is the modal mass; $C_1^* = M_1^* 2\xi_1\omega_1$, $K_1^* = M_1^*\omega_1^2$ and *f(t)* is the modal dynamic excitation force. Here, ξ_1 and ω_1 are respectively the damping ratio and natural frequency of the structure.

System with MTMD

Consider now the simplified SDOF system with n tuned mass dampers with different dynamic characteristics attached to the top floor, as shown in Fig. 1. The main system is characterized by K_1^*, M_1^* and C_1^*, while the j-th TMD is characterized by a mass m_j, damping c_j and stiffness k_j. So, the (n+1) equations of the composite system are given by

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}\dot{\boldsymbol{y}}(t) + \boldsymbol{K}\boldsymbol{y}(t) = \boldsymbol{F}(t) \tag{6}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ are the mass, damping, and stiffness structural matrices, respectively; $\boldsymbol{y}(t)$ is an (n+1) vector which represents the main system and masses displacements relative to the main system; $\boldsymbol{F}(t) = [f_1(\mathrm{t}), f_2(\mathrm{t}), ..., f_n(\mathrm{t})]^T$ is the external excitation.

For analysis, the frequency domain approach will be adopted since the dynamic behavior of the structure can often be described more simply by a transfer function in the frequency domain, and the excitations, such as wind loads, are often modeled as stochastic processes characterized by their spectral density functions in the frequency domain. Adopting this approach, $f_i(\mathrm{t}) = f_i\mathrm{e}^{-i\omega t}$ and $\boldsymbol{y}(t) = \boldsymbol{Y}(\omega)\mathrm{e}^{-i\omega t}$. Substituting these expressions in Eq. (6), one obtains

$$\boldsymbol{Y}(\omega) = (\omega^2\boldsymbol{M} - i\omega\boldsymbol{C} + \boldsymbol{K})^{-1}\boldsymbol{F} \tag{7}$$

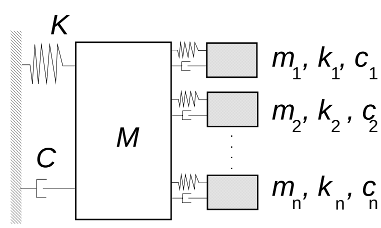


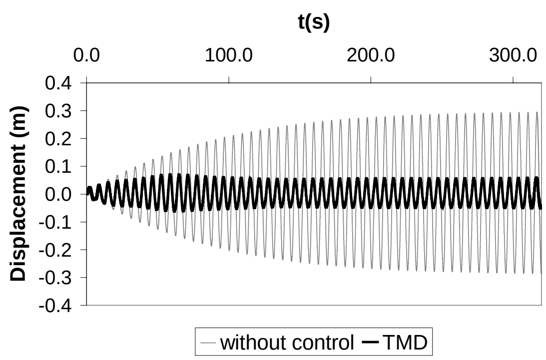
Figure 1. Structural model: multiple tuned mass damper (MTMD) attached to the main structure.

R_d is defined in this work as the magnitude of the first element of the complex-frequency response transfer matrix $(\omega^2\boldsymbol{M} - i\omega\boldsymbol{C} + \boldsymbol{K})^{-1}$ in Eq. (7). The ij element of this matrix represents the permanent response of coordinate i due to a harmonic load applied to coordinate j. The variation of the absolute value of the matrix first element is observed because it represents the permanent response of the main system due to a harmonic load applied to it.

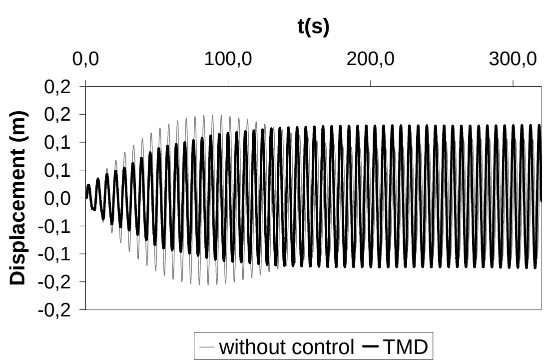
To demonstrate the effectiveness of a MTMD, consider a tall building modeled as a SDOF system with modal parameters, relative to the first lateral bending mode, $M^*_1 = 1.8 \times 10^4$ t; $K^*_1 = 1.82 \times 10^4$ kN/m; $\xi_S = 0.01$ and $\omega_S = 1.01$ rad/s. Consider also a TMD with $m = 3.6 \times 10^2$ t; $k = 3.03 \times 10^2$ kN/m; $\xi = 0.036$; $\omega_{TMD} = 0.917$ rad/s and $\alpha = 0.908$. These dynamic characteristics are related to the 274 m tall Citicorp Center in New York (Soong & Dargush, 1997).The wind load is approximately described by the periodic force:

$$F(t) = P(3sen\omega_e t + 7sen2\omega_e t + 5sen3\omega_e t + 4sen4\omega_e t) \tag{8}$$

where P is the magnitude of the excitation and ω_e is the excitation frequency. Figure 2(a) shows the time response of the building displacement with and without a TMD for P = 40 kN and ω_e = 1.0 rad/s. This shows clearly the advantages of using a TMD in a practical case. If the frequency of the periodic force in Eq. (8) is increased to ω_e = 1.035 rad/s, slightly higher than the lowest natural frequency of the building, one can observe in Fig. 2(b) that the control system looses its efficiency and the amplitude of steady-state response of the controlled system is much higher than the uncontrolled one. In fact, as shown in Fig. 3, where the magnitude of R_d is given as a function of the frequency parameter $\beta = \omega_e / \omega_s$, the response of the building attains its maximum value at ω_e = 1.035 rad/s. This confirms the limitations of a single damper when the excitation frequency is slightly different of that considered in design.



(a)



(b)

Figure 2. Controlled and uncontrolled time response of the main system varying excitation frequency: (a) ω_e = 1.0 rad/s; (b) ω_e = 1.035 rad/s.

To study the efficiency of the MTMD concept, let us consider a MTMD with two equal masses ($m_1 = m_2$ = 180 t), each half of that of the original TMD, with the following dynamic characteristics: ω_1 = 0.9 rad/s; ω_2 = 0.9 rad/s; ξ_1 = 0.1; ξ_2 = 0.05. The response of the structure in the frequency domain with a single and two independent dampers is shown in Fig. 3. It is clear that the MTMD is less sensitive to variations in the excitation frequency and is more efficient in the vicinity of the natural frequency of the structure.

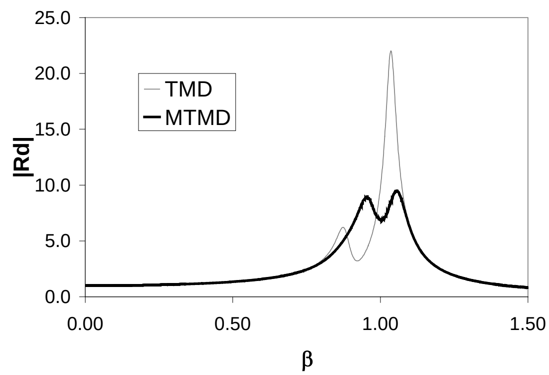


Figure 3. Frequency response of the structure.

The efficiency of the MTMD can be even better by optimizing its parameters and/or by connecting the two masses in different ways as shown in Fig. 4. It is also shown in Fig. 4, for each of the four possible configurations studied in this paper, the total and relative displacement of each mass, respectively d_i, and q_i.

The normalized mass, stiffness and damping matrices in Eq. (6) for each configuration are:

Configuration 1

$$\boldsymbol{M} = \begin{bmatrix} 1+\mu_1+\mu_2 & \mu_1 & \mu_2 \\ \mu_1 & \mu_1 & 0 \\ \mu_2 & 0 & \mu_2 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 2\xi_s\omega_s & 0 & 0 \\ 0 & 2\xi_1\mu_1\alpha_1\omega_s & 0 \\ 0 & 0 & 2\xi_2\mu_2\alpha_2\omega_s \end{bmatrix}$$

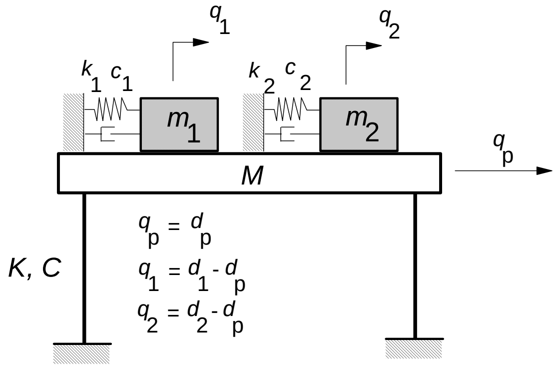
$$\boldsymbol{K} = \begin{bmatrix} \omega_s^2 & 0 & 0 \\ 0 & \mu_1\omega_s^2\alpha_1{}^2 & 0 \\ 0 & 0 & \mu_2\omega_s^2\alpha_2{}^2 \end{bmatrix} \tag{9}$$

Configuration 2

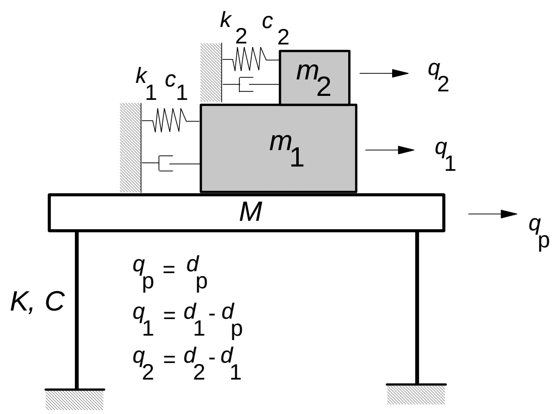
$$\boldsymbol{M} = \begin{bmatrix} 1+\mu_1+\mu_2 & \mu_1+\mu_2 & \mu_2 \\ \mu_1+\mu_2 & \mu_1+\mu_2 & \mu_2 \\ \mu_2 & \mu_2 & \mu_2 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 2\xi_s\omega_s & 0 & 0 \\ 0 & 2\xi_1\mu_1\alpha_1\omega_s & 0 \\ 0 & 0 & 2\xi_2\mu_2\alpha_2\omega_s \end{bmatrix}$$

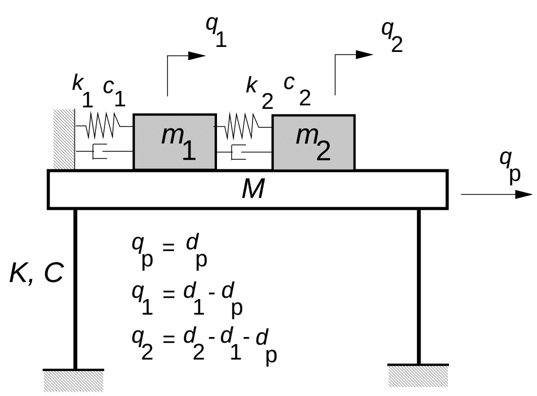
$$\boldsymbol{K} = \begin{bmatrix} \omega_s^2 & 0 & 0 \\ 0 & \mu_1\omega_s^2\alpha_1{}^2 & 0 \\ 0 & 0 & \mu_2\omega_s^2\alpha_2{}^2 \end{bmatrix} \tag{10}$$



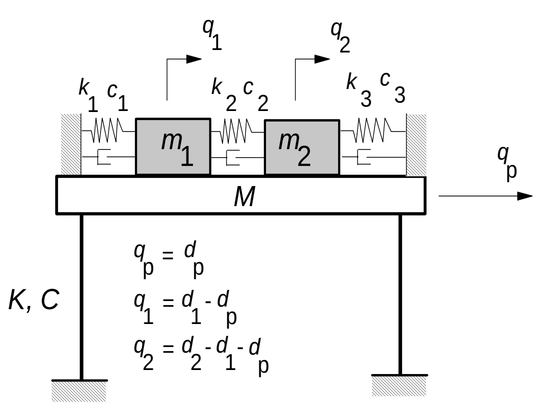
Configuration 1



Configuration 2



Configuration 3



Configuration 4

Figure 4. Configurations of the composite damper analyzed here.

Configuration 3

$$\boldsymbol{M} = \begin{bmatrix} 1+\mu_1 & \mu_1 & 0 \\ \mu_1 & \mu_1+\mu_2 & \mu_2 \\ 0 & \mu_2 & \mu_2 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 2\xi_s\omega_s + 2\xi_2\mu_2\alpha_2\omega_s & 0 & -2\xi_2\mu_2\alpha_2\omega_s \\ 0 & 2\xi_1\mu_1\alpha_1\omega_s & 0 \\ -2\xi_2\mu_2\alpha_2\omega_s & 0 & 2\xi_2\mu_2\alpha_2\omega_s \end{bmatrix}$$

$$\boldsymbol{K} = \begin{bmatrix} \omega_s^2 + \mu_2\omega_s^2\alpha_2^{\,2} & 0 & -\mu_2\omega_s^2\alpha_2^{\,2} \\ 0 & \mu_1\omega_s^2\alpha_1^{\,2} & 0 \\ -\mu_2\omega_s^2\alpha_2^{\,2} & 0 & \mu_2\omega_s^2\alpha_2^{\,2} \end{bmatrix} \tag{11}$$

Configuration 4

$$\boldsymbol{M} = \begin{bmatrix} 1+\mu_1 & \mu_1 & 0 \\ \mu_1 & \mu_1+\mu_2 & \mu_2 \\ 0 & \mu_2 & \mu_2 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 2\xi_s\omega_s + 4\xi_2\mu_2\alpha_2\omega_s & -2\xi_2\mu_2\alpha_2\omega_s & -4\xi_2\mu_2\alpha_2\omega_s \\ -2\xi_2\mu_2\alpha_2\omega_s & 2\xi_1\mu_1\alpha_1\omega_s + 2\xi_2\mu_2\alpha_2\omega_s & 2\xi_2\mu_2\alpha_2\omega_s \\ -4\xi_2\mu_2\alpha_2\omega_s & 2\xi_2\mu_2\alpha_2\omega_s & 4\xi_2\mu_2\alpha_2\omega_s \end{bmatrix}$$

$$\boldsymbol{K} = \begin{bmatrix} \omega_s^2 + 2\mu_2\omega_s^2\alpha_2^{\,2} & -\mu_2\omega_s^2\alpha_2^{\,2} & -2\mu_2\omega_s^2\alpha_2^{\,2} \\ -\mu_2\omega_s^2\alpha_2^{\,2} & \mu_1\omega_s^2\alpha_1^{\,2} + \mu_2\omega_s^2\alpha_2^{\,2} & \mu_2\omega_s^2\alpha_2^{\,2} \\ -2\mu_2\omega_s^2\alpha_2^{\,2} & \mu_2\omega_s^2\alpha_2^{\,2} & 2\mu_2\omega_s^2\alpha_2^{\,2} \end{bmatrix} \tag{12}$$

where ω_s and ξ_s are, respectively, the natural frequency and damping ratio of the main system,

$$\alpha_1 = \frac{\omega_1}{\omega_s} \tag{13}$$

$$\alpha_2 = \frac{\omega_2}{\omega_s} \tag{14}$$

where ω_1 and ω_2 are the frequencies of the two masses of the damper, and

$$\mu_1 = \frac{m_1}{M_1^*} \tag{15}$$

$$\mu_2 = \frac{m_2}{M_1^*} \tag{16}$$

The *minimax* procedure as employed by Tsai & Lin (1993) consists in searching numerically for the design parameters that yield the lowest peak of R_d. The present work is based on this procedure, but here the maximum value of the response is obtained from the following equation

$$\frac{\partial R_d}{\partial \beta} = 0 \tag{17}$$

$$\frac{\partial^2 R_d}{\partial \beta^2} < 0 \tag{18}$$

which defines the maximum of R_d. Here β is the ratio of frequency ω to the excitation frequency. Since Eq. (17) is highly non-linear, the Newton-Raphson method is employed to obtain the system parameters.

Numerical Example

Consider the ten-story building analyzed previously by Villaverde & Koyama (1993). The modal characteristics of the reduced SDOF system associated with the fundamental mode are: $M_1^* = 589.1$ t; $K_1^* = 5.94 \times 10^3$ kN/m and $C_1^* = 74.8$ kNs/m.

Initially the ratio of the damper's mass to the structural mass is taken as: $\mu_1 = 0.061$ and $\mu_2 = 0.042$ and the frequency ratios α_1 e α_2 and damping ratios ξ_1 and ξ_2 are obtained by the *minimax* procedure. The optimal parameters for each configuration are presented in Tab. 1. The corresponding optimum R_d values are compared with that obtained for a single TMD with an equivalent total mass $\mu = 0.103$, using Den Hartog's procedure ($\alpha = 0.906$; $\xi = 0.104$) and with the optimal value for a MTMD designed according to Jangid's equations (Jangid, 1999), also in Tab. 1. The optimization procedure as implemented here leads to more efficient dampers, with configurations 2 and 3 leading to the better results. The harmonic response of the main oscillator is plotted as a function of the frequency parameter β in Fig. 5.

Table 1. MTMD optimum parameters and Rd maximum values.

	α_1	α_2	ξ_1	ξ_2	$R_{d\,\text{máx}}$
Config. 01	0.8500	1.000	0.1378	0.1600	0.368
Config. 02	1.3375	0.9625	0.0400	0.5330	0.365
Config. 03	1.6250	1.1750	0.0050	0.0050	0.365
Config. 04	0.8500	0.8750	0.3389	0.0050	0.372
Jangid (1999)					0.392
TMD					0.409

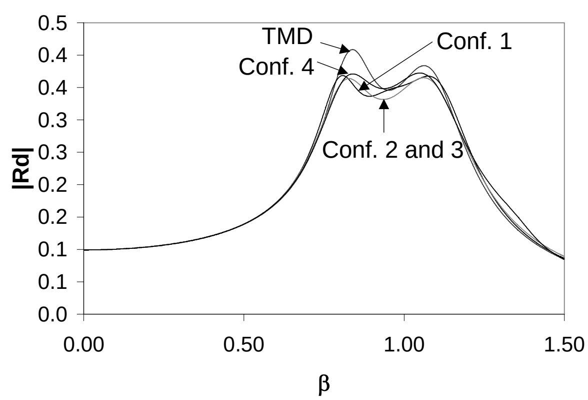


Figure 5. Frequency response for harmonic load.

Two important issues in a TMD design are its travel relative to the floor and the enclosure of space occupied by the TMD. To understand the influence of the damper configuration on these issues, the response of each damper is computed considering a harmonic force $F(t) = 10^3 \sin(\omega t)$ N with $\omega = 2.8566$ rad/s ($\beta = 0.9$). Figure 6 shows the maximum displacements for each mass of the double damper (d_{1max} and d_{2max}) for each configuration. The best configuration in terms of travel space is configuration 3, but configuration 2 could lead to less enclosure space.

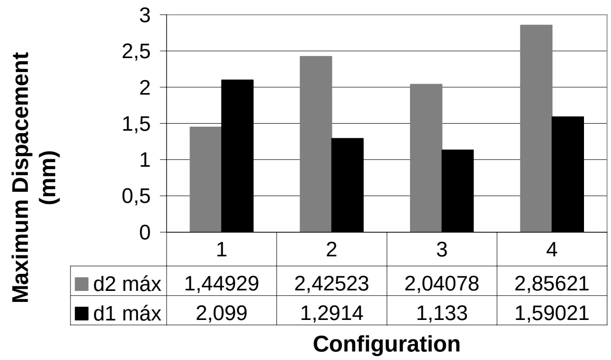


Figure 6. Maximum displacement for each mass of the damper obtained from the time response.

In the previous example the mass ratios μ_i were kept constant in the optimization procedure. Now the procedure is repeated considering also the parameters μ_1 and μ_2 as design variables. The optimum parameters are shown is Tab. 2 together with the corresponding minimum value for the maximum magnification factor R_d. In each case the total mass ratio is kept constant and equal to $\mu = 0.1$. Again configurations 2 and 3 are more efficient. In this optimization process the response of configuration 1 is independent from the ratios μ_i. The inclusion of the mass ratios in the optimization process leads to even better results for $R_{d\,máx}$.

Table 2. Optimum parameters for $\mu = 0.1$.

	α_1	α_2	ξ_1	ξ_2	μ_1	μ_2	$R_{d\,\text{máx}}$
Config. 01	0.9	0.9	0.2	0.2	-	-	0.3977
Config. 02	1.1	0.9	0.05	0.35	0.08	0.02	0.3560
Config. 03	1.1	0.9	0.05	0.35	0.08	0.02	0.3560
Config. 04	1.1	0.8	0.05	0.25	0.03	0.07	0.3942

Again, the equations of motion of each optimized system considering $F(t) = 10^3 \sin(\omega t)$ N with $\omega = 3.174$ rad/s ($\beta = 1.0$) were integrated numerically. The maximums displacements for each mass of the damper are presented in Fig. 7. The response for the mass m_1 is practically the same, independent of the type of connection between the masses. For m_2 as configurations 1 and 4 leads to the best results, but is less efficient in terms of the main system results, while configurations 2 and 3 are more effective in reducing the displacements of the main systems, but exhibit higher displacements relative to the main system.

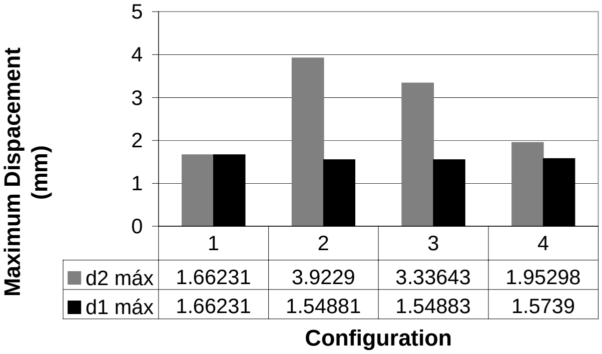


Figure 7. Maximum displacement.

Now consider a MTMD with total mass ratio μ = 0.05. The optimized parameters are presented in Tab. 3. Figure 8 shows the frequency response for this case. Here for the mass ratio is more evident than the results for a MTMD, which are less sensitive and more effective than the results obtained for a single TMD.

Table 3. Optimum parameters for μ = 0.05.

	α_1	α_2	ξ_1	ξ_2	μ_1	μ_2	$R_{d\,\text{máx}}$
Config. 01	0.9	1.0	0.1	0.1	0.025	0.025	0.5041
Config. 02	1.0	0.8	0.05	0.35	0.045	0.005	0.5042
Config. 03	1.0	0.8	0.05	0.35	0.045	0.005	0.5042
Config. 04	1.0	0.8	0.05	0.30	0.03	0.02	0.5171

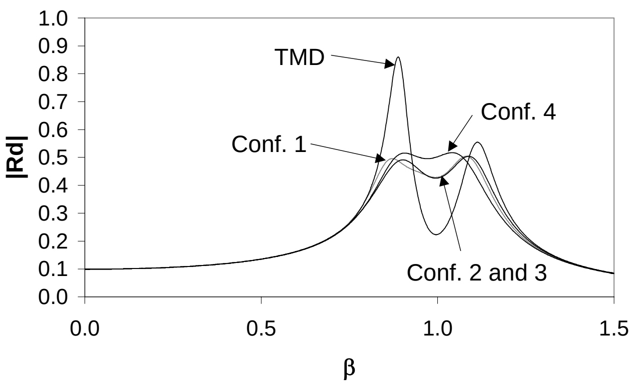


Figure 8. Frequency response for harmonic excitation with passive element (μ = 0.05)

The influence of the ratio between the mass of the two dampers μ_1 / μ_2 on the value of $R_{d\,\text{máx}}$ is shown in Fig. 9 for a total mass μ = 0.05. To obtain the optimal value for R_d, for configuration 1, the best option is to adopt $\mu_1 = \mu_2$; for configurations 2 and 3 m_1 must be larger than m_2, while for configuration 4 one mass must be approximately 50% higher than the other.

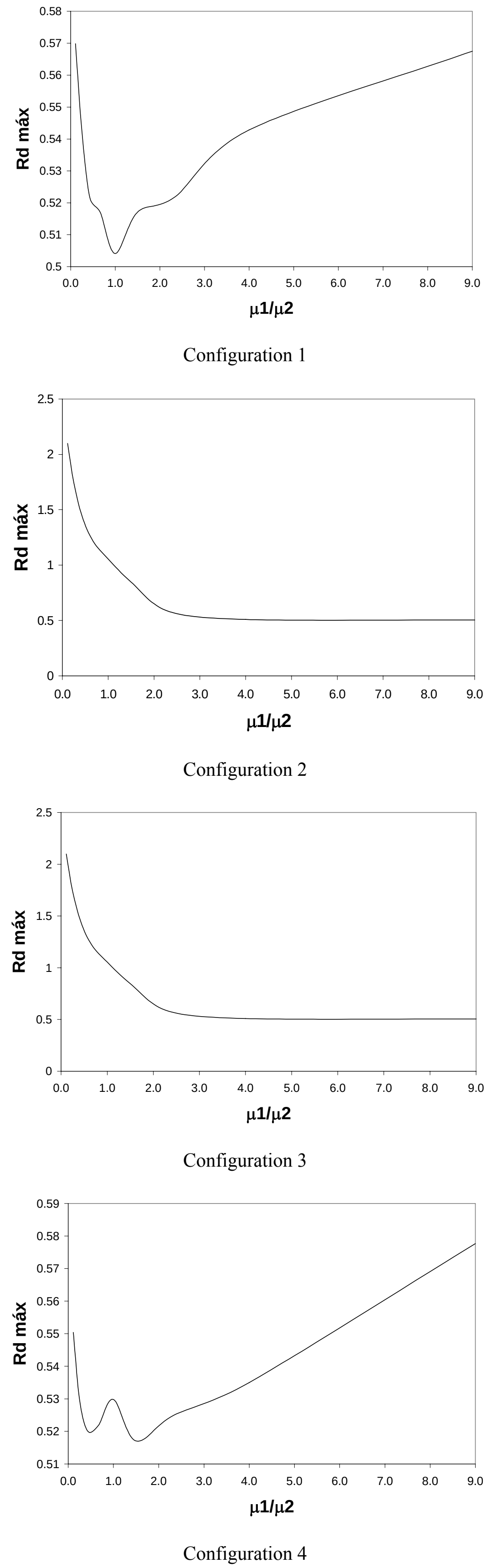


Figure 9. Influence of the dampers mass ratio (μ_1/μ_2) on the minimum value of the maximum magnification factor $R_{d\,max}$.

The use of multiple tuned mass dampers to control the global response of a structure has been investigated in this paper. In the present study four different configurations of a double damper system are studied and their optimal properties are numerically obtained for minimum displacement of the main structure. The *minimax* procedure as implemented here considers as design variables the mass ratios, the damping ratios and tuning frequencies of the dampers. The performance of each control system is assessed by detailed parametric studies. These results show that, independent from the damper configuration, MTMDs can enhance the effectiveness of the control system when compared with the performance of a single optimized TMD. Results also show that the system response is sensitive to small variations in the mass, stiffness and damping parameters. The consideration of all these parameters as design variables leads to the smallest peaks possible of the response in the frequency domain.

The connection between the masses of the dampers has a measurable influence on the performance of the composite damper giving the designer a certain flexibility in important design issues, such as the amount of added mass placed on the top story of a building, TMD travel relative to the floor, the amount of spring force on the building and enclosure of space occupied by the TMD.

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References

Abé, M. and Fujino, Y., 1994, "Dynamic characterization of multiple tuned mass dampers and some design formulas", *Earthquake Engineering and Structural Dynamics*, Vol.23, pp. 813-835.

Carneiro, R.B., 2004, Vibration control of high buildings using multiple tuned mass dampers (MTMD) (in Portuguese), MSc Thesis, University of Brasilia, Brasilia.

Den Hartog, J.P., 1956, "Mechanical Vibrations", McGraw-Hill, New York.

Gu, M., Chen, M.R. and Chang, C.C., 2001, "Parametric study on multiple tuned mass dampers for buffeting control of Yangpu bridge", *Journal of Wind Engineering and Industrial Aerodynamics*, Vol. 89, pp. 987-1000.

Hoang N. and Warnitchai P., 2005, "Design of multiple tuned mass dampers by using a numerical optimizer", *Earthquake Engineering and Structural Dynamics*, Vol. 34, No. 2, pp. 125-144.

Holmes, J.D., 1995, "Listing of installations", *Engineering Structures*, Vol. 17, No. 9, pp. 676-678.

Igusa, T. and Xu, K., 1994, "Vibration control using multiple tuned mass dampers", *Journal of Sound and Vibration*, Vol. 175, No. 4, pp. 491-503.

Jangid, R. S. and Datta, T.K., 1997, "Performance of multiple tuned mass dampers for torsionally coupled systems", *Earthquake Engineering and Structural Dynamics*, Vol. 26, pp. 307-317.

Jangid, R. S., 1999, "Optimum multiple tuned mass dampers for based-excited undamped system", *Earthquake Engineering and Structural Dynamics*, Vol. 28, pp. 1041-1049.

Kareem, A. and Kline, S., 1995, "Performance of multiple mass dampers under random loading", *Journal of Structural Engineering*, Vol. 121, No. 2, pp. 348-361.

Li, C., 2000, "Performance of multiple tuned mass dampers for attenuating undesirable oscillations of structures under the ground acceleration", *Earthquake Engineering and Structural Dynamics*, Vol. 29, pp. 1405-1421.

Li, C. and Qu, W., 2006, "Optimum properties of multiple tuned mass dampers for reduction of translational and torsional response of structures subject to ground acceleration", *Engineering Structures*, Vol. 28, pp. 472-494.

Magluta, C. Ainsworth Jr., G.O. and Roitman, N., 2003, "Comparison between multiple vibration absorbers and single absorbers systems". In: Proc. of COBEM XVII Int. Cong. Mech Eng., São Paulo.

Park, J. and Reed, D., 2001, "Analysis of uniformly and linearly distributed mass dampers under harmonic and earthquake excitation", *Engineering Structures*, Vol. 23, pp. 802-814.

Petersen, N.R., 1980, "Design of large scale TMD". In: Proceedings of Structural Control, North Holland, Leipholz, H.H.E. ed., pp. 581-596.

Poovarodom, N., Kanchanosot, S. and Warnitchai, P., 2003, "Application of non-linear multiple tuned mass dampers to suppress man-induced vibrations of a pedestrian bridge", *Earthquake Engineering and Structural Dynamics,* Vol. 32, pp. 1117-1131.

Soong T.T. and Dargush S.F., 1997,"Passive Energy Dissipation Systems in Structural Engineering", John Wiley & Sons, Chichester.

Tsai, H. and Lin, G., 1993, "Optimum tuned-mass dampers for minimizing steady-state response of support-excited and damped systems", *Earthquake Engineering and Structural Dynamics,* Vol. 22, pp. 957-973.

Villaverde, R. and Koyama, L.A, 1993, "Damped resonant appendages to increase inherent damping in buildings", *Earthquake Engineering and Structural Dynamics*, Vol. 22, pp. 491-507.

Warburton, G.B., 1982, "Optimum absorber parameters for various combinations of response and excitation parameters", *Earthquake Engineering and Structural Dynamics,* Vol. 10, pp. 381-401.

Xu, K. and Igusa, T., 1992, "Dynamic characteristics of multiple substructures with closely spaced frequencies", *Earthquake Engineering and Structural Dynamics*, Vol. 21, pp. 1059-1070.

Yamaguchi, H. and Harnpornchai, N., 1993, "Fundamental characteristics of multiple tuned mass dampers for supressing harmonically forced oscillations", *Earthquake Engineering and Structural Dynamics*, Vol. 22, pp. 51-62.

Yau, J. and Yang Y., 2004, "A Wideband MTMD System for Reducing the Dynamic Response of Continuous Truss Bridges to Moving Train Loads", *Engineering Structures*, Vol. 26, pp. 1795-1807.