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Stagnation Point Flow and Heat Transfer of a Micropolar Fluid with Uniform Suction or Blowing

The steady laminar flow of an incompressible non-Newtonian micropolar fluid impinging on a permeable flat plate with heat transfer is investigated. A uniform suction or blowing is applied normal to the plate, which is maintained at a constant temperature. Numerical solution for the governing nonlinear momentum and energy equations is obtained. The effect of the uniform suction or blowing and the characteristics of the non-Newtonian fluid on both the flow and heat transfer is presented and discussed.

Keywords: stagnation point flow, non-Newtonian fluid, heat transfer, suction, numerical solution

Introduction

The two-dimensional flow of a fluid near a stagnation point is a classical problem in fluid mechanics. It was first examined by Hiemenz (1911) who demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary differential equation of third order using similarity transformation. Owing to the nonlinearities in the reduced differential equation, no analytical solution is available and the nonlinear equation is usually solved numerically subject to two-point boundary conditions, one of which is prescribed at infinity.

Later the problem of stagnation point flow was extended in numerous ways to include various physical effects. The axisymmetric three-dimensional stagnation point flow was studied by Homann (1936). The results of these studies are of great technical importance, for example in the prediction of skin-friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows. Either in the two or three-dimensional case Navier-Stokes equations governing the flow are reduced to an ordinary differential equation of third order using a similarity transformation. The effect of suction on Hiemenz problem has been considered in the literature. Schlichting and Bussman (1943) gave the numerical results first. More detailed solutions were later presented by Preston (1946). An approximate solution to the problem of uniform suction is given by Ariel (1994a). The effect of uniform suction on Homann problem where the flat plate is oscillating in its own plane is considered by Weidman and Mahalingam (1997). In hydromagnetics, the problem of Hiemenz flow was chosen by Na (1979) to illustrate the solution of a third-order boundary value problem using the technique of finite differences. An approximate solution of the same problem has been provided by Ariel (1994b). The effect of an externally applied uniform magnetic field on the two or three-dimensional stagnation point flow was given, respectively, by Attia (2003a) and Attia(2003b) in the presence of uniform suction or injection.

The study of heat transfer in boundary layer flows is of importance in many engineering applications such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, thermal recovery of oil, etc. Massoudi and Ramezan (1990) used a perturbation technique to solve for the stagnation point flow and heat transfer of a non-Newtonian fluid of second grade. Their analysis is valid only for small values of the parameter that determines the behavior of the non-Newtonian fluid. Later

Massoudi and Ramezan (1992) extended the problem to nonisothermal surface. Garg (1994) improved the solution obtained by Massoudi and Ramezan (1992) by computing numerically the flow characteristics for any value of the non-Newtonian parameter using a pseudo-similarity solution.

Many researchers considered non-Newtonian fluids. Thus, among the non-Newtonian fluids, the solution of the stagnation point flow, for viscoelastic fluids, has been given by Rajeshwari and Rathna (1962), Beard and Walters (1964), Teipel (1986), Ariel (1992), and others; for power-law fluid by Djukic (1974); and for second grade fluids by Teipel (1988) and Ariel (1995) in the hydrodynamic case and by Attia (2000) in the hydromagnetic case. Stagnation point flow of a non-Newtonian micropolar fluid was studied by Nath (1975) and Nazar et al. (2004) with zero vertical velocity at the surface. The potential importance of micropolar fluids in industrial applications has motivated these studies. The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equations for Newtonian fluids so that more complex fluids such as particle suspensions, liquid crystals, animal blood, lubrication and turbulent shear flows can be described by this theory. The theory of micropolar fluids, first proposed by Eringen (1966), is capable of describing such fluids. In practice, the theory of micropolar fluids requires that one must add a transport equation representing the principle of conservation of local angular momentum to the usual transport equations for the conservation of mass and momentum, and additional local constitutive parameters are also introduced (Nazar et al., 2004). The key points to note in the development of Eringen's microcontinuum mechanics are the introduction of new kinematics variables, e.g. the gyration tensor and microinertial moment tensor, and the addition of the concept of body moments, stress moments, and microstress averages to classical continuum mechanics. However, a serious difficulty is encountered when this theory is applied to real, non-trivial flow problems; even for the linear theory, a problem dealing with simple microfluids must be formulated in terms of a system of nineteen partial differential equations in nineteen unknowns and the underlying mathematical problem is not easily amenable to solution. These special features of micropolar fluids were discussed in a comprehensive review paper of the subject and application of micropolar fluid mechanics by Arimen et al. (1973).

The purpose of the present paper is to study the effect of uniform suction or blowing directed normal to the wall on the steady laminar flow of an incompressible non-Newtonian micropolar fluid at a two-dimensional stagnation point with heat transfer. The wall and stream temperatures are assumed to be constants. A numerical solution is obtained for the governing momentum and energy equations using finite difference approximations, which takes into

account the asymptotic boundary conditions. The numerical solution computes the flow and heat characteristics for the whole range of the non-Newtonian fluid characteristics, the suction or blowing parameter and the Prandtl number.

Formulation of the Problem

Consider the two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid impinging perpendicular on a permeable wall and flows away along the *x*-axis. This is an example of a plane potential flow that arrives from the *y*-axis and impinges on a flat wall placed at *y*=0, divides into two streams on the wall and leaves in both directions. The viscous flow must adhere to the wall, whereas the potential flow slides along it. (*u,v*) are the components for the viscous flow of velocity at any point (*x,y*) for the viscous flow whereas (*U,V*) are the velocity components for the potential flow. A uniform suction or blowing is applied at the plate with a transpiration velocity at the boundary of the plate given by *-v_o*, where *v_o*>0 for suction. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point is given by

$$U(x)=ax, V(y)=-ay$$

where the constant *a*(>0) is proportional to the free stream velocity far away from the surface. The simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar and incompressible micropolar fluid are (Nath, 1975) (Nazar et al., 2004):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = U \frac{dU}{dx} + (\mu + h) \left(\frac{\partial^2 u}{\partial y^2} \right) + h \frac{\partial N}{\partial y}, \tag{2}$$

$$\rho \left(u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \frac{\gamma}{j} \frac{\partial^2 N}{\partial y^2} - \frac{h}{j} \left(2N + \frac{\partial u}{\partial y} \right), \tag{3}$$

where *N* is the component of microrotation vector normal to the *x*-*y* plane or the angular velocity of the microelements whose direction of rotation is in the *x*-*y* plane, *μ* is the viscosity of the fluid, *ρ* is the density and *j* is the microinertia density, *γ* is the spin-gradient viscosity and *h* is the vortex viscosity. We follow the work of many recent authors by assuming that *γ* is constant and given by (Nath, 1975) (Nazar et al., 2004) (Rees and Pop, 1998):

$$\gamma = (\mu + h/2) j \tag{4}$$

and we take *j=v/a* as a reference length where *v* is the kinematic viscosity. Relation (4) is invoked to allow Eqs. (1)-(3) to predict the correct behaviour in the limiting case when microstructure effects become negligible, and the microrotation, *N*, reduces to the angular velocity (Nazar et al., 2004).

The appropriate physical boundary conditions of Eqs. (1)-(3) are

$$u(x,0) = 0, v(x,0) = -v_o, N(x,0) = -n \frac{\partial u}{\partial y}, \tag{5a}$$

$$y \rightarrow \infty : u(x, y) \rightarrow U(x) = ax, v(x, y) \rightarrow 0, N(x, y) \rightarrow 0, \tag{5b}$$

where *n* is a constant and 0 ≤ *n* ≤ 1. The case *n*=0, which indicates *N*=0 at the wall, represents concentrated particle flows in which the

microelements close to the wall surface are unable to rotate (Nazar et al., 2004). This case is also known as the strong concentration of microelements (Guram and Smith, 1980). The case *n*=1/2 indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration (Ahmadi, 1976) of microelements which will be considered here. The case *n*=1 is used for the modeling of turbulent boundary layer flows (Nazar et al., 2004). The governing equations (1)-(4) subject to the boundary conditions (5) can be expressed in a simpler form by introducing the following transformation

$$\eta = \sqrt{\frac{a}{\nu}} y, u = axf'(\eta), v = -\sqrt{a\nu} f(\eta), N = ax\sqrt{\frac{a}{\nu}} g(\eta), \tag{6}$$

equations (2) and (3) for the functions *f*(*η*) and *g*(*η*) take the form

$$\left(1 + \frac{K}{2} \right) f''' + ff'' - f'^2 + Kg' + 1 = 0 \tag{7}$$

$$\left(1 + \frac{K}{2} \right) g'' + fg' - f'g - K(2g + f'') = 0 \tag{8}$$

It is worth mentioning that when *n*=1/2, we can take (Nazar et al., 2004)]

$$g(\eta) = -\frac{1}{2} f''(\eta) \tag{9}$$

then Eqs. (7) and (8) can be reduced the single equation

$$\left(1 + \frac{K}{2} \right) f''' + ff'' - f'^2 + 1 = 0 \tag{10}$$

subject to the boundary conditions

$$f(0) = A, f'(0) = 0, f'(\infty) = 1, \tag{11}$$

where *K* = *h/μ* (>0) is the material parameter, *A* = *v_o*/√*aν* is the suction parameter and primes denote differentiation with respect to *η*. For micropolar boundary layer flow, the wall skin friction *τ_w* is given by

$$\tau_w = \left[(\mu + h) \frac{\partial u}{\partial y} + hN \right]_{y=0} \tag{12}$$

Using *U(x)=ax* as a characteristic velocity, the skin friction coefficient *C_f* can be defined as

$$C_f = \frac{\tau_w}{\rho U^2}, \tag{13}$$

Substituting (6) and (12) into (13), we get

$$C_f \text{Re}_x^{1/2} = (1 + K/2) f''(0) \tag{14}$$

where *Re_x*^{1/2} = *xU/ν* is the local Reynolds number.

Using the boundary layer approximations and neglecting the dissipation, the equation of energy for temperature *T* is given by (Massoudi and Ramezan, 1990) (Massoudi and Ramezan, 1992),

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} \quad (15)$$

where c_p is the specific heat capacity at constant pressure of the fluid, and k is the thermal conductivity of the fluid. A similarity solution exists if the wall and stream temperatures, T_w and T_∞ are constants—a realistic approximation in typical stagnation point heat transfer problems (Massoudi and Ramezan, 1990) (Massoudi and Ramezan, 1992).

The boundary conditions for the temperature field are

$$y = 0 : T = T_w, \quad (16a)$$

$$y \rightarrow \infty : T \rightarrow T_\infty, \quad (16b)$$

Introducing the non-dimensional variable

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad (17)$$

and using the similarity transformations given in Eq. (6), we find that Eqs. (12) and (13) reduce to,

$$\theta'' + Pr f \theta' = 0 \quad (18)$$

$$\theta(0) = 1, \theta(\infty) = 0, \quad (19)$$

where $Pr = \mu c_p / k$ is the Prandtl number.

The heat transfer at the wall is computed from Fourier's law (Massoudi and Ramezan, 1990) (Massoudi and Ramezan, 1992) as follows;

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} = -k(T_\infty - T_w) \sqrt{\frac{a}{v}} G(Pr) \quad (20)$$

where G is the dimensionless heat transfer rate which is given by

$$G^{-1} = \int_0^\infty d\eta \exp(-2Pr \int_0^\eta f ds) \quad (21)$$

The flow Eqs. (10) and (11) are decoupled from the energy Eqs. (18) and (19), and need to be solved before the latter can be solved. The flow Eq. (10) constitutes a non-linear, non-homogeneous boundary value problem (BVP). In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. The boundary value problem given by Eqs. (10) and (11) may be viewed as a prototype for numerous other situations which are similarly characterized by a boundary value problem having a third order differential equation with an asymptotic boundary condition at infinity. Therefore, its numerical solution merits attention from a practical point of view. The flow Eqs. (10) and (11) are solved numerically using finite difference approximations. A quasi-linearization technique is first applied to replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence. The quasi-linearized form of Eq. (10) is,

$$(1 + K/2) f_{n+1}''' + f_n f_{n+1}'' + f_n'' f_{n+1} - f_n'' f_n - 2f_n' f_{n+1}' + f_n'^2 + 1 = 0 \quad (22)$$

where the subscript n or $n+1$ represents the n th or $(n+1)$ th approximation to the solution. Then, Crank-Nicolson method is used to replace the different terms by their second order central difference approximations. An iterative scheme is used to solve the quasi-linearized system of difference equations. The solution for the Newtonian case is chosen as an initial guess and the iterations are continued till convergence within prescribed accuracy. Finally, the resulting block tri-diagonal system was solved using generalized Thomas' algorithm.

The energy Eq. (18) is a linear second order ordinary differential equation with variable coefficient, $f(\eta)$, which is known from the solution of the flow Eqs. (10) and (11) and the Prandtl number Pr is assumed constant. Equation (18) is solved numerically under the boundary condition (19) using central differences for the derivatives and Thomas' algorithm for the solution of the set of discretized equations. The resulting system of equations has to be solved in the infinite domain $0 < \eta < \infty$. A finite domain in the η -direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid-independence studies show that the computational domain $0 < \eta < \eta_\infty$ can be divided into intervals each is of uniform step size which equals 0.02. This reduces the number of points between $0 < \eta < \eta_\infty$ without sacrificing accuracy. The value $\eta_\infty = 10$ was found to be adequate for all the ranges of parameters studied here. Convergence is assumed when the ratio of every one of f , f' , f'' , or f''' for the last two approximations differed from unity by less than 10^{-5} at all values of η in $0 < \eta < \eta_\infty$.

Results and Discussion

Figures 1 and 2 present the profiles of f and f' , respectively, for various values of K and A . The figures show that increasing the parameter K decreases both f and f' due to the increase in the damping effect of the viscous forces. On the other hand, increasing A increases them which is expected since increasing suction opens an easier path for the incoming flow towards the wall and, in turn, increases both f and f' . The figures indicate also that the effect of K on f and f' is more pronounced for higher values of A (case of suction). However, the effect of A on f and f' becomes more pronounced for smaller values of K . Also, increasing K increases the velocity boundary layer thickness while increasing A decreases it.

Figure 3 presents the profile of temperature θ for various values of K and A and $Pr=0.5$. It is clear that increasing K increases θ and the thickness of the thermal boundary layer. Increasing A decreases θ for all K and its influence becomes more apparent for smaller K . This emphasizes the influence of the injected flow in the cooling process. The action of fluid injection ($A < 0$) is to fill the space immediately adjacent to the disk with fluid having nearly the same temperature as that of the disk. As the injection becomes stronger, so that does the blanket extends to greater distances from the surface. As shown in Fig. 3, the progressive flattening of the temperature profile adjacent to the disk manifests these effects. Thus, the injected flow forms an effective insulating layer, decreasing the heat transfer from the disk. Suction, on the other hand, serves the function of bringing large quantities of ambient fluid into the immediate neighborhood of the disk surface. As a consequence of the increased heat-consuming ability of this augment flow, the temperature drops quickly as we proceed away from the disk. The presence of fluid at near-ambient temperature close to the surface increases the heat transfer.

Figures 4 and 5 present the temperature profiles for various values of K and Pr and for $A=-0.5$ and 0.5 , respectively. The figures bring out clearly the effect of the Prandtl number on the thermal boundary layer thickness. As shown in Figs. 4 and 5, increasing Pr

decreases the thermal boundary layer thickness for all K and A . It is shown in Fig. 4 the influence of blowing in flattening of the temperature profiles adjacent to the disk for higher Pr . The effect of K on θ is more pronounced for higher values of Pr for the blowing case (see Fig. 4).

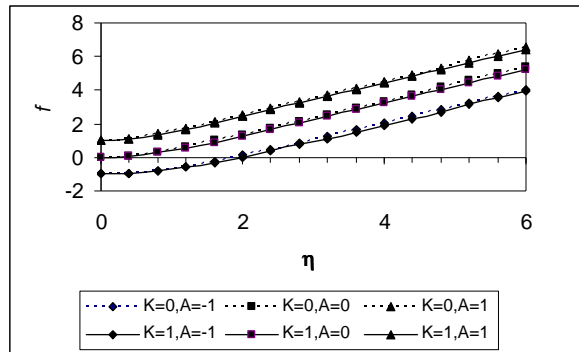


Figure 1. Effect of the parameters K and A on the profile of f .

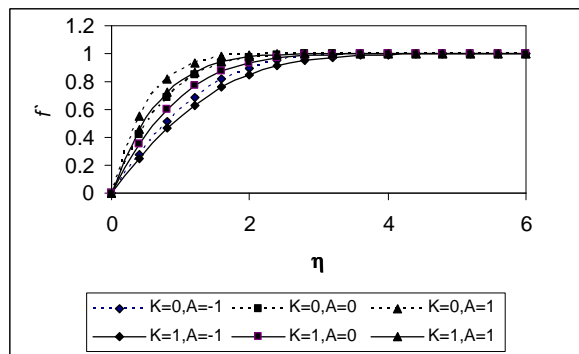


Figure 2. Effect of the parameters K and A on the profile of f .

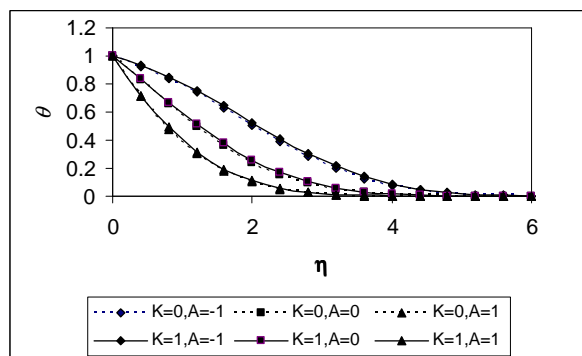


Figure 3. Effect of the parameters K and A on the profile of θ ($Pr=0.5$).

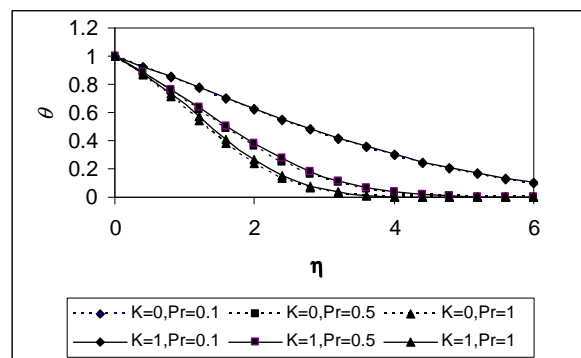


Figure 4. Effect of the parameters K and Pr on the profile of θ ($A=-0.5$).

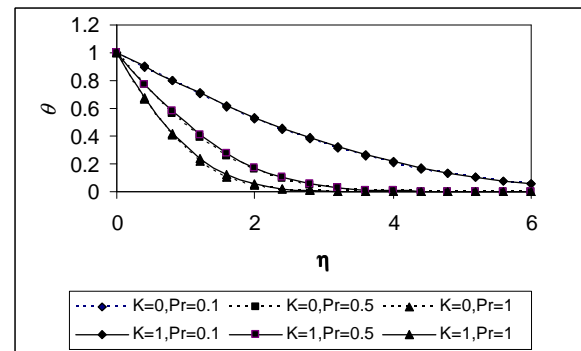


Figure 5. Effect of the parameters K and Pr on the profile of θ ($A=0.5$).

Tables 1 and 2 present the variation of the wall shear stress $C_f Re_x^{1/2}$ and the heat transfer rate at the wall $G(Pr)$, respectively, for various values of K and A and for $Pr=0.5$. Table 1 shows that, for $A < 0$, increasing K increases $C_f Re_x^{1/2}$ steadily. However, for $A \geq 0$, increasing K increases $C_f Re_x^{1/2}$ and then increasing K more decreases $C_f Re_x^{1/2}$. Increasing A increases $C_f Re_x^{1/2}$ for all K and its effect is more apparent for smaller K . Table 2 shows that increasing K decreases $G(Pr)$ due to its damping affect for the coming flow towards the wall. Increasing A increases $G(Pr)$ for all K as increasing suction helps bringing fluid at near-ambient towards the surface of the wall which increases the heat transfer.

Table 1 Variation of the wall shear stress $C_f Re_x^{1/2}$ with K and A .

A	$K=0$	$K=0.5$	$K=1$	$K=1.5$	$K=2$	$K=10$
-2	0.4758	0.7574	0.7986	0.8377	0.8751	1.3531
-1	0.7566	1.1586	1.1839	1.2103	1.2372	1.6556
0	1.2326	1.7915	1.7612	1.7469	1.7431	2.0128
1	1.8892	2.6352	2.5116	2.4312	2.3779	2.4199
2	2.6699	3.6299	3.3899	3.2271	3.1124	2.8704

Table 2 Variation of the wall heat transfer $G(Pr)$ with K and A ($Pr=0.5$).

A	$K=0$	$K=0.5$	$K=1$	$K=1.5$	$K=2$
-2	0.0339	0.0323	0.0313	0.0303	0.0294
-1	0.1649	0.1600	0.1557	0.1519	0.1486
0	0.4352	0.4258	0.4178	0.4109	0.4050
1	0.8042	0.7928	0.7833	0.7752	0.7681
2	1.2269	1.2157	1.2062	1.1982	1.1912

Table 3 presents the effect of K on $G(Pr)$ for various values of Pr and for $A=0$. Increasing K decreases $G(Pr)$ for all Pr and its effect is more for higher Pr . Increasing Pr increases $G(Pr)$ for all K . Table 4 shows the variation of $G(Pr)$ for various values of Pr and A

and for $K=1$. Increasing A increases $G(Pr)$ and its effect is more apparent for higher Pr . For the suction case ($A \geq 0$), increasing Pr increases G steadily. On the other hand, for the large blowing case, increasing Pr decreases G steadily. But for moderate blowing velocity ($A=1$), increasing Pr increasing G and increasing Pr more decreases G .

Table 3 Variation of the wall heat transfer $G(Pr)$ with K and Pr ($A=-0.5$).

K	$Pr=0.05$	$Pr=0.1$	$Pr=0.5$	$Pr=1$	$Pr=1.5$
0	0.2021	0.2935	0.8042	1.3280	1.8258
0.5	0.2003	0.2901	0.7928	1.3112	1.8058
1	0.1988	0.2872	0.7833	1.2975	1.7898
1.5	0.1974	0.2846	0.7752	1.2860	1.7767
2	0.1961	0.2822	0.7681	1.2762	1.7656

Table 4 Variation of the wall heat transfer $G(Pr)$ with A and Pr ($K=1$).

A	$Pr=0.05$	$Pr=0.1$	$Pr=0.5$	$Pr=1$	$Pr=1.5$
-2	0.1013	0.0966	0.0323	0.0054	0.0008
-1	0.1317	0.1519	0.1600	0.1126	0.0727
0	0.1651	0.2176	0.4258	0.5594	0.6535
1	0.2003	0.2901	0.7928	1.3112	1.8058
2	0.2371	0.3674	1.2157	2.2008	2.8862

Conclusions

The two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid with heat transfer is studied in the presence of uniform suction or blowing. A numerical solution for the governing equations is obtained which allows the computation of the flow and heat transfer characteristics for various values of the non-Newtonian parameter K , the suction parameter A , and the Prandtl number Pr . The results indicate that increasing the parameter K increases both the velocity and thermal boundary layer thickness while increasing A decreases the thickness of both layers. The effect of the parameter K on the velocity is more apparent for suction than blowing. The influence of the parameter K on the temperature is more apparent for higher values of Prandtl number. The effect of the suction velocity on the shear stress at the wall depends on the value of the non-Newtonian parameter K . On the other hand, the influence of the blowing velocity on the heat transfer rate at the wall depends on the value of the non-Newtonian parameter K .

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