

# Electromagnetic Transients in Multiconductor Transmission Lines

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**Abstract**— The theory of multiconductor transmission lines is based on the principle of superposition. According to this principle, any electric or magnetic field can be represented as the sum of individual electric or magnetic fields. This theory is used to analyze and design electrical systems involving multiple conductors. In this work, the procedures for analyzing electromagnetic transients in 3-conductor transmission lines are presented, which is the last step for generalization in the case of lines to "n" conductors. The paper presents elementary cases and a more general one to illustrate such an approach.

**Index Terms**— Electromagnetic Transients, Power Systems analysis, Electric Circuits

## I. INTRODUCTION

The seminal work by Dommel [1] in the late 1960s fostered what was considered one of the greatest revolutions in power system analysis. Both the high efficiency and robustness of the developed methodology enabled the release of several high-performance power systems computational tools, such as EMTP, ATP, PSCAD/EMTDC, and others. Such tools have promoted the retirement of the old analog transient network analyzers and launched the power and control systems analysis in the digital age. Dommel's approach evaluates the step-by-step evolution of both voltages and currents through an assembly of resistances and current sources network that take into account the history of these values in the last computed step. The power of this formulation is that the linear equation system is stable once the matrix is positive defined. This characteristic is suitable for applying several optimized and fast-solver techniques [2], [3].

In [4] was proposed an approach that reaches the same goal as Dommel's by evaluating the "increments" of both voltages and currents with an assembly of resistances and sources network that take into account the "increment" history of these values in the last step. It presented a technique for the treatment of transients in lossless transmission lines for two conductors.

Clayton Paul [5], which is a classic reference in this line of research, presents two techniques. The first, called "Recursive Solution", use the Laplace transform, whose solution leads to a series that can be solved by recursive replacement. The difficulty of this technique in the treatment of networks with nonlinear characteristics, which do not adapt to the transformation of Laplace. The second, called "Decoupling", very popular with developers of computational tools for this purpose, replace the transmission line with "n" conductors, in a set of "n-1" lines with two conductors. In the "Decoupling" mode, the obtaining of the disengaged lines is obtained from an elegant and complex development of matrix differential equations, which requires obtaining the eigenvalues and eigenvectors of the resulting matrix, which is still a considerable computational effort when it comes to lines with

many conductive conductors. In both cases, some approximations need to be made, to accommodate the solution to situations that require symmetric arrays. We'll take care of that again in the next few sections.

## II. TRANSIENTS IN A 3-CONDUCTOR TRANSMISSION LINE

The case of the 3-conductor transmission line was the first step to generalizing the case of "n" conductors. This choice was adopted to identify the mathematical difficulties involved and the ease of the physical analysis of phenomena.

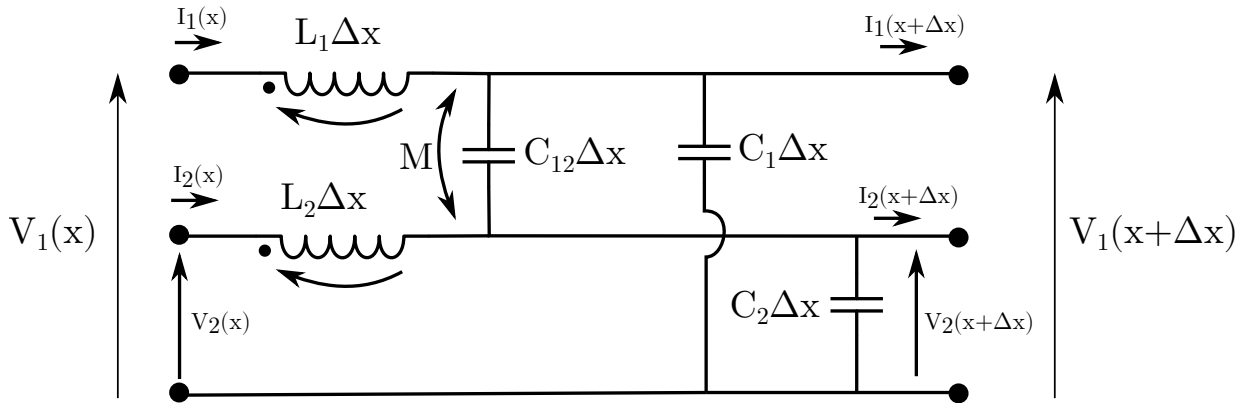


Fig. 1. Equivalent circuit for a 3-conductors TL for an elementary stretch.

Figure 1 shows the equivalent circuit of an elementary stretch of a three-wire transmission line, and we will apply the voltage and circuit equations for such a circuit.

### A. Voltage equations

For conductor 1 the voltage balance provides us with:

$$v_1(x) - L_1 \Delta x \frac{\partial i_1}{\partial t} - M \Delta x \frac{\partial i_2}{\partial t} - v_1(x + \Delta x) = 0 \quad (1)$$

which mathematically manipulated results in:

$$\frac{\partial v_1}{\partial x} = -L_1 \frac{\partial i_1}{\partial t} - M \frac{\partial i_2}{\partial t} \quad (2)$$

Conductor 2, similarly, provides us with:

$$v_2(x) - M \Delta x \frac{\partial i_1}{\partial t} - L_2 \Delta x \frac{\partial i_2}{\partial t} - v_2(x + \Delta x) = 0 \quad (3)$$

which can be rewritten as:

$$\frac{\partial v_2}{\partial x} = -M \frac{\partial i_1}{\partial t} - L_2 \frac{\partial i_2}{\partial t} \quad (4)$$

### B. Current equations

From the balance of the currents in conductor 1 we can write:

$$i_1(x) - C_{12} \Delta x \frac{\partial (v_1 - v_2)}{\partial t} - C_1 \Delta x \frac{\partial v_1}{\partial t} - i_1(x + \Delta x) = 0 \quad (5)$$

whose mathematical manipulation leads us to:

$$\frac{\partial i_1}{\partial x} = -C_{1eq} \frac{\partial v_1}{\partial t} + C_{12} \frac{\partial v_2}{\partial t} \quad (6)$$

and for conductor 2:

$$i_2(x) + C_{12} \Delta x \frac{\partial(v_1 - v_2)}{\partial t} - C_2 \Delta x \frac{\partial v_2}{\partial t} - i_2(x + \Delta x) = 0 \quad (7)$$

which can be written as:

$$\frac{\partial i_2}{\partial x} = C_{12} \frac{\partial v_1}{\partial t} - C_{2eq} \frac{\partial v_2}{\partial t} \quad (8)$$

In which:  $C_{1eq} = C_1 + C_{12}$  e  $C_{2eq} = C_2 + C_{12}$ .

### C. Voltage wave equations

The wave equation for this transmission line configuration is obtained according to a procedure similar to that applied in the single-phase transmission line. Because we have four state variables, two voltages, and two currents, an additional term will be inserted into the final result. The procedure begins by deriving both members of (2) concerning "x" and both members of (6) and (8) concerning "t" and making the appropriate substitutions so that it results:

$$\frac{\partial^2 v_1}{\partial x^2} = A \frac{\partial^2 v_1}{\partial t^2} + B \frac{\partial^2 v_2}{\partial t^2} \quad (9)$$

in which:

$$A = L_1 C_{1eq} - M C_{12} \quad (10)$$

$$B = -L_1 C_{12} + M C_{2eq} \quad (11)$$

Next, a similar procedure applies to the equations of conductor 2, so that it results:

$$\frac{\partial^2 v_2}{\partial x^2} = C \frac{\partial^2 v_1}{\partial t^2} + D \frac{\partial^2 v_2}{\partial t^2} \quad (12)$$

in which:

$$C = M C_{1eq} - L_2 C_{12} \quad (13)$$

$$D = L_2 C_{2eq} - M C_{12} \quad (14)$$

### D. Current wave equations

Following the identical procedure with the equations (6) and (8) we obtain the wave equations for the currents, which are given by:

$$\frac{\partial^2 i_1}{\partial x^2} = A \frac{\partial^2 i_1}{\partial t^2} + F \frac{\partial^2 i_2}{\partial t^2} \quad (15)$$

$$\frac{\partial^2 i_2}{\partial x^2} = G \frac{\partial^2 i_1}{\partial t^2} + D \frac{\partial^2 i_2}{\partial t^2} \quad (16)$$

in which:

$$F = -L_2 C_{12} + M C_{1eq} \quad (17)$$

$$G = M C_{2eq} - L_1 C_{12} \quad (18)$$

*E. First solution of the wave equation*

As it is a propagation phenomenon, the first solution set for the wave equations of voltages (9) and (12) is of the type:

$$V_{1+} = f_1 \left( t - \frac{x}{v_1} \right) \quad (19)$$

$$V_{2+} = f_2 \left( t - \frac{x}{v_2} \right) \quad (20)$$

For the wave equations of the currents (15) and (16) the first solution set is given by:

$$I_{1+} = g_1 \left( t - \frac{x}{v_1} \right) \quad (21)$$

$$I_{2+} = g_2 \left( t - \frac{x}{v_2} \right) \quad (22)$$

The expressions (19) to (22) are representations of voltage and current waves that propagate in the direction of  $x > 0$ , with their respective velocities  $v_1$  and  $v_2$  to be determined. The terms corresponding to the propagation speeds are identical two by two, because voltage and current waves from the same conductor, which travel in the same direction, do so with the same speed.

*F. Relationship between voltage and currents waves*

By conveniently replacing the solution proposals from (19) to (22) in (2), results:

$$-\frac{1}{v_1} \dot{f}_1 = -L_1 \dot{g}_1 - M \dot{g}_2 \quad (23)$$

By the characteristics of the  $f$ 's and  $g$ 's functions, this same relationship reproduces with the primary functions, so we can write:

$$-\frac{1}{v_1} V_{1+} = -L_1 I_{1+} - M I_{2+} \quad (24)$$

or even:

$$V_{1+} = Z_{11} I_{1+} + Z_{m1} I_{2+} \quad (25)$$

By applying the same procedure, from (4), we obtain:

$$V_{2+} = Z_{m2} I_{1+} + Z_{22} I_{2+} \quad (26)$$

in which:

$$Z_{11} = v_1 L_1 \quad Z_{22} = v_2 L_2 \quad Z_{m1} = v_1 M \quad Z_{m2} = v_2 M \quad (27)$$

are the characteristic impedances of the 3-conductor line.

It is appropriate to represent the equations (25) and (26), as follows:

$$\begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{m1} \\ Z_{m2} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} \quad (28)$$

*G. Second solution of the wave equation*

The other set of solutions satisfying wave equations (9) and (12) is of the type:

$$V_{1-} = h_1 \left( t + \frac{x}{v_1} \right) \quad (29)$$

$$V_{2-} = h_2 \left( t + \frac{x}{v_2} \right) \quad (30)$$

For the current wave equations (15) and (16), the second solution set is given by:

$$I_{1-} = k_1 \left( t + \frac{x}{v_1} \right) \quad (31)$$

$$I_{2-} = k_2 \left( t + \frac{x}{v_2} \right) \quad (32)$$

The expressions from (29) to (32) are representations of voltage and current waves that propagate in the direction of  $x < 0$ , with their respective speeds  $v_1$  and  $v_2$ .

#### H. Relation between voltages and currents for the second solution

From (2) we can write:

$$\frac{1}{v_1} \dot{h}_1 = -L_1 \dot{k}_1 - M \dot{k}_2 \quad (33)$$

In view of the characteristics of the functions  $g$ 's and  $h$ 's this same relationship is reproduced with the primary functions, that is:

$$\frac{1}{v_1} V_{1-} = -L_1 I_{1-} - M I_{2-} \quad (34)$$

or also:

$$V_{1-} = -Z_{11} I_{1-} - Z_{m1} I_{2-} \quad (35)$$

Applying the same procedure, starting from (4), we obtain:

$$V_{2-} = -Z_{m2} I_{1-} - Z_{22} I_{2-} \quad (36)$$

Representing (35) and (36) in a matrix form:

$$\begin{bmatrix} V_{1-} \\ V_{2-} \end{bmatrix} = - \begin{bmatrix} Z_{11} & Z_{m1} \\ Z_{m2} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{1-} \\ I_{2-} \end{bmatrix} \quad (37)$$

The final solution is the sum of the two sets of solutions presented, so that the voltage at any point on the transmission line is given by:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} + \begin{bmatrix} V_{1-} \\ V_{2-} \end{bmatrix} \quad (38)$$

or also:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{m1} \\ Z_{m2} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} - \begin{bmatrix} Z_{11} & Z_{m1} \\ Z_{m2} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{1-} \\ I_{2-} \end{bmatrix} \quad (39)$$

Simply, we can write:

$$[V] = [Z_0] \cdot [I_+] - [Z_0] \cdot [I_-] \quad (40)$$

For currents the development is similar. From (6), we get for the traveling wave:

$$\frac{1}{v_1} \dot{g}_1 = -C_{1eq} \dot{f}_1 + C_{12} \dot{f}_2 \quad (41)$$

or also:

$$-\frac{1}{v_1} I_{1+} = -C_{1eq} V_{1+} + C_{12} V_{2+} \quad (42)$$

which can be expressed as:

$$I_{1+} = Y_{11}V_{1+} - Y_{12}V_{2+} \quad (43)$$

Being the characteristic admittances of conductor 1:

$$Y_{11} = v_1 C_{1eq}$$

$$Y_{12} = v_1 C_{12}$$

For the traveling current wave for conductor 2, we obtain from (8):

$$-\frac{1}{v_2} I_{2+} = C_{12}V_{1+} - C_{2eq}V_{2+} \quad (44)$$

or also:

$$I_{2+} = -Y_{12}V_{1+} + Y_{22}V_{2+} \quad (45)$$

Being the characteristic admittances of the conductor 2.

$$Y_{22} = v_2 C_{2eq}$$

$$Y_{21} = v_2 C_{12}$$

Following the same procedure, a similar result is obtained for the regressive wave currents:

$$I_{1-} = -Y_{11}V_{1-} + Y_{12}V_{2-} \quad (46)$$

and

$$I_{2-} = -Y_{21}V_{1-} - Y_{22}V_{2-} \quad (47)$$

In a matrix, we can write:

$$\begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} = \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} \quad (48)$$

and

$$\begin{bmatrix} I_{1-} \\ I_{2-} \end{bmatrix} = \begin{bmatrix} -Y_{11} & Y_{12} \\ Y_{21} & -Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_{1-} \\ V_{2-} \end{bmatrix} \quad (49)$$

So the complete solution results:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} - \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_{1-} \\ V_{2-} \end{bmatrix} \quad (50)$$

or also:

$$[I] = [Y_0] \cdot [V_+] - [Y_0] \cdot [V_-] \quad (51)$$

### 1. Propagation speeds

The propagation speed is a physical property of the Transmission Line. In the case of the multiconductor line, this speed does not depend on the distribution of currents between its conductors, but on its parameters. To evaluate the propagation speed of Line 1, it is enough to assume that only this line is energized and the others are empty, as shown in Figure 2.

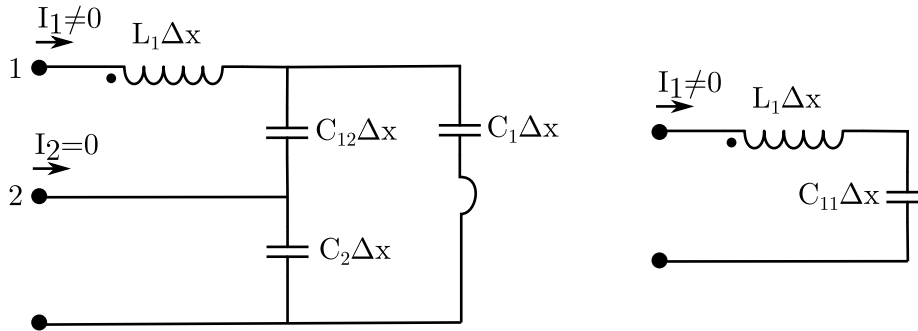


Fig. 2. Three-conductor line with  $i_1 \neq 0$  and  $i_2 = 0$  and equivalent single-phase line

The figure also shows the single-phase transmission line equivalent to this condition, in which:

$$C_{11} = C_1 + \frac{C_2 C_{12}}{C_2 + C_{12}} \quad (52)$$

For a single-phase line with these parameters, the propagation speed will be given by:

$$v_1 = \frac{1}{\sqrt{L_1 C_{11}}} \quad (53)$$

To evaluate the propagation speed of Line 2, we assume that only this line is energized and the others are empty, as shown in Figure 3.

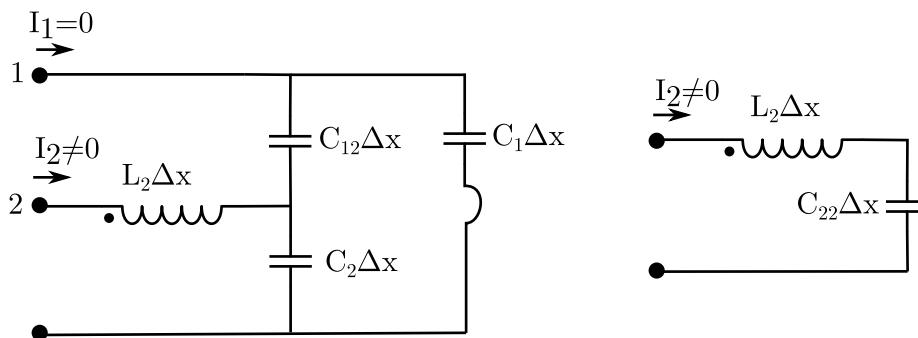


Fig. 3. Three-conductor line with  $i_1 = 0$  and  $i_2 \neq 0$  and equivalent single-phase line

For the case where the line is fed only through conductor 2, the characteristics of the equivalent single-phase line result in a propagation speed given by:

$$v_2 = \frac{1}{\sqrt{L_2 C_{22}}} \quad (54)$$

in which:

$$C_{22} = C_2 + \frac{C_1 C_{12}}{C_1 + C_{12}} \quad (55)$$

Substituting the propagation velocities in the expressions of the characteristic impedances results:

$$Z_{11} = \sqrt{\frac{L_1}{C_{11}}} \quad Z_{22} = \sqrt{\frac{L_2}{C_{22}}} \quad Z_{m1} = \frac{M}{\sqrt{L_1 C_{11}}} \quad Z_{m2} = \frac{M}{\sqrt{L_2 C_{22}}} \quad (56)$$

Being:

- $Z_{11}$ : Characteristic impedance of conductor 1
- $Z_{22}$ : Characteristic impedance of conductor 2
- $Z_{m1}$ : Characteristic mutual impedance of conductor 1

- $Z_{m2}$ : Characteristic mutual impedance of conductor 2

### III. ELEMENTARY CASES

#### A. Elementary Case 1

To understand the procedure, we will analyze the evolution of the voltage at the ends of a 3-conductor transmission line, which is excited by a voltage step  $E_0$  with the other terminals open. The parameters are indicated in Figure 4.

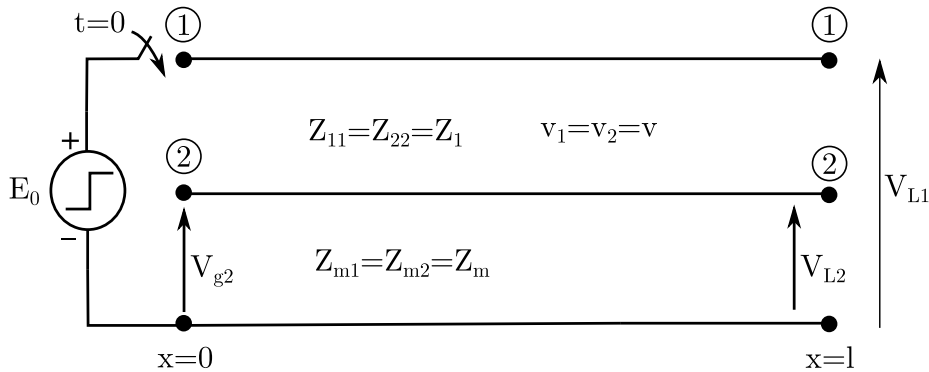


Fig. 4. Circuit for the Elementary's Case 1

At the instant  $t = 0_+$  only the progressive solution is admitted, so we can write:

$$\begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{m1} \\ Z_{m2} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} \quad (57)$$

Identifying the parameters and quantities with the problem data, we have at the first terminal the imposed voltage, an open terminal for the load terminal.

$$V_{1+} = E_0 \quad I_{2+} = 0 \quad Z_{11} = Z_1 \quad Z_{22} = Z_2 \quad Z_{m1} = Z_{m2} = Z_m \quad (58)$$

So the matrix system results:

$$\begin{bmatrix} E_0 \\ V_{2+} \end{bmatrix} = \begin{bmatrix} Z_1 & Z_m \\ Z_m & Z_1 \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} \quad (59)$$

whose solution is given by:

$$I_{1+} = I_{G1}(0) = \frac{E_0}{Z_1} \text{ and } V_{2+} = V_{G2}(0) = \frac{Z_m}{Z_1} E_0$$

These voltage and current waves are launched towards the end of the line and propagate with speed  $v_1 = v_2 = v$ , so they will reach the end of the line at instant  $t = \tau = l/v$ . At this instant, the voltages at the line terminals will be incremented with values, such as:

$$\begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} - \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix}$$

Substituting the known quantities, we get:

$$\begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = \begin{bmatrix} 2E_0 \\ \frac{2Z_m}{Z_1} E_0 \end{bmatrix} \quad (60)$$



or

$$\begin{bmatrix} V_{L1}(\tau) \\ V_{L2}(\tau) \end{bmatrix} = \begin{bmatrix} V_{L1}(0) \\ V_{L2}(0) \end{bmatrix} + \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2E_0 \\ \frac{2Z_m}{Z_1} E_0 \end{bmatrix} \quad (61)$$

or also:

$$\begin{bmatrix} V_{L1}(\tau) \\ V_{L2}(\tau) \end{bmatrix} = \begin{bmatrix} 2E_0 \\ \frac{2Z_m}{Z_1} E_0 \end{bmatrix} \quad (62)$$

At this instant, a regressive wave is generated, whose increment will be:

$$[\Delta V_-] = [\Delta V_L] - [V_+] \quad (63)$$

Which results:

$$[\Delta V_-] = \begin{bmatrix} 2E_0 \\ \frac{2Z_m}{Z_1} E_0 \end{bmatrix} - \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} = \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (64)$$

This regressive wave will perturb the voltages at the generator terminals by introducing an increment such that:

$$2[\Delta V_-] + [Y_0]^{-1} [\Delta I_G] = [\Delta V_G] \quad (65)$$

So it results:

$$\begin{bmatrix} 2E_0 \\ \frac{2Z_m}{Z_1} E_0 \end{bmatrix} + \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta I_{G1} \\ \Delta I_{G2} \end{bmatrix} = \begin{bmatrix} \Delta V_{G1} \\ \Delta V_{G2} \end{bmatrix} \quad (66)$$

Since  $\Delta I_{G2} = 0$  (open terminal) and  $\Delta V_{G1} = 0$  (imposed voltage) we obtain  $\Delta I_{G1} = -\frac{2E_0}{Z_1}$  and  $\Delta V_{G2} = 0$ . Thus, the new voltages and currents in the generator will be given by:

$$\begin{bmatrix} V_{G1}(2\tau) \\ V_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} V_{G1}(0) \\ V_{G2}(0) \end{bmatrix} + \begin{bmatrix} \Delta V_{G1} \\ \Delta V_{G2} \end{bmatrix} \quad (67)$$

or

$$\begin{bmatrix} V_{G1}(2\tau) \\ V_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (68)$$

We can note that the voltages at the beginning of the line remain constant, since  $\Delta V_{G1}(t = 2\tau) = \Delta V_{G2}(t = 2\tau) = 0$ .

$$\begin{bmatrix} I_{G1}(2\tau) \\ I_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} I_{G1}(0) \\ I_{G2}(0) \end{bmatrix} + \begin{bmatrix} \Delta I_{G1} \\ \Delta I_{G2} \end{bmatrix} = \begin{bmatrix} \frac{E_0}{Z_1} \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{2E_0}{Z_1} \\ 0 \end{bmatrix} \quad (69)$$

or

$$\begin{bmatrix} I_{G1}(t = 2\tau) \\ I_{G2}(t = 2\tau) \end{bmatrix} = \begin{bmatrix} -\frac{E_0}{Z_1} \\ 0 \end{bmatrix} \quad (70)$$

At this instant, new progressive waves are generated, which reach the end of the line at the instant  $t = 3\tau$ , such that:

$$\begin{bmatrix} \Delta V_{1+} \\ \Delta V_{2+} \end{bmatrix} = \begin{bmatrix} \Delta V_{G1}(t = 2\tau) \\ \Delta V_{G2}(t = 2\tau) \end{bmatrix} - \begin{bmatrix} \Delta V_{1-} \\ \Delta V_{2-} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (71)$$

or

$$\begin{bmatrix} \Delta V_{1+} \\ \Delta V_{2+} \end{bmatrix} = - \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (72)$$

The new voltages are obtained from:

$$-2 \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} - \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} \quad (73)$$

Remembering that, as the line is open, results:

$$\begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

we get

$$\begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = -2 \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (74)$$

Thus, the new voltages at the end of the transmission line, at this instant, will be:

$$\begin{bmatrix} V_{L1}(3\tau) \\ V_{L2}(3\tau) \end{bmatrix} = \begin{bmatrix} V_{L1}(\tau) \\ V_{L2}(\tau) \end{bmatrix} + \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (75)$$

The graphs in Figure 5 show the evolution of voltages at the source and at the end of the line. This sequence is permanently maintained, as they are loss-free lines.

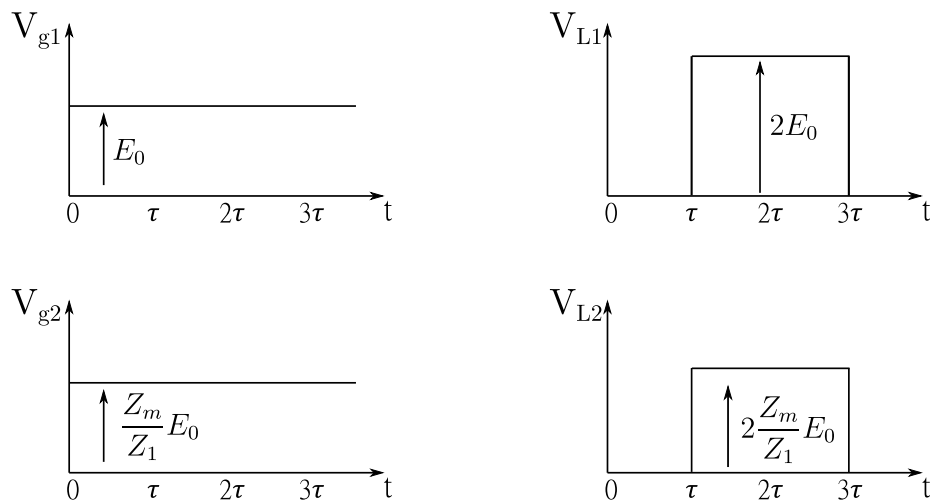


Fig. 5. Voltage at both ends for the line for the first elementary case.

One can see the coupling effect between both circuits and the similarity between the waveforms of both active and passive circuits. Even when there is no generator at the second line, there is an induced voltage originated from the first transmission line.

### B. Elementary Case 2

The second elementary case, although simple, is an example that has great practical application in electrical grounding of electrical cable shields. In this case, an ideal source is connected to one of the three-conductor transmission line conductors and another terminal, which may be a cable shield, is grounded at this end. The objective is to determine the evolution of the voltages and currents in both conductors at the other ends. In this elementary case there is no additional grounding resistance for the shielding, so we can consider a simplification for the results concerning a very low grounding resistance.

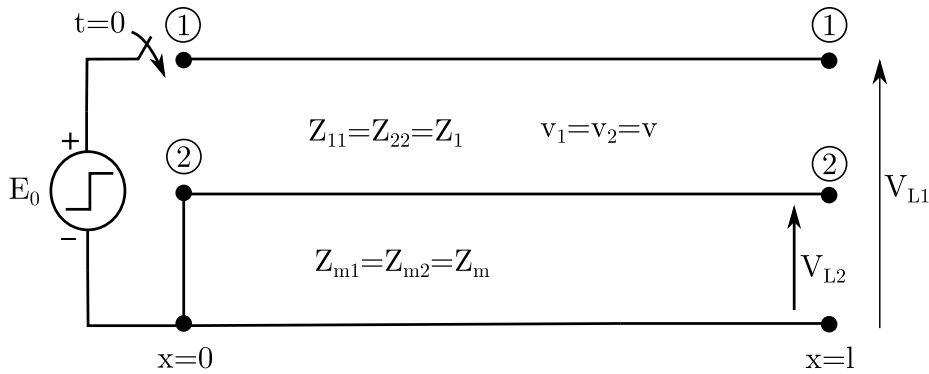


Fig. 6. Terminal grounded at one end

At the instant  $t = 0_+$  only the progressive solution is admitted, so we can write:

$$\begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{m1} \\ Z_{m2} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} \quad (76)$$

Identifying the parameters and quantities with the problem data, we have  $V_{1+} = E_0$  (imposed voltage);  $V_{2+} = 0$  (short-circuited terminal);  $Z_{11} = Z_1$ ;  $Z_{22} = Z_2$  and  $Z_{m1} = Z_{m2} = Z_m$ . So the matrix system results:

$$\begin{bmatrix} E_0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_1 & Z_m \\ Z_m & Z_1 \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} \quad (77)$$

Whose solution is

$$I_{1+} = I_{G1}(0) = \frac{z_1}{Z_1^2 - z_m^2} E_0$$

and

$$I_{2+} = I_{G2}(0) = -\frac{z_m}{Z_1^2 - z_m^2} E_0$$

These voltage and current waves are launched towards the end of the line and propagate with speed  $v_1 = v_2 = v$ , so they will reach the end of the line at instant  $t = \tau = l/v$ . At this instant, the voltages at the line terminals will be incremented with values, such as:

$$2 \begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} - \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} \quad (78)$$

Substituting the known quantities, we get:

$$\begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = \begin{bmatrix} 2E_0 \\ 0 \end{bmatrix} \quad (79)$$

then:

$$\begin{bmatrix} V_{L1}(\tau) \\ V_{L2}(\tau) \end{bmatrix} = \begin{bmatrix} V_{L1}(0) \\ V_{L2}(0) \end{bmatrix} + \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2E_0 \\ 0 \end{bmatrix} \quad (80)$$

or also:

$$\begin{bmatrix} V_{L1}(\tau) \\ V_{L2}(\tau) \end{bmatrix} = \begin{bmatrix} 2E_0 \\ 0 \end{bmatrix} \quad (81)$$

At this instant, a regressive wave is generated, whose increment will be:

$$[\Delta V_-] = [\Delta V_L] - [V_+] \quad (82)$$

Which results:

$$[\Delta V_-] = \begin{bmatrix} 2E_0 \\ 0 \end{bmatrix} - \begin{bmatrix} E_0 \\ 0 \end{bmatrix} = \begin{bmatrix} E_0 \\ 0 \end{bmatrix} \quad (83)$$

This regressive wave will perturb the voltages at the generator terminals at  $t = 2\tau$  by introducing an increment such that:

$$2[\Delta V_-] + [Y_0]^{-1} [\Delta I_G] = [\Delta V_G] \quad (84)$$

So it results:

$$\begin{bmatrix} 2E_0 \\ 0 \end{bmatrix} + \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta I_{G1} \\ \Delta I_{G2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (85)$$

Since  $\Delta V_{G1} = \Delta V_{G2} = 0$ , since the voltages at these terminals are imposed, so we get  $\Delta I_{G1} = -\frac{2E_0}{Z_1}$  and  $\Delta I_{G2} = 0$ . Thus, the new voltages and currents in the generator will be given by:

$$\begin{bmatrix} V_{G1}(2\tau) \\ V_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} V_{G1}(0) \\ V_{G2}(0) \end{bmatrix} + \begin{bmatrix} \Delta V_{G1} \\ \Delta V_{G2} \end{bmatrix} \quad (86)$$

or

$$\begin{bmatrix} V_{G1}(2\tau) \\ V_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (87)$$

We can note here that the voltages at the beginning of the line remain constant, since  $\Delta V_{G1}(t = 2\tau) = \Delta V_{G2}(t = 2\tau) = 0$ .

$$\begin{bmatrix} I_{G1}(2\tau) \\ I_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} I_{G1}(0) \\ I_{G2}(0) \end{bmatrix} + \begin{bmatrix} \Delta I_{G1} \\ \Delta I_{G2} \end{bmatrix} = \begin{bmatrix} \frac{E_0}{Z_1} \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{2E_0}{Z_1} \\ 0 \end{bmatrix} \quad (88)$$

or

$$\begin{bmatrix} I_{G1}(t = 2\tau) \\ I_{G2}(t = 2\tau) \end{bmatrix} = \begin{bmatrix} -\frac{E_0}{Z_1} \\ 0 \end{bmatrix} \quad (89)$$

At this instant, new progressive waves are generated, which reach the end of the line at the instant  $t = 3\tau$ , such that:

$$\begin{bmatrix} \Delta V_{1+} \\ \Delta V_{2+} \end{bmatrix} = \begin{bmatrix} \Delta V_{G1}(t = 2\tau) \\ \Delta V_{G2}(t = 2\tau) \end{bmatrix} - \begin{bmatrix} \Delta V_{1-} \\ \Delta V_{2-} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (90)$$

or

$$\begin{bmatrix} \Delta V_{1+} \\ \Delta V_{2+} \end{bmatrix} = - \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (91)$$

The new voltages are obtained from:

$$-2 \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} - \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} \quad (92)$$

Remembering that, as the line is open:

$$\begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

so we have:

$$\begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = -2 \begin{bmatrix} E_0 \\ \frac{Z_m}{Z_1} E_0 \end{bmatrix} \quad (93)$$

Thus, the new voltages at the end of the transmission line, at this instant, will be:

$$\begin{bmatrix} V_{L1}(3\tau) \\ V_{L2}(3\tau) \end{bmatrix} = \begin{bmatrix} V_{L1}(\tau) \\ V_{L2}(\tau) \end{bmatrix} + \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (94)$$

The graphs in Figure 7 show the evolution of the currents at the source and the voltage at the terminals of conductor 1.

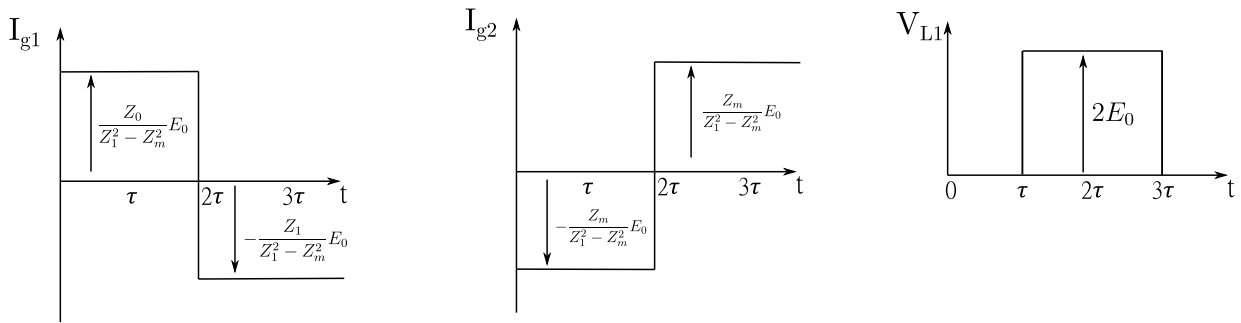


Fig. 7. Currents and voltage waveforms at both ends for the second elementary case.

#### IV. GENERAL CASE

Figure 8 shows a generic Three-conductor line connecting the source, fault location, to the load. The modeled fault is an ideal short-circuit and it is further replaced by a generic impedance between the same terminals. This can cause a device to malfunction or even fail because it can cause an electrical current to take a path that it is not supposed to, which can lead to damage to equipment. A short circuit is modeled as an ideal fault between node 1 and the reference as shown in the figure. The second terminal can be open-circuited between the reference terminal or an impedance can be coupled as well.

A traveling wave is a wave that propagates through a medium, such as a transmission line. The wave travels through the medium at a certain speed and its amplitude and phase change as it moves. In this General Case, we first represent a zero-impedance defect which leads the traveling wave to be fully reflected, and further a general impedance network. At the second terminal, we can observe the coupling effect between the energized line and a passive one.

At the instant immediately after the fault, the progressive waves of voltage and current are  $[\mathbf{V}_+]$  and  $[\mathbf{I}_+]$ . These quantities are imposed by the characteristics of the defect.

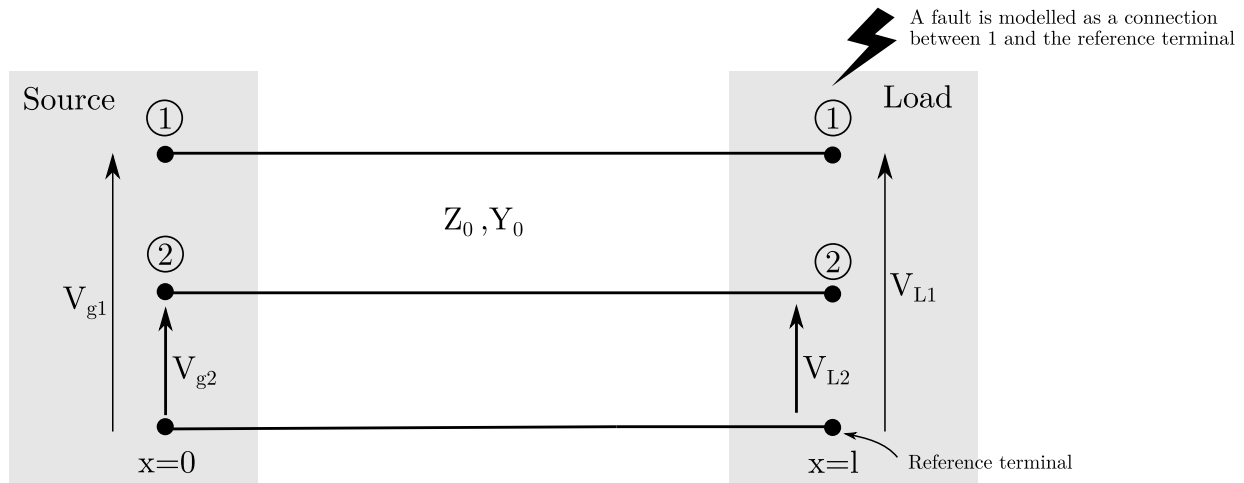


Fig. 8. General three-conductor transmission line

After a transit time, that is, in  $t = \tau$ , these waves reach the end of the line, when, due to electrical discontinuity, the regressive waves are generated in such a way that:

$$[I_+] + [I_-] = [I_L] \quad (95)$$

In which  $[V_L]$  and  $[I_L]$  are the voltages and currents at the load. From (40) and (51), we can write:

$$[Y_0] \cdot [V_+] - [Y_0] \cdot [V_-] = [I_L] \quad (96)$$

or also:

$$[V_+] - [V_-] = [Y_0]^{-1} [I_L] \quad (97)$$

Adding these two expressions gives:

$$2[V_+] - [Y_0]^{-1} [I_L] = [V_L] \quad (98)$$

We can associate this last result with a generalized electrical circuit, composed of a Thévenin equivalent generator connected to the load at the end of the line.

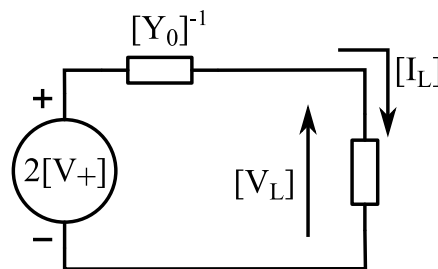


Fig. 9. Generalized equivalent Thévenin of LT in charge

The set of reflected voltages that return to the generator is such that:

$$[V_-] = [V_L] - [V_+] \quad (99)$$

This new set of voltages that act on the generator at the instant  $t = 2\tau$  composes a new Thevenin equivalent in the generator. This new set of voltages that act on the generator at the instant  $t = 2\tau$  composes a new Thevenin equivalent in the generator.

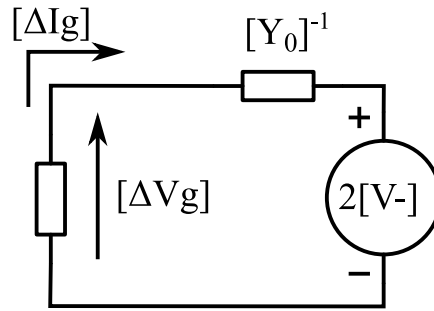


Fig. 10. Generalized equivalent Thévenin of LT in generator

Deactivating the generators, the set of voltage increases at the generator terminals and the new set of progressive voltages that return to the load are determined.

$$2[V_-] + [Y_0]^{-1}[I_G] = [V_G] \quad (100)$$

$$[V_+] = [V_G] - [V_-] \quad (101)$$

#### A. Case Study

This is a typical case of impedance grounding of shields. In this case, a voltage step of 1000V is applied to the terminals of one of the conductors, which, when propagating, induces voltages in conductor 2. The objective is to observe the evolution of voltages and currents in the conductors in the first moments of the transient. The parameters were chosen so that the propagation speeds of the conductors are equal.

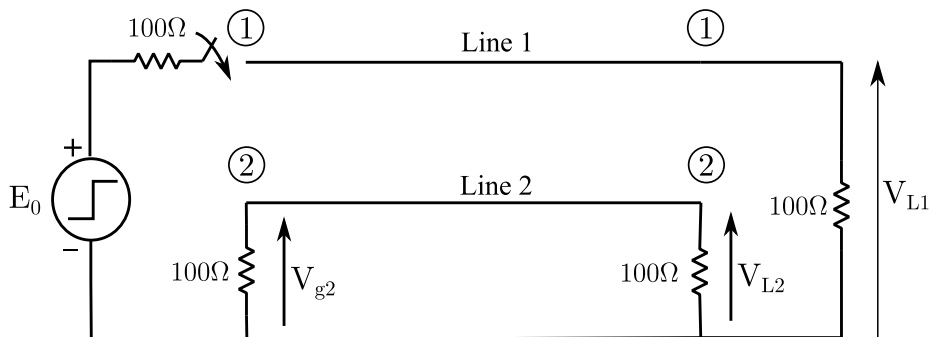


Fig. 11. Voltages and Currents at the ends of the lines

The line parameters are  $Z_{11} = 318\Omega$ ,  $Z_{m1} = 97.7\Omega$ ,  $Z_{22} = 294.3\Omega$ ,  $Z_{m2} = 106.5\Omega$  and the propagation speed is  $2.38 \times 10^{-8}$  m/s for both lines. The procedure begins by determining the voltages and currents injected into the line that will compose the progressive waves that are directed towards the load. As we only have traveling waves, we can write:

$$\begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{m1} \\ Z_{m2} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix}$$

therefore:

$$\begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} = \begin{bmatrix} 318 & 97.7 \\ 106.5 & 294.3 \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} \quad (102)$$

On the generator side, the relationship between these same quantities is extracted from the circuit in Figure 14.

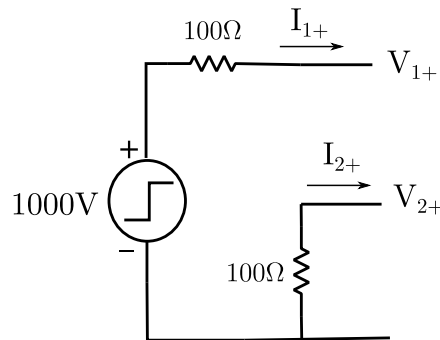


Fig. 12. Cálculo da tensão progressiva inicial.

We can write:

$$\begin{bmatrix} 1000 \\ 0 \end{bmatrix} - \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \cdot \begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} = \begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} \quad (103)$$

From (102) and (103) we extract:

$$\begin{bmatrix} I_{1+} \\ I_{2+} \end{bmatrix} = \begin{bmatrix} 2.553 \\ -0.690 \end{bmatrix} \quad \begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} = \begin{bmatrix} 744.645 \\ 68.971 \end{bmatrix}$$

The voltages and currents determined are also the voltages and currents at the beginning of the transmission line, so we can write that:

$$\begin{bmatrix} I_{G1}(0) \\ I_{G2}(0) \end{bmatrix} = \begin{bmatrix} 2.553 \\ -0.690 \end{bmatrix} \quad \begin{bmatrix} V_{G1}(0) \\ V_{G2}(0) \end{bmatrix} = \begin{bmatrix} 744.645 \\ 68,971 \end{bmatrix}$$

At the instant  $t = \tau$  these voltages and currents reach the load and will cause a disturbance in these quantities such that:

$$2 \begin{bmatrix} V_{1+} \\ V_{2+} \end{bmatrix} - \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix}$$

that is

$$2 \begin{bmatrix} 744.645 \\ 68.971 \end{bmatrix} - \begin{bmatrix} 318 & 97.7 \\ 106.5 & 294.3 \end{bmatrix} \begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix} = \begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix}$$

To calculate the current and voltage increments we Analyze the travelling wave load circuit.

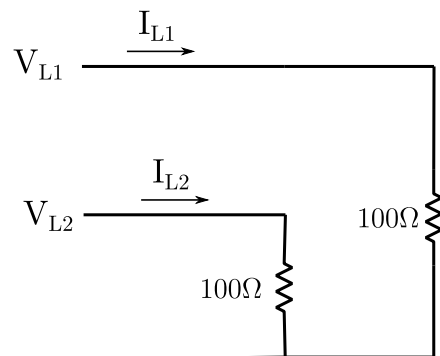


Fig. 13. Load side circuit viewed by the travelling wave.



where the voltage and current increment is then calculated by such circuit.

$$\begin{bmatrix} \Delta V_{L1} \\ \Delta V_{L2} \end{bmatrix} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{bmatrix} \Delta I_{L1} \\ \Delta I_{L2} \end{bmatrix}$$

The regressive voltage and current waves generated at this instant are such that:

$$\begin{bmatrix} \Delta V_{-}(\tau) \\ \Delta V_{-}(\tau) \end{bmatrix} = \begin{bmatrix} \Delta V_{L1}(\tau) \\ \Delta V_{L2}(\tau) \end{bmatrix} - \begin{bmatrix} V_{1+}(0) \\ V_{2+}(0) \end{bmatrix}$$

Substituting their values gives:

$$\begin{bmatrix} \Delta V_{-}(\tau) \\ \Delta V_{-}(\tau) \end{bmatrix} = \begin{bmatrix} -373.075 \\ -134.348 \end{bmatrix}$$

At  $2\tau$  these voltages and currents reach the generator, disturbing their magnitudes, so that applying the principle of line-side superposition we can write:

$$2 \begin{bmatrix} \Delta V_{1-}(\tau) \\ \Delta V_{2-}(\tau) \end{bmatrix} + \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta I_{G1}(2\tau) \\ \Delta I_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} \Delta V_{G1}(2\tau) \\ \Delta V_{G2}(2\tau) \end{bmatrix}$$

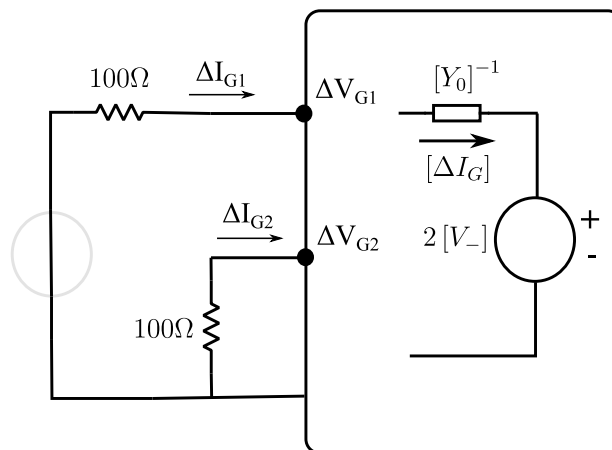


Fig. 14. Illustration of the superposition principle applied to the problem.

Substituting the known quantities for their values, results:

$$2 \begin{bmatrix} -373.075 \\ -134.348 \end{bmatrix} + \begin{bmatrix} 318 & 97,7 \\ 106,5 & 294,3 \end{bmatrix} \begin{bmatrix} \Delta I_{G1}(2\tau) \\ \Delta I_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} \Delta V_{G1}(2\tau) \\ \Delta V_{G2}(2\tau) \end{bmatrix}$$

On the generator side it results:

$$\begin{bmatrix} \Delta V_{G1}(2\tau) \\ \Delta V_{G2}(2\tau) \end{bmatrix} = - \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{bmatrix} \Delta I_{G1}(2\tau) \\ \Delta I_{G2}(2\tau) \end{bmatrix}$$

Being the sources deactivated in for the superposition, we can apply the increments to the former currents and voltages.

$$\begin{bmatrix} I_{G1}(2\tau) \\ I_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} I_{G1}(0) \\ I_{G2}(0) \end{bmatrix} + \begin{bmatrix} \Delta I_{G1}(2\tau) \\ \Delta I_{G2}(2\tau) \end{bmatrix} \quad \begin{bmatrix} V_{G1}(2\tau) \\ V_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} V_{G1}(0) \\ V_{G2}(0) \end{bmatrix} + \begin{bmatrix} \Delta V_{G1}(2\tau) \\ \Delta V_{G2}(2\tau) \end{bmatrix}$$

The generator voltages at the time  $t = 2\tau$  are then

$$\begin{bmatrix} V_{G1}(2\tau) \\ V_{G2}(2\tau) \end{bmatrix} = \begin{bmatrix} 571.112 \\ 47.697 \end{bmatrix}$$

The new progressive wave is given by;

$$\begin{bmatrix} V_{+1}(2\tau) \\ V_{+2}(2\tau) \end{bmatrix} = \begin{bmatrix} \Delta V_{G1}(2\tau) \\ \Delta V_{G2}(2\tau) \end{bmatrix} - \begin{bmatrix} \Delta V_{1-}(\tau) \\ \Delta V_{2-}(\tau) \end{bmatrix} = \begin{bmatrix} 199.542 \\ 113.074 \end{bmatrix}$$

At  $t = 3\tau$  the disturbance generated at the beginning of the line reaches its terminals. Following the same procedure adopted at the instant  $t = \tau$ , we obtain:

$$2 \begin{bmatrix} 199.542 \\ 113.074 \end{bmatrix} - \begin{bmatrix} 318 & 97,7 \\ 106,5 & 294,3 \end{bmatrix} \begin{bmatrix} \Delta I_{L1}(3\tau) \\ \Delta I_{L2}(3\tau) \end{bmatrix} = \begin{bmatrix} \Delta V_{L1}(3\tau) \\ \Delta V_{L2}(3\tau) \end{bmatrix} \quad (104)$$

and

$$\begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{bmatrix} \Delta I_{L1}(3\tau) \\ \Delta I_{L2}(3\tau) \end{bmatrix} = \begin{bmatrix} \Delta V_{L1}(3\tau) \\ \Delta V_{L2}(3\tau) \end{bmatrix} \quad (105)$$

Finally, from (104) and (105) we have:

$$\begin{bmatrix} V_{L1}(3\tau) \\ V_{L2}(3\tau) \end{bmatrix} = \begin{bmatrix} V_{L1}(\tau) \\ V_{L2}(\tau) \end{bmatrix} + \begin{bmatrix} \Delta V_{L1}(3\tau) \\ \Delta V_{L2}(3\tau) \end{bmatrix} = \begin{bmatrix} 459.170 \\ -65.376 \end{bmatrix}$$

The same procedure repeats, and we perform calculations until we achieve the steady state. As we are dealing with a process that can be automatized, we developed a software to calculate the voltage and currents for the Multiconductor Transmission Line. One can see in both Figures 15 and 16 the generator and load voltages along several transit times. Such an automation facilitates to enable this paper approach to be used in real world problems.

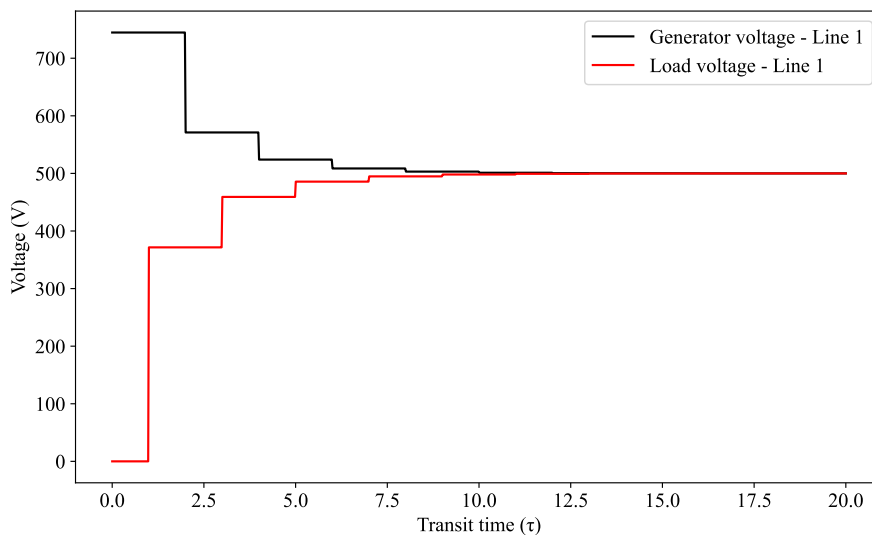


Fig. 15. Voltage transients for Case Study's line 1.

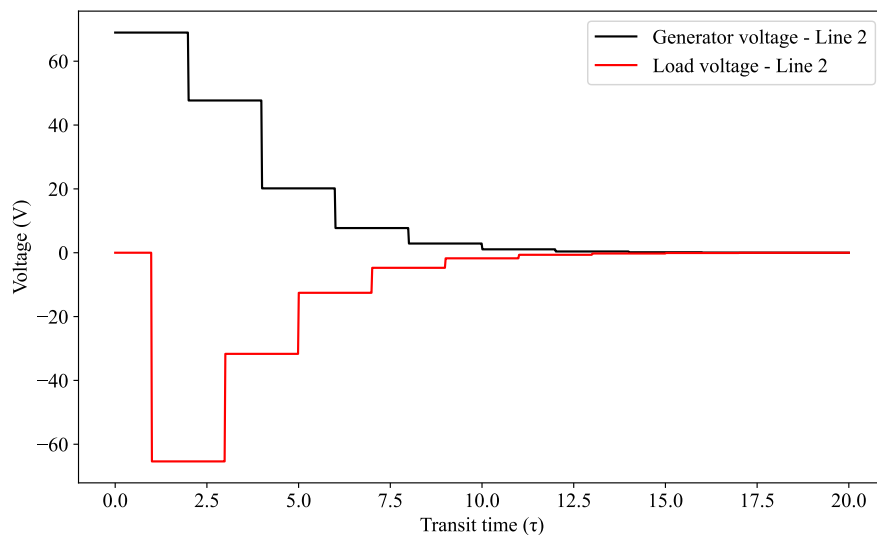


Fig. 16. Voltage transients for Case Study's line 2.

## CONCLUSION

The transmission line theory is a critical tool for understanding and designing electrical systems. It can be used to predict and mitigate the effects of electromagnetic interference, and to optimize the performance of electrical systems. We have studied in this paper the 3-conductor case, by an analytical formulation and also generalizing such solution by a software.

We have also testes such procedure with adaptations to the single phase transmission line and the results were in perfect agreement with the literature, such as [6]. This approach will make possible to the "increment" transient analysis to include multiconductor transmission lines, being possible to include capacitors and inductors as loads and to simulate non-linear loads.

This paper formulation was also used for a software to compute the transients automatically. We intend to expand such software to analyze transients of a general form waveform, apply it to an n-conductor line such as a printed circuit board. Another direct application of this paper is to evaluate the grounding of shielded cables.

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