Statistical Modeling of Rain Attenuation in Tropical Terrestrial Links

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***Abstract*— This paper presents a statistical modeling of rain attenuation in terrestrial links located in a tropical region using different types of distribution. Each distribution is tested against long-term statistics of data collected during up to two years of experimental measurements in links operating at 15 GHz. Statistics obtained from each distribution are tested against the corresponding ones derived from the experimental data using a test variable described in the ITU recommendation P.311-13 and an alternative test variable.**

***Index Terms*—** radio propagation, rain attenuation, statistical modeling, terrestrial links.

I. INTRODUCTION

Radio links operating at frequencies above 10 GHz are subject to severe propagation impairments. The main cause of unavailability in wireless systems operating outdoor at these frequencies is the attenuation caused by rainfall. To counteract rain effect, different fade mitigation techniques (FMT) may be used: adaptive transmission power control, route diversity or adaptive modulation and coding schemes. An accurate prediction of the attenuation caused by rain is essential for the design of these systems.

To predict the percentage of time during which a system will be unavailable it is necessary to study the stationary behavior of the rain attenuation in terms of the percentage of time that a specific level of attenuation is exceeded. In this case, the knowledge of the statistics of rain attenuation is enough. However, to develop and test FMT, the dynamic behavior of rain attenuation must be known. Unfortunately, long-term propagation measurements of this effect are scarce and not available for any location, frequency and link geometry. Prediction models based in the statistical modeling of rain attenuation must be employed. Among the prediction models, there are the purely stochastic ones, for which statistical modeling is indispensable since it provides statistics parameters to feed them.

In the seventies, S. H. Lin [1] showed that the conditional long-term distribution of rain attenuation is approximately lognormal. The study was based on thirty-one sets of experimental data collected in terrestrial and earth-space links operating above 10 GHz in five different countries. Since then, the

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assumption of a lognormal behavior of rain attenuation for both slant paths [2]-[5] and terrestrial links [6] has widely been adopted. This assumption provides good results for many temperate climates.

There are, however, important exceptions and other distributions must be considered. In [7]-[8], the Weibull distribution is used to model rain attenuation in slant paths, whereas [8]-[9] consider Gamma distribution. The Lognormal distribution is used to model unconditional attenuation distributions for satellite and terrestrial links in tropical regions in [10] and [11], respectively, and a Bi-Lognormal distribution was analyzed in [12] for slant-paths, but there is evidence that the Gamma distribution performs better for terrestrial links in such regions [13]. The synthesizers models in these references generate time series containing clear sky and rain attenuation periods and unconditional distributions are used to derive statistics parameters.

Recently, the Inverse Gaussian distribution has been proposed by [14] to model the conditional distribution of rain attenuation in earth-space and terrestrial links located in both temperate and tropical climates.

In this paper, several types of distribution representing the statistical behavior of rain attenuation in terrestrial links are examined and tested using long-term experimental data obtained from five links converging at a tropical site [15]. The statistical modeling considers the experimental unconditional distributions of rain attenuation in order to help the development of time series synthesizers. Also, a new test variable [16] is used to evaluate the errors between predicted and experimental statistics of rain attenuation.

II. EXPERIMENTAL SETUP

The experimental data of rain attenuation were collected in five convergent line-of-sight terrestrial links located in São Paulo, Brazil, for periods between one and two years depending on the link. The links characteristics are given in Table I [15] and their spatial distribution is shown in Fig.1.

TABLE I. LINK CHARACTERISTICS

Link	Path length (km)	Frequency (GHz)	Sampling rate (Hz)	Time period (months)	Up-time (%)
BD	12.8	14.55	0.1	24	88.6
CN15	12.8	14.55	0.1	24	91.8
SC	18.4	14.50	0.1	12	90.6
BA	21.7	14.53	0.1	12	89.2
PR	43.0	14.52	1.0	24	91.9

The experimental setup included a tipping bucket raingauge located at the common end of the links, with 0.1 mm capacity, and a data acquisition unit that samples and records the AGC voltage of each receiver, as well as the date and time of each raingauge tip. Calibration curves obtained previously enable to convert AGC voltage data in received power levels. Further processing leads to the corresponding rain attenuation time series values. The raingauge data helps to identify the dynamic behavior of rain-fading events and to eliminate power level variations not related to rainfall.

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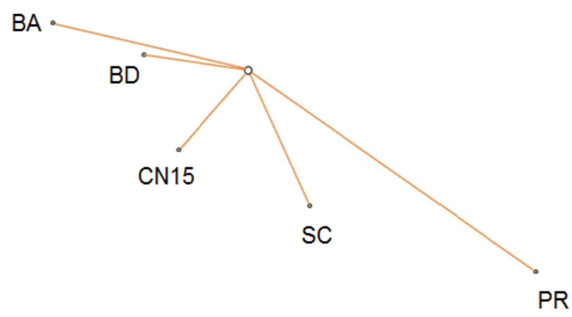

Fig. 1. Spatial distribution of the convergent links.

III. STATISTICAL MODELING

The lognormal behavior of rain attenuation might be justified, according to Lin [1], considering that the attenuation $A(t)$ is influenced by many effects of the propagation medium. These effects can be modeled by random time-varying multiplicative functions that depend on climatologic characteristics of the link site.

(1)

where each $\{S_i\}$, with $1 \leq i \leq n$, represents one random influence of an environmental parameter [1].

Taking logarithms on both sides of (1) it is possible to observe that *log A* is a sum of many random variables. If the number of variables is large and there is no dominant influence, the Central Limit Theorem states that the distribution of *log A* approaches a Normal distribution which means that the rain attenuation is log normally distributed.

However, the best performance of Gamma distribution for terrestrial links located in tropical regions when compared to Lognormal [13] suggests that, for high intensity of rainfall regimes, there may exist a dominant component in the attenuation effect.

Besides the Lognormal (Ln) and Gamma (Ga), four additional types of distribution were used to model the measured rain attenuation statistics: Pareto (Pa), Weibull (We), Inverse Gaussian (IG) and Nakagami (Na) [17]. Only the two-parameter versions of these distributions were considered. Table II presents the Probability Density Functions (PDF) of those distributions.

The parameters of each type of distribution are derived from the experimental unconditional long-term Complementary Cumulative Distribution Function (CCDF) of rain attenuation by a curve fitting in specific ranges of time percentage that minimizes the r.m.s error in dB between experimental and theoretic attenuation CCDFs. Since long-term cumulative statistics are being considered, the observation time is an integer multiple of 12 months as recommended by ITU [18].

The upper limit of the range of time percentage used in the curve fitting is 5%, for which attenuation levels are always higher than 1 dB. The minimum value of time percentage was chosen according to the margin of each link: 0.01% for BD link, 0.02% for CN15, 0.03% for BA, 0.05% for

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SC and 0.1% for PR.

TABLE II. STATISTICAL DISTRIBUTIONS

Distribution	Probability density function (PDF)	Parameters
Gamma	$p_{Gamma}(x) = \frac{1}{b^c \Gamma(c)} x^{c-1} e^{\frac{-x}{b}}$	$x \geq 0, b > 0, c > 0$, $\Gamma(c)$ is the Gamma function
Inverse Gaussian	$p_{InvGaussian}(x) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} e^{\left(\frac{-\lambda(x-\mu)^2}{2\mu^2 x}\right)}$	$x > 0, \mu > 0, \lambda > 0$
Lognormal	$p_{Lognormal}(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\left(\frac{-(log(x/m))^2}{2\sigma^2}\right)}$	$x \geq 0, m > 0, \sigma > 0$
Nakagami	$p_{Nakagami}(x) = \frac{2\mu^\mu}{\Gamma(\mu)\omega^\mu} x^{2\mu-1} e^{\frac{-\mu}{\omega}x^2}$	$x > 0, \mu > 0.5, \omega > 0$ $\Gamma(\mu)$ is the Gamma function
Pareto	$p_{Pareto}(x) = \frac{c.a^c}{x^{c+1}}$	$x \geq a, a > 0, c > 0$,
Weibull	$p_{Weibull}(x) = \frac{\beta}{\eta^\beta} x^{\beta-1} e^{-\left(\frac{x}{\eta}\right)^\beta}$	$x \geq 0, \eta > 0, \beta > 0$

Three different fittings were done with the lognormal distribution (Ln1, Ln2 and Ln3). In the case of Ln1, a linear fitting of the logarithm of attenuation CCDF was performed [3]. Ln2 is similar to Ln3, but an attenuation offset parameter is included in the fitting [11].

The fitted distributions were compared to the experimental ones at time percentages [0.01; 0.02; 0.03; 0.05; 0.1; 0.2; 0.3; 0.5; 1; 2; 3; 5] using the test variable V_i defined by ITU-R Rec. P.311-13 for each of the i radio links [18]:

$$V_i = \begin{cases} (A_{m,i}/10)^{0.2} \ln(A_{p,i}/A_{m,i}) & \text{for } A_{m,i} < 10\ dB \\ \ln(A_{p,i}/A_{m,i}) & \text{for } A_{m,i} \geq 10 \text{ dB} \end{cases} \tag{2}$$

where A_m (dB) is the measured attenuation and A_p (dB) is the predicted attenuation.

To evaluate the results over a range of time percentages, the r.m.s. value of all V_i values in the range is calculated. Two ranges of time percentage were considered: below 1% and below 5%. Table III shows the r.m.s. values obtained for each distribution and each link.

The results indicate a better performance of the Gamma and Inverse Gaussian distributions for the range of time percentages is below 1%. If the range below 5% is considered, the Inverse Gaussian has the best performance, except for SC link. It is possible to observe that the performance of the distributions, except Ln1, is degraded in this range. The measured values of exceeded attenuation for time percentages greater than 1% are always lower than 10 dB. The tapering factor of ITU test variable that reduces the influence of low attenuation levels was developed assuming a lognormal distribution of rain attenuation. Thus, the performance comparison of the different distributions is impaired in low attenuation levels range because ITU method will favour Lognormal distribution.

TABLE III. STATISTICAL MODELING – R.M.S. ERRORS

Link	**Curve fitting range (%)**	**Distribution**	**Curve fitting r.m.s error (dB)**	**ITU (below 1%)**		**ITU (below 5%)**		**Alternative method**		
				Range (%)	r.m.s.	Range (%)	r.m.s	Range (dB)	Range (%)	r.m.s
BD	0.01-5	Pa	2.24	0.01-1	0.20	0.01-5	0.40	2-32	0.01-1.80	0.75
		Ga	0.58		0.06		0.23	2-32	0.01-1.80	0.10
		We	1.28		0.14		0.23	2-32	0.01-1.80	0.32
		IG	0.71		0.08		0.08	2-32	0.01-1.80	0.15
		Na	0.82		0.21		0.41	2-32	0.01-1.80	0.21
		Ln1	4.78		0.19		0.17	2-32	0.01-1.80	0.28
		Ln2	1.21		0.15		0.17	2-32	0.01-1.80	0.29
		Ln3	1.67		0.17		0.31	2-32	0.01-1.80	0.46
CN15	0.02-5	Pa	1.92	0.02-1	0.13	0.02-5	0.30	2-32	0.03-2.91	0.60
		Ga	0.64		0.04		0.28	2-32	0.03-2.91	0.13
		We	1.01		0.09		0.14	2-32	0.03-2.91	0.21
		IG	0.55		0.05		0.06	2-32	0.03-2.91	0.08
		Na	1.13		0.16		0.58	2-32	0.03-2.91	0.29
		Ln1	3.13		0.13		0.11	2-32	0.03-2.91	0.17
		Ln2	0.93		0.09		0.13	2-32	0.03-2.91	0.17
		Ln3	1.38		0.11		0.22	2-32	0.03-2.91	0.33
BA	0.03-5	Pa	1.78	0.03-1	0.13	0.03-5	0.31	2-32	0.05-2.45	0.45
		Ga	0.76		0.08		0.38	2-32	0.05-2.45	0.16
		We	0.91		0.08		0.12	2-32	0.05-2.45	0.15
		IG	0.49		0.04		0.10	2-32	0.05-2.45	0.07
		Na	1.28		0.26		0.56	2-32	0.05-2.45	0.31
		Ln1	3.05		0.11		0.09	2-32	0.05-2.45	0.11
		Ln2	0.92		0.09		0.12	2-32	0.05-2.45	0.15
		Ln3	1.25		0.10		0.21	2-32	0.05-2.45	0.25
SC	0.05-5	Pa	1.30	0.05-1	0.09	0.05-5	0.22	2-32	0.05-2.86	0.26
		Ga	1.03		0.10		0.47	2-32	0.05-2.86	0.21
		We	0.76		0.06		0.05	2-32	0.05-2.86	0.08
		IG	0.68		0.05		0.17	2-32	0.05-2.86	0.12
		Na	1.42		0.27		0.66	2-32	0.05-2.86	0.34
		Ln1	1.47		0.06		0.06	2-32	0.05-2.86	0.08
		Ln2	0.82		0.06		0.15	2-32	0.05-2.86	0.09
		Ln3	0.94		0.07		0.11	2-32	0.05-2.86	0.12
PR	0.1-5	Pa	2.06	0.1-1	0.08	0.1-5	0.27	3-32	0.15-3.55	0.40
		Ga	0.48		0.03		0.15	3-32	0.15-3.55	0.05
		We	1.10		0.05		0.14	3-32	0.15-3.55	0.15
		IG	0.61		0.03		0.06	3-32	0.15-3.55	0.07
		Na	1.10		0.04		0.43	3-32	0.15-3.55	0.13
		Ln1	3.15		0.12		0.10	3-32	0.15-3.55	0.11
		Ln2	0.78		0.04		0.12	3-32	0.15-3.55	0.09
		Ln3	1.50		0.06		0.20	3-32	0.15-3.55	0.24

The ITU test variable considers the ratio between measured and predicted attenuations to evaluate the accuracy of a prediction method. However, as pointed out by [16], "*links are designed to meet availability objectives, usually expressed in terms of percentages of one year or one month, i.e., percentages of time*". Considering that, an alternative test variable proposed in [16] was also considered:

$$V_{i2}(A) = \log\left(\frac{P_p(A)}{P_m(A)}\right) \tag{3}$$

where P_m is the measured exceedance probability and P_p is the predicted exceedance probability for each attenuation threshold A.

To evaluate the results over a range of attenuation levels, the r.m.s. value of all V_{i2} values in the range is calculated.

The alternative test variable requires values for measured and predicted exceedance probabilities associated to an attenuation threshold. To apply the test variable to ITU databanks it is necessary to invert the ITU prediction method which can be done using numerical methods [16].

The main advantage of the alternative test variable is to test the performance of different prediction methods using experimental data with different range of time percentages.

The attenuation levels [2; 3; 4; 5; 6; 8; 10; 12; 15; 17.5; 20; 22.5; 25; 27.5; 30; 32] were considered, except for the PR link which starts at 3 dB level since 2 dB corresponds to a time percentage out of the range used in the fitting.

Table III shows the r.m.s. values for each distribution and each link. The results indicate a better performance of Inverse Gaussian and Gamma distributions.

Table IV shows a comparison of the overall performance of the various distributions considering all 5 links. The Inverse Gaussian distribution has the best overall performance followed by Gamma, which does not perform so well for attenuation levels lower or equal to 4 dB.

TABLE IV. COMPARISON OF OVERALL PERFORMANCE WITH NEW TEST VARIABLE

Att. level (dB)	Pa	Ga	We	IG	Na	Ln1	Ln2	Ln3
2	1.45	0.45	0.37	0.19	0.84	0.03	0.21	0.69
3	1.02	0.27	0.33	0.14	0.57	0.05	0.24	0.56
4	0.76	0.18	0.32	0.12	0.38	0.05	0.27	0.47
5	0.61	0.10	0.30	0.14	0.23	0.06	0.29	0.41
6	0.45	0.09	0.25	0.13	0.15	0.08	0.27	0.33
8	0.24	0.10	0.19	0.12	0.03	0.14	0.22	0.22
10	0.06	0.08	0.09	0.07	0.05	0.19	0.12	0.08
12	0.10	0.08	0.05	0.06	0.08	0.22	0.07	0.06
15	0.22	0.06	0.11	0.06	0.09	0.24	0.08	0.15
17.5	0.26	0.05	0.14	0.07	0.09	0.21	0.12	0.18
20	0.28	0.04	0.16	0.09	0.07	0.18	0.14	0.21
22.5	0.27	0.06	0.17	0.10	0.05	0.13	0.16	0.21
25	0.23	0.06	0.14	0.09	0.04	0.07	0.14	0.18
27.5	0.15	0.05	0.09	0.06	0.04	0.11	0.09	0.11
30	0.07	0.03	0.04	0.03	0.04	0.22	0.04	0.05
32	0.07	0.03	0.05	0.04	0.05	0.31	0.05	0.06
2 to 32	0.52	0.14	0.20	0.10	0.27	0.17	0.18	0.30

The lognormal (Ln1) has the best performance for levels lower or equal to 6 dB, but does not perform so well for higher levels, whereas Nakagami has a good performance for levels greater or equal to 8 dB, but does not perform well for lower levels. When comparing the Inverse Gaussian and Lognormal (Ln1) distributions, the results confirm the ones obtained for conditional distributions in [14].

The analyzed data provides no insight on a possible relation between the path-length and the most

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appropriate type of distribution.

Fig. 2 shows the curve fittings for BD link using Inverse Gaussian, Gamma and Lognormal (Ln1) distributions.

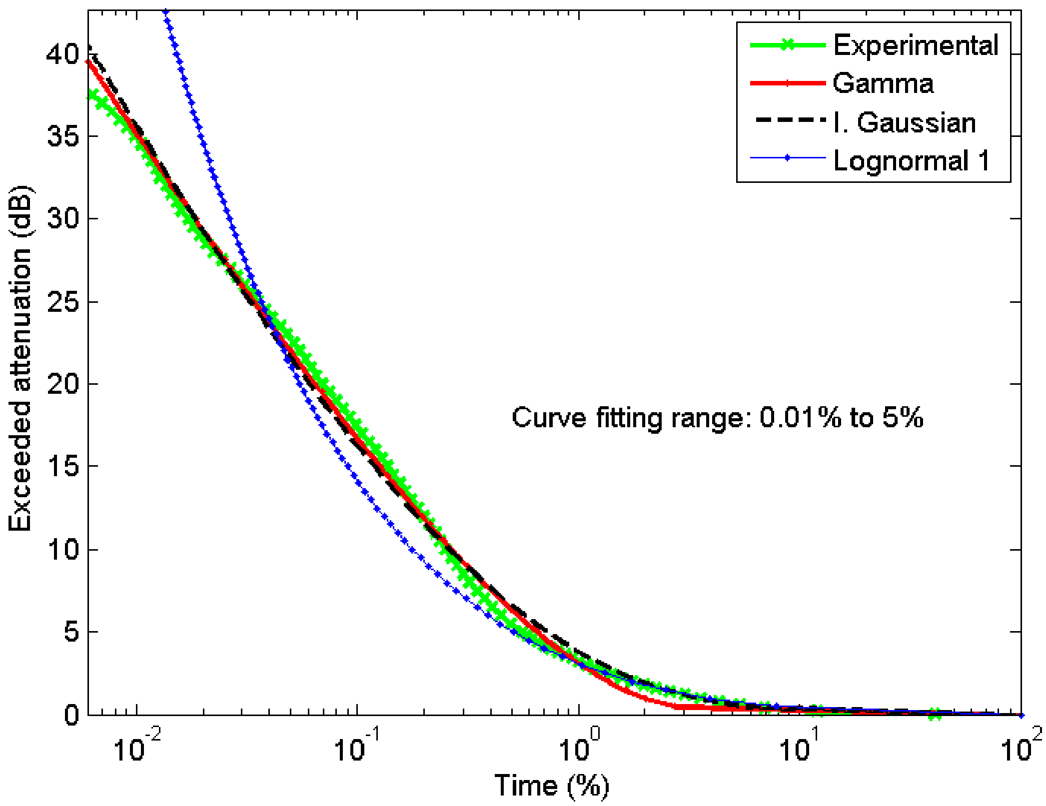

Fig. 2. Statistical modeling using Inverse Gaussian, Gamma and Lognormal (Ln1) distributions.

IV. CONCLUSION

The results indicate that Inverse Gaussian and Gamma are the most adequate distributions to model statistics of rain attenuation in terrestrial links located in a region with tropical climate. Both distributions perform well for medium and high levels of attenuation, but both the Inverse Gaussian and the Lognormal outperform the Gamma distribution at low levels.

A new test variable was used to evaluate the fitted attenuation distributions in comparison to measured ones. It has the advantage of evaluating the errors in terms of predicted unavailability time percentages, which corresponds the actual performance objectives in the process of radio links design.

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