

# Utilizing Shift-or-Shrink Method to Optimize Multisegment Structures of Bent Optical Waveguides

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**Abstract**— The authors utilized a heuristic shift-or-shrink method that is associated with the beam propagation method to design an optimal multisegment bent optical waveguide. By transforming the shape of each segment, the multisegment structure of the bent waveguide can be optimized. In addition to achieving maximal transmission efficiency, only four segments are sufficient to satisfactorily improve the transmission performance. Moreover, the optimal path of each of these multisegment bent waveguides remains similar to each other waveguide's optimal path even though the number of the segments increases.

**Index Terms**—shift-or-shrink method, beam propagation method, BPM, bent waveguide.

## I. INTRODUCTION

Optimization algorithms and their applications in various fields have been studied for a long time. One of the crucial optimization algorithms presented over the past two hundred years was the least-square approximation proposed by Legendre, which finds the most approximate solution for a set of equations. Currently, with the assistance of computers, a new family of stochastic optimization algorithms called evolutionary algorithms (EAs) has been created. The most well-known EA is the genetic algorithm (GA) [1], which contains three main classes of operations: selection, crossover, and mutation. Another familiar method is particle swarm optimization (PSO) [2], [3]. In PSO, each particle is stochastically accelerated toward its own extremum locations, that is, global or local extrema. Moreover, the present authors proposed the shift-or-shrink (SOS) method to obtain the optimal directional coupling efficiency from an optical fiber to a diffused waveguide [4] and attempted to find the solution of a calculus-of-variation equation by maximizing the propagation constant of the fundamental mode in an optical waveguide [5]. The proposed SOS algorithm performs the shift or the shrink operation in each round of the procedure. The SOS method is like a simplified version of the downhill simplex method [6]. The latter is more complicated because it includes as many as four different operations: reflection, expansion, contraction, and shrinking. Compared with the aforementioned optimization algorithms, the SOS method is more interpretable and easier in terms

of computer programming. For example, although the PSO algorithm is capable of optimizing the function of a higher number of variables, it requires much more complicated conditions for making decisions such that the entire execution duration consumes a higher amount of time. Conversely, the optimal solution can be quickly obtained by the proposed SOS method because it converges to a stable value just by executing a few rounds in simple loop structures. Furthermore, to obtain the global extrema of a function, the SOS method should be repeated by resetting the distinct initial guesses and then comparing the respective magnitudes of the results corresponding to these initial guesses. In this study, the authors utilized the SOS method to optimize a specific type of bent waveguide structure in photonic integrated circuits.

Photonic integrated circuitry manipulates lightwaves, rather than electrons or holes, to perform various optical functions. The circuitry integrates multiple photonic devices to control optical signals and provides optical paths to distribute lightwaves to different destinations. For designing photonic circuits, bent waveguide structures are necessary to connect optical paths in different directions. However, considerable power losses may occur at the bent corners and bent sections, thus lowering the transmission efficiencies of lightwaves. Some optimization methods have been proposed for minimizing the power losses due to the bent optical waveguides. For instance, Khanzadeh et al. proposed a square connection zone, which is divided into several smaller parts with variable refractive indices, that connects two orthogonal straight waveguides. They utilized the GA to search for the most appropriate refractive index distribution among these tiny parts within the connection zone such that their designed structure has the lowest bending loss [7]. Xiao et al. employed the random orthogonal axial gradient method to optimize the S-shaped optical waveguide bend for obtaining the most efficient transmission of lightwave [8]. Conversely, several researchers have designed optimal bent waveguides by using the beam propagation method (BPM) [9]-[12]. In this study, the SOS algorithm and the BPM were combined to design a new type of bent waveguide structure and optimized the structure. The design principle and numerical results are presented in the following paragraphs.

## II. DESCRIPTION OF SOS METHOD AND ITS APPLICATION IN OPTIMIZING MULTISEGMENT STRUCTURES OF BENT OPTICAL WAVEGUIDES

The basic principle of the SOS method is briefly described as follows. For searching the extremum of a multivariable function  $f(x_1, x_2, x_3, \dots, x_n)$ , a searching polyhedron can be established in the  $n$ -dimensional space. The space comprises one central point  $C$  and  $2n$  vertices, that is,  $P_1, P_2, \dots, P_{2n}$ , that are adjacent to  $C$ . The vector  $\vec{CP}_i$  is orthogonal to the vector  $\vec{CP}_j$  for  $i \neq j$ . Each vertex has the same distance  $h$  to  $C$ . If the initial value of  $h$  and the coordinate of the center  $C \equiv (x_1, x_2, x_3, \dots, x_n)$  are given, the coordinates of the  $2n$  vertices can be set as  $P_1 \equiv (x_1 + h, x_2, x_3, \dots, x_n)$ ,  $P_2 \equiv (x_1 - h, x_2, x_3, \dots,$

$x_n$ ),  $P_3 \equiv (x_1, x_2 + h, x_3, \dots, x_n)$ ,  $P_4 \equiv (x_1, x_2 - h, x_3, \dots, x_n)$ ,  $P_5 \equiv (x_1, x_2, x_3 + h, \dots, x_n)$ ,  $P_6 \equiv (x_1, x_2, x_3 - h, \dots, x_n)$ , ...,  $P_{2n-1} \equiv (x_1, x_2, x_3, \dots, x_n + h)$ , and  $P_{2n} \equiv (x_1, x_2, x_3, \dots, x_n - h)$ . After computing the  $2n + 1$  values of  $f(x_1, x_2, x_3, \dots, x_n)$  at the center and the total vertices of the searching polyhedron, the extreme value can be determined by comparing their respective values. In case the extreme value of  $f(x_1, x_2, x_3, \dots, x_n)$  lies on one of the vertices, the searching polyhedron is shifted such that this vertex becomes the new central point of the shifted polyhedron. Conversely, if the extreme value lies on the center of the searching polyhedron,  $h$  is shrunk by a factor such that all the vertices of the searching polyhedron uniformly move toward this center. Once the new searching polyhedron is generated, the next round of SOS procedure is conducted. The entire procedure is repeated until the final searching polyhedron converges to a tiny region (the value of  $h$  is smaller than a predetermined value that represents the precision). Meanwhile, the extremum of  $f(x_1, x_2, x_3, \dots, x_n)$  of the final searching polyhedron and the desired accuracy are obtained. The flowchart of the SOS algorithm is illustrated in Fig. 1. Especially, a two-dimensional (2D) searching polyhedron becomes a parallelogram in a plane, and the polyhedron is a square in the special case. For easier interpretation, the establishment of the searching polyhedron, the shift, and the shrinking procedure in the 2D case are displayed in Fig. 2(a)–(c), respectively.

Consider a paraboloid function  $z = f(x, y) = (x - 2)^2 + (y - 3)^2$  as an example for observing the SOS method operation. The authors selected an initial  $h$  value of 3 and the center of the initial searching square  $C$  as (5.2, 4.6). Then, a MATLAB program was developed based on the SOS algorithm to seek the minimum zero located at (2, 3). Let the integer  $N$  be labeled on the central point of the searching square in the  $n$ th round of the SOS procedure. Fig. 3 presents the locations of the searching square centers from the first round to the 11th round of the SOS procedure. For easier observation, all the vertices of the searching squares were omitted. It was found that these centers approach the location of the minimum of  $f(x, y) = (x - 2)^2 + (y - 3)^2$  gradually. Moreover, the searching square centers of the tenth and the 11th rounds of the SOS procedure are located in the vicinity of the coordinate (2, 3). Fig. 3 also presents that the shrink operation is from the third to the fourth rounds of the SOS procedure. Thus, the centers of the searching squares are maintained at the same coordinate. Moreover, the shrink operation is also from the fifth to the seventh rounds and from the eight to the ninth rounds of the SOS procedure. The other rounds of the SOS procedure are the shift operations; thus, the centers of the searching squares move toward the point (2, 3) of the function minimum.

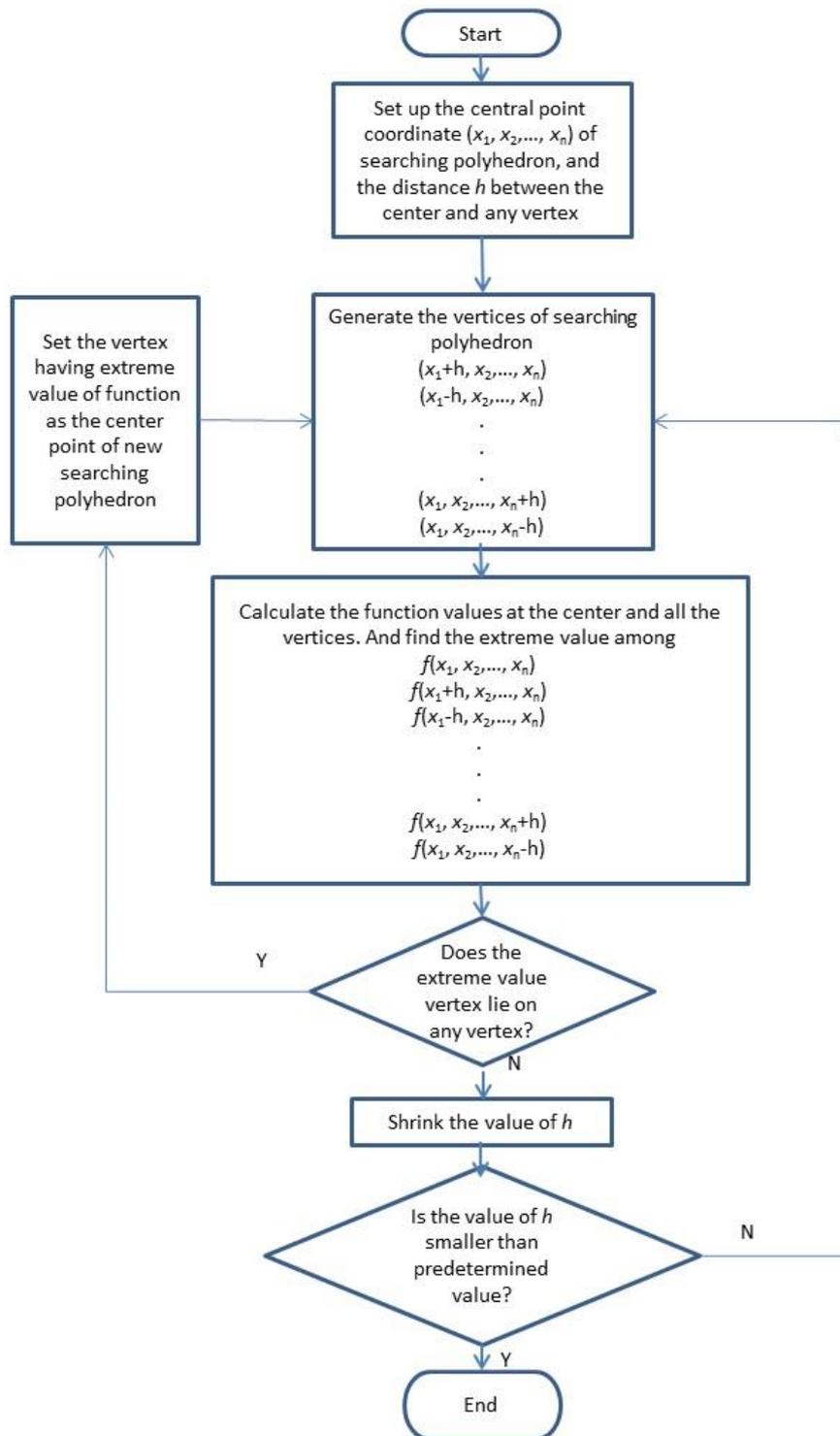


Fig. 1. Flowchart of the SOS algorithm.

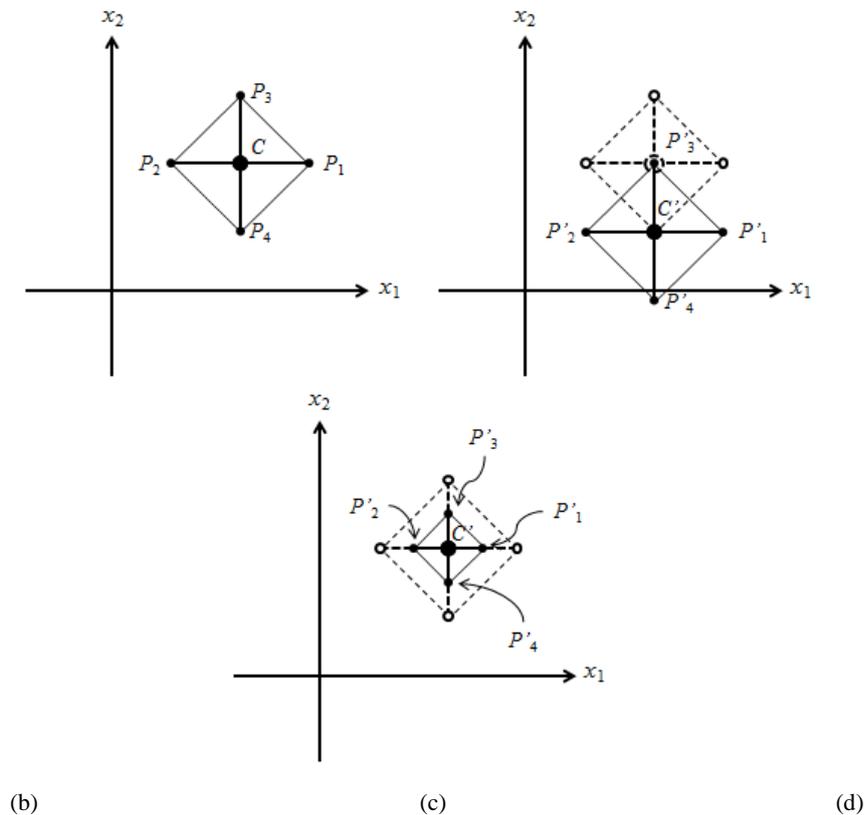


Fig. 2. (a) A searching polyhedron for obtaining the extremum of  $f(x_1, x_2)$  in the 2D space. In this special case, the searching polyhedron is a square. (b) Shift operation. The value of  $f(x_1, x_2)$  at the vertex  $P_4$  was assumed to be the extremum. The original searching square is depicted using dashed squares. The new square is depicted as a solid square. (c) Shrink operation. All the new vertices uniformly move toward the center.

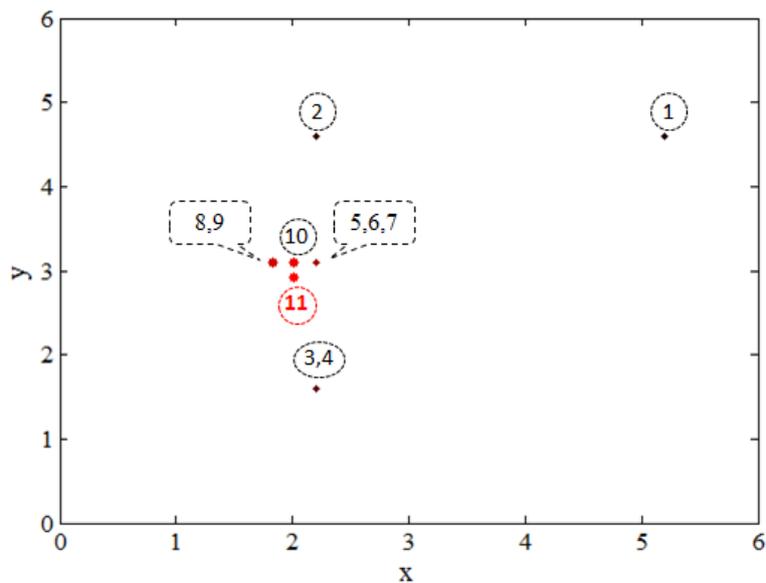


Fig. 3. Centers of the searching squares from the first round to the 11th round of the SOS procedure for determining the location of the minimum of  $f(x, y) = (x - 2)^2 + (y - 3)^2$ .

Consider the 2D conventional bent waveguide structure, as shown in Fig. 4(a). The structure has a straight-but-oblique section that is connected to two upright waveguides with the same width  $W$ . The refractive indices of the core region and the cladding region of the waveguide are  $n_1$  and  $n_2$ , respectively. The lateral displacement between the center of the input and that of the output waveguide is  $D$ , and the vertical offset between them is  $L$ . The loss of the conventional bent waveguide increases with the bent angle. Some improvements for enhancing the transmission performance have been proposed. For example, the  $S$ -shaped bent structure can be substituted for the straight-but-oblique section of the conventional bent waveguide to reduce the radiation power loss [7], [8]. For a practical fabrication, the  $S$ -shaped waveguide bend may have several imperfections such as nonuniformity and curvature radius deviation. In this study, the authors proposed an alternate design of the bent structure—a multisegment section of the bent waveguide. All these segments are parallelograms. Initially, the straight-but-oblique section was divided into  $2m$  equal segments ( $m$  is an arbitrary integer). Each segment is a parallelogram that has the same height  $L/2m$ . The bottom and the top sides of each parallelogram have an identical length  $W$  that is the same as the width of the upright input and output waveguides. Let  $C_j$  denote the center of the top of the  $j$ th parallelogram ( $1 \leq j \leq 2m$ ) and  $C_0$  represent the central point of the bottom in the first parallelogram. If the position of  $C_0$  is selected as the origin of the coordinate system, the initial coordinate  $C_j = (\frac{jD}{2m}, \frac{jL}{2m})$  is obtained, as presented in Fig. 4(a), when  $1 \leq j \leq 2m$ . For reducing the number of variables of the objective function to optimize the multisegment bent waveguide by using the SOS method, the authors proposed the following two regulations: (i)  $C_j$  can move horizontally during each round of the SOS procedure; however,  $C_0 = (0,0)$ ,  $C_m = (\frac{D}{2}, \frac{L}{2})$ , and  $C_{2m} = (D, L)$  are always fixed. As long as  $C_j$  ( $1 \leq j \leq 2m$  but  $j \neq 0, m, \text{ and } 2m$ ) moves in the horizontal direction, each parallelogram (segment) is transformed to its new shape. However, the width  $W$  and the height  $L/2m$  of the segment are invariant. Simultaneously, the multisegment structure of the bent waveguide changes to a new pattern. (ii) During each round of the SOS procedure for optimizing the multisegment waveguide structure, the horizontal movements of  $C_k$  and  $C_{2m-k}$  ( $1 \leq k \leq m-1$ ) are mutually dependent. Their movement distances are kept the same but are maintained antidiagonal to each other. This implies that  $C_{2m-k}$  moves from  $(\frac{(2m-k)D}{2m}, \frac{(2m-k)L}{2m})$  to  $(\frac{(2m-k)D}{2m} - \Delta_k, \frac{(2m-k)L}{2m})$  while  $C_k$  moves from  $(\frac{kD}{2m}, \frac{kL}{2m})$  to  $(\frac{kD}{2m} + \Delta_k, \frac{kL}{2m})$ , where  $\Delta_k$  is the respective increment and decrement (or decrement and increment) in the abscissa of  $C_k$  and  $C_{2m-k}$ . Based on the above two regulations, only  $(m-1)$  variables are required for a case that has  $2m$  parallelograms. In this case, it is necessary to compute  $2(m-1) + 1 = (2m-1)$  values for each searching polyhedron. Fig. 4(b) presents an example of the six-segment ( $m=3$ ) structures of bent optical waveguides. In this case, only  $C_1, C_2, C_4,$  and  $C_5$  move horizontally. Mutual dependence exists

between the horizontal movements of  $C_1$  and  $C_5$ . There is another mutual dependence between the horizontal movements of  $C_2$  and  $C_4$ . Each segment is transformed to a new shape, but the initial width  $W$  and initial height  $L/6$  are maintained during the SOS optimization process.

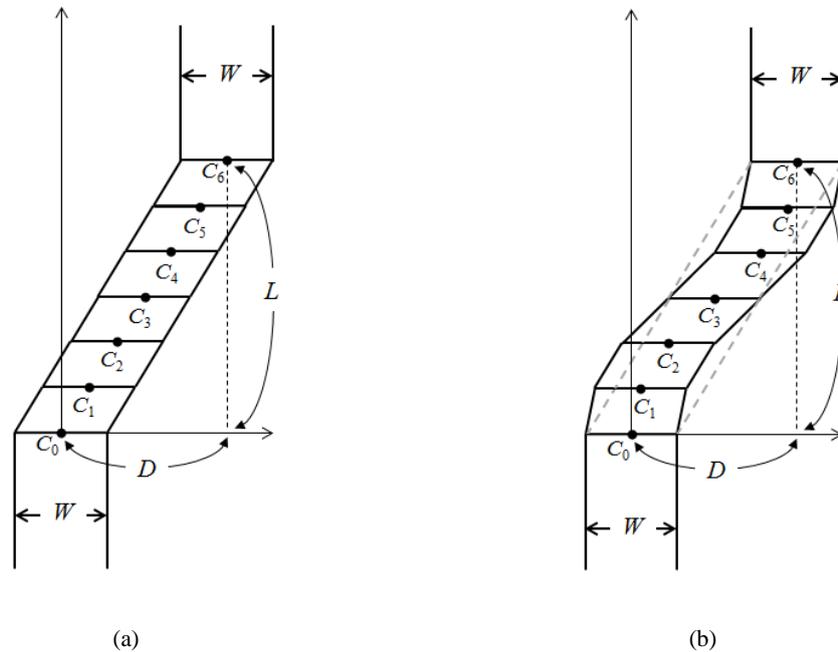


Fig. 4. (a) Conventional 2D optical waveguide. The straight-but-oblique section is connected to both upright input and output waveguides with the same width  $W$ . The straight-but-oblique section was divided into six ( $m = 3$ ) segments. (b)  $C_k$  and  $C_{6-k}$  ( $1 \leq k \leq 2$ ) move horizontally. Each segment is transformed into a new shape, but the initial width  $W$  and original height  $L/6$  are maintained. The dash lines in this subfigure represent the pattern of the initial and conventional 2D optical waveguide.

By using the SOS method and the previously mentioned regulations to optimize the multisegment bent optical waveguide structure, only the abscissas of  $C_k$  and  $C_{2m-k}$  ( $1 \leq k \leq m-1$ ) have to be determined in the case of  $2m$  segments because their respective abscissas are mutually dependent. Let  $C_0 = (0, 0)$ ,  $C_m = (\frac{D}{2}, \frac{L}{2})$ , and  $C_{2m} = (D, L)$  be invariant. The initial and the predetermined values of  $h$  can be set. Based on the SOS method, the selected  $(2m-1)$  initial guesses comprising the abscissas of  $C_1, C_2, \dots, C_{m-1}, C_{m+1}, \dots, C_{2m-1}$  for the optimization process are as follows:  $(\frac{D}{2m}, \dots, \frac{(m-1)D}{2m}, \frac{(m+1)D}{2m}, \dots, \frac{(2m-1)D}{2m})$ ,  $(\frac{D}{2m} + h, \dots, \frac{(m-1)D}{2m}, \frac{(m+1)D}{2m}, \dots, \frac{(2m-1)D}{2m} - h)$ ,  $(\frac{D}{2m} - h, \dots, \frac{(m-1)D}{2m}, \frac{(m+1)D}{2m}, \dots, \frac{(2m-1)D}{2m} + h)$ ,  $(\frac{D}{2m}, \dots, \frac{(m-1)D}{2m} + h, \frac{(m+1)D}{2m} - h, \dots, \frac{(2m-1)D}{2m})$ , and  $(\frac{D}{2m}, \dots, \frac{(m-1)D}{2m} - h, \frac{(m+1)D}{2m} + h, \dots, \frac{(2m-1)D}{2m})$ . The objective function for optimization is the transmission efficiency of the bent optical waveguide, which is defined as the ratio of the input and the output power. The BPM can be employed to simulate the incident optical beams propagating along these

multisegment structures of the bent waveguides and can calculate the corresponding transmission efficiencies. Initially, the authors used the aforementioned initial guesses comprising the abscissas of  $C_1, C_2, \dots, C_{m-1}, C_{m+1}, \dots,$  and  $C_{2m-1}$  to obtain the shape of each segment in the bent section and to determine the corresponding multisegment structure of the bent optical waveguide. Then, BPM was utilized to calculate the transmission efficiencies for these  $(2m - 1)$  cases and to determine the largest efficiency value among them. Subsequently, the next round of the SOS procedure was conducted. Finally, the entire procedure terminates when  $h$  becomes smaller than the predetermined value. When the maximal transmission efficiency is obtained, each segment (parallelogram) is transformed to its final shape and the optimal multisegment structure of the bent waveguide is determined. In this study, the authors developed an optimization program based on the SOS algorithm, combined with BPM, to obtain the most suitable abscissa of  $C_j$  ( $1 \leq j \leq m - 1$  or  $m + 1 \leq j \leq 2m - 1$ ) to form the optimal structure of the multisegment bent waveguide. The BPM simulation results of propagating lightwaves and transmission efficiencies under different conditions were obtained by using the Rsoft-BeamPROP software.

### III. NUMERICAL SIMULATION RESULTS

In the simulation, the following structural parameters of the multisegment bent optical waveguide were selected:  $n_1 = 1.502$ ,  $n_2 = 1.5$ ,  $W = 4 \mu\text{m}$ ,  $D = 20.95 \mu\text{m}$ , and  $L = 600 \mu\text{m}$ . The optical wavelength was  $\lambda = 0.633 \mu\text{m}$ , and the incident light beam was the fundamental mode the waveguide.

Fig. 5(a) presents the conventional bent waveguide structure. A straight-but-oblique section was connected to the two upright waveguides. The corresponding propagating lightwave intensity distribution simulated by the BPM is presented in Fig. 5(b). Two large power losses occur at the two bending corners. In this case, the transmission efficiency was only 24.1%. The optimal structure of the four-segment bent waveguide obtained when the straight-but-oblique section was divided into four segments ( $m = 2$ ) and optimized based on the proposed SOS algorithm and the two earlier mentioned regulations is presented in Fig. 5(c). The corresponding BPM simulation of the incident lightwave propagating along the optimal four-segment bent waveguide is displayed in Fig. 5(d). The improvement in reducing the optical power loss is very obvious. Due to this improvement, the transmission efficiency increases up to 61.1%, more than 2.5 times of that of the conventional bent structure. Fig. 5(e) demonstrates that the calculated transmission efficiencies obtained using the SOS method converge to the same maximum regardless of whether the initial value of  $h$  is  $2.4 \mu\text{m}$  or  $3.2 \mu\text{m}$ . The transmission efficiency can be obtained after the seventh round of the SOS procedure when the initial value of  $h$  is  $3.2 \mu\text{m}$ , and the identical transmission efficiency can be obtained after the fifth round of the SOS procedure when the initial value of  $h$  is  $2.4 \mu\text{m}$ . The same convergence for distinct initial values of  $h$  reveals that the SOS method is robust.

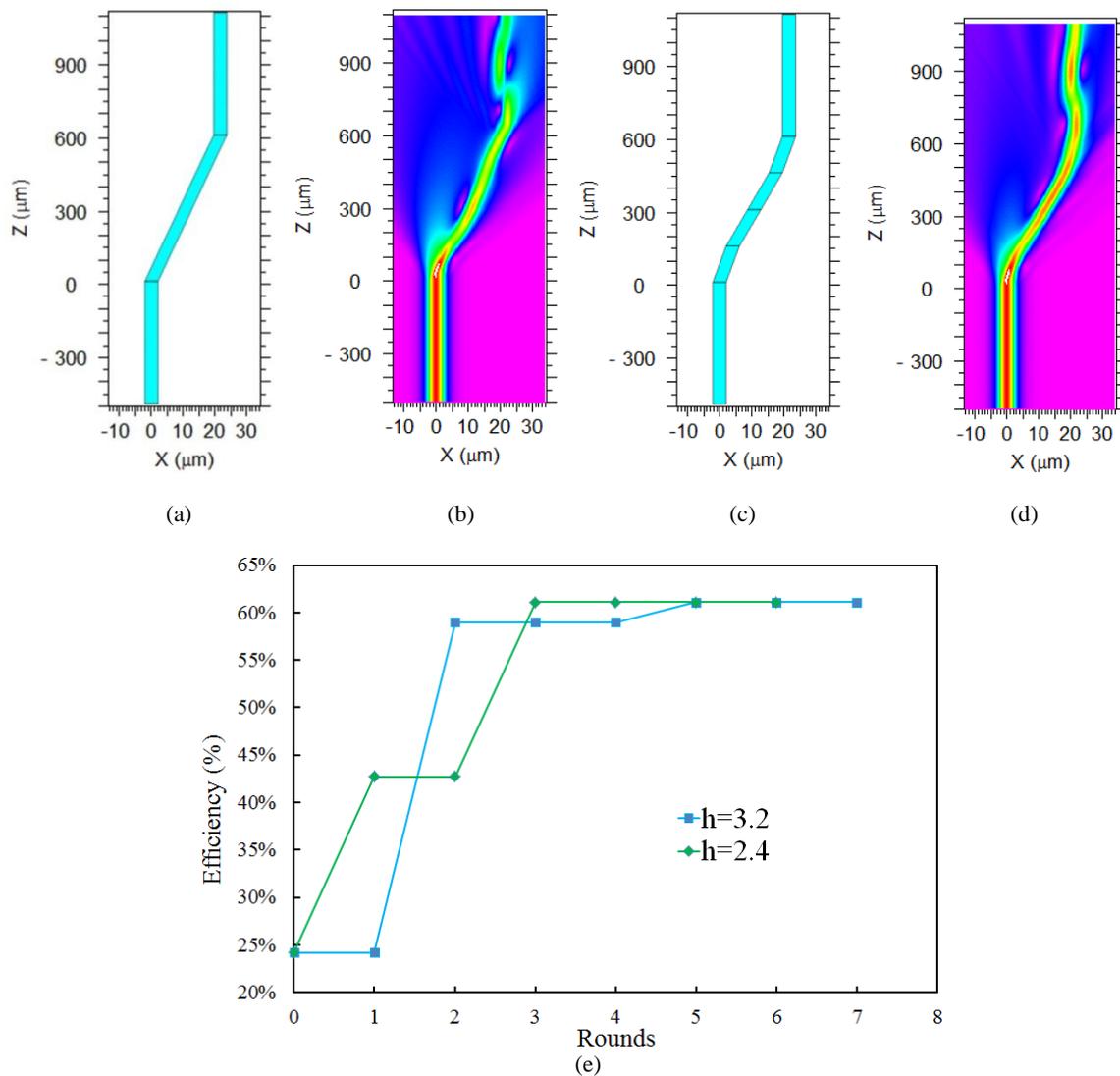


Fig. 5. (a) Initial bent waveguide structure. (b) BPM simulation of the propagating lightwave intensity distribution along the initial bent waveguide structure. (c) The optimal four-segment structure of the bent waveguide. (d) BPM simulation of the incident lightwave propagating along the optimal four-segment structure of the bent waveguide. (e) The transmission efficiencies calculated using the SOS method converges to the same maximum regardless of whether the initial value of  $h$  is  $3.2 \mu\text{m}$  (blue dot) or  $2.4 \mu\text{m}$  (green square).

By combining the SOS method and BPM, the left sides of Fig. 6(a)–(d) present the optimal six-segment, eight-segment, ten-segment, and 12-segment structures of bent waveguides, respectively. Moreover, the right sides of these subfigures reveal the respective BPM simulation results of incident lightwaves propagating along these different multisegment optimal bending structures. Compared with the optical power losses of the conventional bent waveguide structure, the losses of the proposed structure is significantly reduced at the two bending corners. The corresponding optimal transmission efficiencies are 60.8%, 62.4%, and 62.4%, and 63.7%, respectively. These optimal efficiencies are very near to one another even when the number of segments increases. This result reveals that the optimal structure does not need too many segments for optimizing the bent structure of the waveguide, and as few as four segments are sufficient to improve the transmission efficiency satisfactorily.

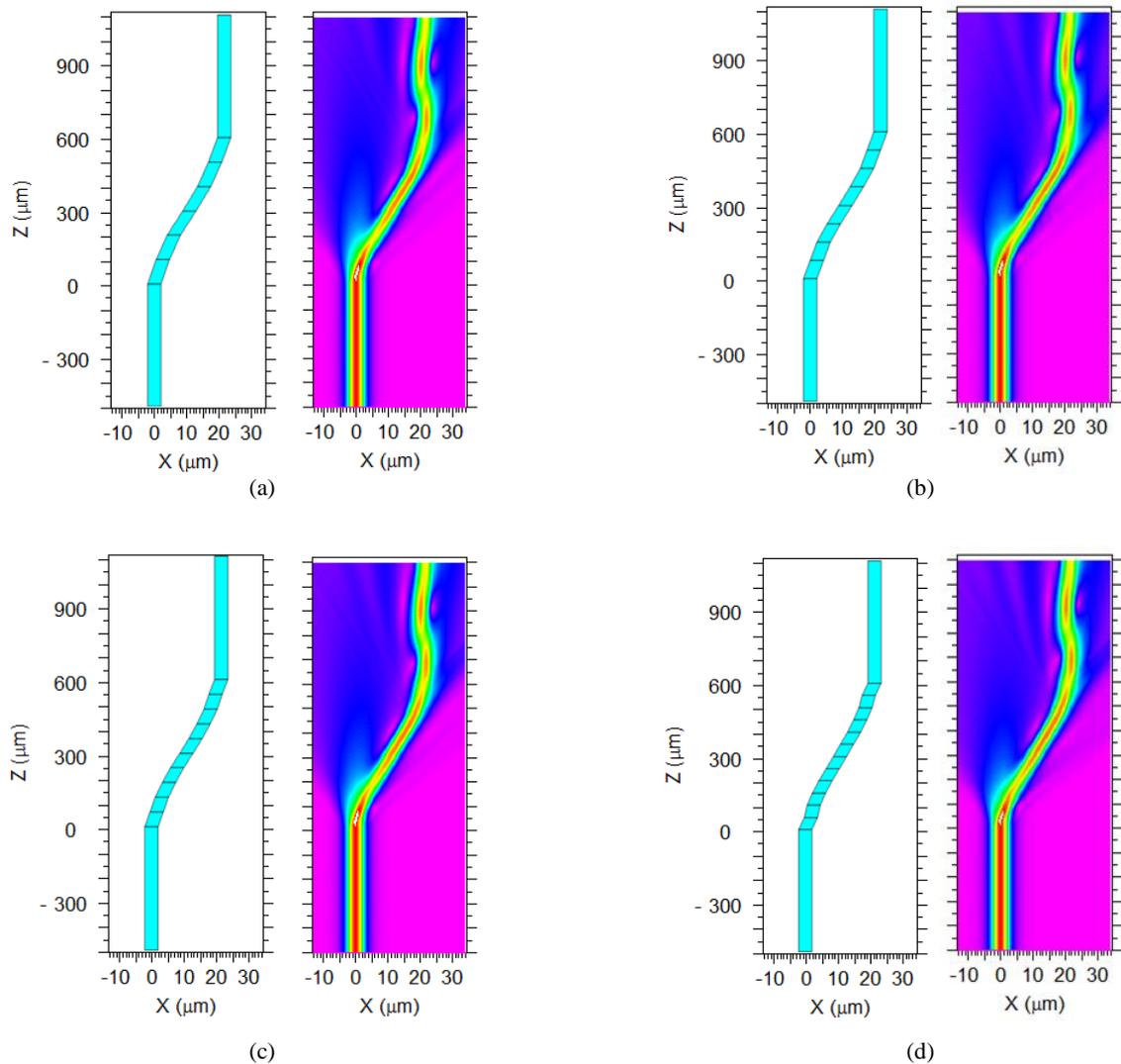


Fig. 6. Optimal multisegment bent waveguides and their respective BPM simulation results of the incident lightwave propagating along them in the case of a (a) six-segment structure, (b) eight-segment structure, (c) ten-segment structure, and (d) 12-segment structure.

By reducing the width of the waveguide such that the bent section of the optical waveguide appears as a line or a curve, it is easier to compare the optical paths of the aforementioned optimal multisegment bent waveguides and observe their differences. Fig. 7 presents a comparison between the optical paths of the conventional, optimal four-segment ( $m = 2$ ), optimal eight-segment ( $m = 4$ ), and optimal 12-segment ( $m = 6$ ) bends. The blue dot line represents the straight-but-oblique optical path of the conventional bent waveguide. The green solid line, red dashed line, and black dashed-dotted curve represent the optical paths of the four-, eight-, and 12-segment optimal bends, respectively. In this figure, each curve is depicted by connecting from  $C_0$ ,  $C_1$ ,  $C_2, \dots$ , to  $C_{2m}$  with a straight line segment for different cases of  $m = 2, 4$ , and  $6$ . Obviously, these optimal paths become similar to one another when the number of segments increases. This result is in agreement with the fact that the optimal transmission efficiency values are very near to each other for these optimal multisegment bent waveguides.

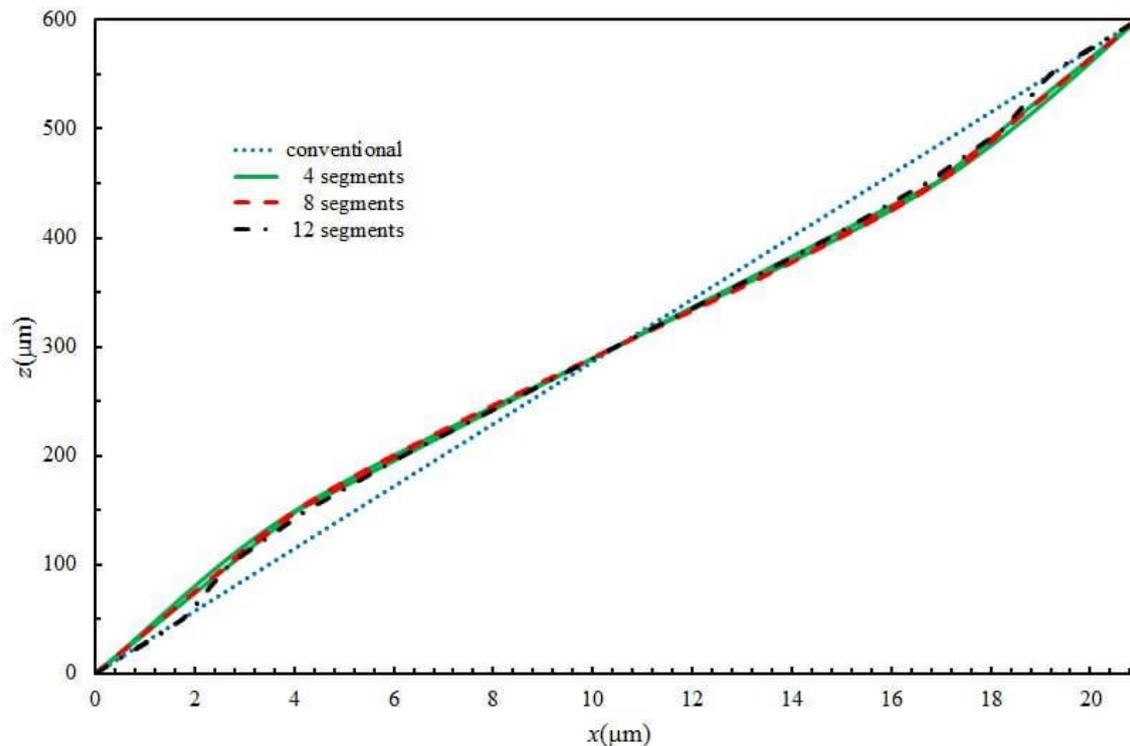


Fig. 7. Comparison between the optical paths of the conventional (blue dotted line), optimal four-segment (green curve), optimal eight-segment (red dash curve), and optimal 12-segment (black dash-dot curve) bent waveguide structures.

#### IV. CONCLUSIONS

A novel method was proposed to design optical bent waveguides by transforming the shape of each segment by using the proposed SOS algorithm and combining with the BPM. The proposed method is sufficient to satisfactorily optimize the transmission efficiency, although the straight-but-oblique section of the conventional bent waveguide was divided into as few as four segments. The transmission efficiency of the optimal multisegment bent waveguide increases up to more than 2.5 times of that of the conventional bent waveguide structure. Although the SOS algorithm does not guarantee the best solution, it can provide a sufficiently good solution. Moreover, the SOS algorithm is simple and easy to implement. In the future, this method may be used by the authors to design more complicated optical devices, such as Y branches and multimode interference waveguides.

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