

Improved Polynomial Rooting of Capon's Algorithm to Estimate the Direction-of-Arrival in Smart Array Antenna

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Abstract— Direction of arrival algorithms are used, in general, to estimate a number of incident plane waves on the antenna array and their angles of incidence. In this paper, firstly, a new approach representing a more extensive version of the conventional Capon method is proposed. The study employs the root-Capon approach based on cross-correlation matrix in extracting the estimated direction-of-arrival of some number of signals to a given uniform linear array (ULA) antenna system. Secondly, a simple modification on the antenna element's number is employed for the Capon and the proposed approach to obtain more performances. The effectiveness of the proposed algorithm is verified through numerical simulation examples, and it is shown that the root version of Capon method can provide more accurate angle estimation with less computational complexity than conventional Capon algorithm. Similarly, it is found also that the used modification plays a significant role in the performance's improvement of Capon and its variant approaches.

Index Terms— Direction of arrival estimation, array signal processing, spatial spectrum (pseudo-spectrum), Capon, root-Capon, improved-root-Capon.

I. INTRODUCTION

Smart Antenna system combines multiple antenna elements with a signal processing capability to optimize its radiation pattern automatically in response to the signal environment. Furthermore, in reality antennas are not smart, it is the digital signal processing along with the antennas that plays a key role to make the system smart. In this kind of antenna array, angle-of-arrival (AOA) estimation and beam-forming are the two main functions that can be carried out, thanks to integrated techniques on sophisticated signal processing. This means that the estimation of the signal direction is the most important signal processing step in smart antennas. In these last, direction of arrival (DOA) algorithms are used, in general, to estimate a number of incident plane waves on the antenna array and their angles of incidence; moreover, DOA estimation is considered as critical technology of array signal processing and it does not lie only to smart antennas but, it is also of great importance in a variety of applications, such as radar, speech signal processing, sonar, astronomy, electronic

surveillance, wireless communication, medical diagnosis and treatment, seismology and other areas [1]-[3].

Many algorithms for DOA estimation have been proposed, and their performances have been studied thoroughly over the years; besides, intensive studies on DOA estimation field were carried out to pinpoint the most suitable algorithm for the application of interest. The most widely used and most popular are: Bartlett method [4], Capon method [5], the maximum entropy method [6], Multiple Signal Classification (MUSIC) [7], the minimum norm technique [1], and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [8] etc. Generally, the DOA estimation techniques can be classified into two main categories: beam-scan approaches and subspace approaches [9]-[10]. The beam-scan techniques form a conventional beam, scans it over the appropriate region and plots the magnitude squared of output. While in subspace techniques which are also referred to as high-resolution algorithms, the orthogonality between the signal and noise subspaces is exploited [10].

Capon approach has been one among many well known algorithms for the angle of arrival (AOA) determination of incident signals using array antenna structure. Since its appearance, the standard Capon algorithm saw some improvements where several modifications were proposed and numerous researches were done in this topic. In [11], a cascaded Capon beamformer termed as m-Capon was proposed so as to avoid exact EVD and sources number estimation. New modified Capon estimator called MCB was proposed for estimating the direction-of-arrivals (DOAs) of multiple noncoherent narrowband signals [12]. George, in his work [13], has derived matched filter-bank-based spectral estimators which are the amplitude spectrum Capon (ASC) and the power spectrum Capon (PSC) estimators. These estimation methods have provided spectra which are characterized by a significantly improved resolution compared to classical approaches. Also the minimum variance distortionless response (MVDR) which is a data-dependent algorithm proposed by Capon was modified in [14] for reducing direction-of-arrival estimation mismatch. The paper objectives are to design and model a new technique in order to overcome the signal suppression phenomena with respect to any deflection in the signature vector and to increase the behaviour freedom degree by reducing the Vandermonde matrix effects. In [15], the authors have used the Levenberg-Marquardt (LM) algorithm to the optimization of the cost function of the Capon for azimuth/elevation angle-of-arrival (AOA) estimation. In the same context, there have been other studies on applying the Newton-type method to the optimization mentioned by Selva [16].

Our quintessential interest in this paper is firstly, to introduce the root version of the classical Capon method in order to reduce its computational cost by obtaining result in a numerical format. In fact, the new method is based on the direct calculation of the directions of arrival by researching the zeros of a polynomial instead of pseudo-spectrum plotting then finding its maximum. Secondly, to

propose a new modification related to the antenna parameters. This proposition leads to improve the Capon method performances in term of resolution, and to obtain more best and accurate numerical results by the root-Capon algorithm.

The remainder of this paper is organized as follows: the signal model is presented in section II. A summary of the conventional Capon algorithm, the proposed polynomial version method, and the modified number of elements are introduced in section III. Section IV provides the simulation results and finally, our conclusion is given in section V.

II. SIGNAL MODEL

The problem of central interest herein is that of the angles of arrival estimation which can be stated in the following way:

A uniform linear array (ULA) composed of N antenna elements receives transmitted signals from M far-field sources ($M < N$). The objective is then to isolate the M unknown sources by exploiting the signals which are received by the array antennas and the available information's in reception. The waves which are transmitted by the sources arrive on the array from various directions $(\theta_1, \dots, \theta_M)$. The received signals are, therefore, linear combinations of the incident signals and noise. By carrying out K observations based on the number of snapshots, the k^{th} received vector at the antennas array output $\mathbf{x}(k) \in \mathbb{C}^{N \times 1}$ can be modeled as [9],[11]:

$$\mathbf{x}(k) = \mathbf{A}(\theta) \cdot \mathbf{s}(k) + \mathbf{v}(k), \quad k = 1, \dots, K \quad (1)$$

Equation (1) can be written in a matrix form as follows:

$$\mathbf{x}(k) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_M)] \cdot \begin{bmatrix} s_1(k) \\ s_2(k) \\ s_3(k) \\ \vdots \\ s_M(k) \end{bmatrix} + \mathbf{v}(k) \quad (2)$$

Where $\mathbf{s}(k)$ is the $[M \times 1]$ complex transmit vector, $\mathbf{A}(\theta)$ is the $[N \times M]$ steering matrix of the sources, it contains information about the angles of arrival, and $\mathbf{a}(\theta_m) = [e^{-j\phi_{1,m}}, e^{-j\phi_{2,m}}, \dots, e^{-j\phi_{N,m}}]^T$ is the $[N \times 1]$ steering vector which is related to the m^{th} source with $\phi_{n,m} = \frac{2\pi d}{\lambda} (n-1) \sin \theta_m$ represents the geometrical phase shift introduced by the n^{th} array element to the m^{th} source according to the angle of incidence. $(.)^T$ denotes the transposition, d and λ are the inter-antenna distance and the wavelength, respectively. $\mathbf{v}(k)$ is the $[N \times 1]$ additive white Gaussian noise (AWGN) vector with zero-mean and variance σ^2 .

The $[N \times N]$ input covariance matrix can be expressed as:

$$\mathbf{R}_{xx} = E[\mathbf{x}(k)\mathbf{x}^H(k)] \quad (3)$$

Where $(.)^H$ indicates the Hermitian transpose.

The estimated spatial correlation matrix $\hat{\mathbf{R}}_{xx}$ for the K number of snapshots is given by:

$$\hat{\mathbf{R}}_{xx} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k)\mathbf{x}^H(k) \quad (4)$$

Since the covariance matrix \mathbf{R}_{xx} is estimated, several methods can be used for the direction of arrival estimation. These various methods were largely studied in the literature of smart antennas and spectral analysis.

III. DIRECTION OF ARRIVAL ALGORITHMS

In this section, it is worth to call back briefly the traditional Capon's algorithm, then explain in detail the proposed polynomial algorithm, and finally clear up the modification which can be applied to the two algorithms.

A. Capon algorithm

Introduced by J. Capon [5] and also called Minimum Variance Distortionless Response (MVDR) algorithm. This spectral-based method is one of the conventional popular techniques. The main idea of Capon's minimum variance algorithm is to minimize the received power of the incoming signal in all direction while maintaining a unity gain in look direction [17]-[18]. The constrained imposed on this algorithm is given as:

$$\min_{\omega} E[|\mathbf{y}(t)|^2] = \min_{\omega} \omega^H \mathbf{R}_{xx} \omega$$

Subject to

$$\omega^H \mathbf{a}(\theta_1) = 1 \quad (5)$$

Solving this constraint optimization problem by the use of Lagrange optimization technique for the weight vector [2], [19] we obtain the Capon's weight vector:

$$\omega_{\text{capon}} = \frac{\mathbf{R}_{xx}^{-1} \mathbf{a}(\theta)}{\mathbf{a}(\theta)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta)} \quad (6)$$

Thus, Capon's output spectrum is:

$$P_{\text{capon}}(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta)} \quad (7)$$

Now, one can use the previous expression to calculate and plot the output power spectrum over the

scanning angles and determine the direction of arrival by locating the peaks in the spectrum. The estimated values of direction of arrival of all signals can be obtained by:

$$\hat{\theta}_m = \arg \max_{\theta_m} P_{\text{capon}}(\theta_m), \quad m = 1, 2, \dots, M \quad (8)$$

B. Proposed Algorithm

The method proposed here is called root-Capon, as its name shown, is a polynomial-rooting version of the classical Capon algorithm. Furthermore, similar direction of arrival estimation method based on the spectral Capon rooting algorithm is developed in a slightly different way by [20]. This real polynomial rooting technique uses a form of derivation on the denominator of the Capon's spectrum expression and some manipulations, and then it carries out a conformal transformation which maps the unit circle onto the real line, to find the polynomial roots. Otherwise, our key idea is the same as that of root-MUSIC [21]. This means that, to write the denominator of the capon spectrum expression which is named the null spectrum as a polynomial and then find the roots of the polynomial.

Starting with the Capon's spectrum expression:

$$P_{\text{capon}}(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta)} \quad (9)$$

Defining $D(\theta) = \mathbf{a}(\theta)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta)$ as the null spectrum, then the denominator of equation (9) above can be rewritten as:

$$P_{\text{capon}}(\theta) = \frac{1}{D(\theta)} \quad (10)$$

In the case of a ULA, the n^{th} element of the array steering vector is given by:

$$\mathbf{a}_n(\theta) = e^{2j\pi n \left(\frac{d}{\lambda}\right) \sin \theta}, \quad n = 0, 1, \dots, N - 1 \quad (11)$$

That means, the null spectrum $D(\theta)$ can be rewritten as:

$$D(\theta) = \sum_{m=1}^N \sum_{n=1}^N e^{-2j\pi m \left(\frac{d}{\lambda}\right) \sin \theta} D_{mn} e^{+2j\pi n \left(\frac{d}{\lambda}\right) \sin \theta} \quad (12)$$

Letting $z = e^{-2j\pi \left(\frac{d}{\lambda}\right) \sin \theta}$, the denominator $D(\theta)$ can be became:

$$D(z) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} z^m D_{mn} z^{-n}$$

$$= \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} D_{mn} z^{m-n} \quad (13)$$

With D_{mn} denotes the m^{th} row and n^{th} column element of \mathbf{R}_{xx}^{-1}

Letting now $l = n - m$ to simplify the double summation of equation (13) to a single one. The range on l is set by the limits on n and m , i.e. $-(N - 1) \leq l \leq (N - 1)$. Then, the polynomial $D(z)$ can be written as:

$$D(z) = \sum_{l=-N+1}^{N-1} D_l z^l \quad (14)$$

Where D_l is the sum of the elements along the l^{th} diagonal such that:

$$D_l = \sum_{n-m=l} D_{mn} \quad (15)$$

Root-Capon solves the rooting problem of a polynomial rather than finding the spectral peaks in the Capon algorithm. Additionally, the roots of the polynomial $D(z)$ that lie closest to the unit circle becomes equivalent to the poles of the Capon's pseudo-spectrum. That is $P_{root-capon}(z)|_{z=e^{j\theta}} = P_{capon}(\theta)$ with $(\theta) = -2\pi \left(\frac{d}{\lambda}\right) \sin \theta$.

The polynomial of equation (14) is of order $2(N-1)$ and thus their roots can be written as follows:

$$\mathbf{z}_i = |z_i| e^{j \cdot arg(z_i)}, \quad i = 1, 2, \dots, 2(N - 1) \quad (16)$$

Where $arg(\mathbf{z}_i)$ is the phase angle of \mathbf{z}_i .

The roots lying close to the unit circle have magnitudes $|z_i| \simeq 1$. One can calculate and find the DOA basing on the relationship between z and θ , $(e^{j \cdot arg(z_i)}$ and $e^{-2j\pi \left(\frac{d}{\lambda}\right) \sin \theta_i}$), which gives:

$$\theta_i = -\sin^{-1} \left(\frac{\lambda}{2\pi d} \cdot arg(\mathbf{z}_i) \right) \quad (17)$$

C. Modified Number of Array Elements

The idea to modify the number of array elements is extracted from the modification proposed by Liao and Abouzaid [22] for improving the resolution of MUSIC and root-MUSIC Algorithms. In fact, the authors have used and added a three principal modified factors which are related generally to signal and antenna parameters. However, in our work, we are satisfied with the proposition of only one modification which depends on antenna parameters and precisely concerns the number of array elements. Basing almost on the same idea, we have also recently proposed a work [23] to improve the

resolution performance of Min-Norm and Root-Min-Norm algorithms.

In order to make the peaks of the Capon spectrum sharper or to reach less error in root-Capon algorithm, one of the best solutions is to increase the number of antennas. This is realizable, not only by increasing the array size but also, by the modification of the real array element's number by multiplying it with a multiplicative factor (MF). The new number of array elements N' can then be written as:

$$N' = N \times MF \quad (18)$$

The new elements number, referred by equation (18), can be easily substituted in the improved Capon and root-Capon approaches. This new number of antenna array elements is of outstanding importance during the signal processing stage of the array system. Indeed, the multiplicative factor gives a number of elements offering more signal component for the followed algorithm. In addition, the selection of a suitable multiplicative factor easily leads to determine the actual and appropriate number of elements that can be required for the design of the antenna array.

IV. SIMULATION RESULTS

In this section, we work on the improvement of the performances of the capon and its rooting version where we make a simple modification by using a multiplicative factor related to the number of array elements. The direction of arrival performances of the studied methods can be demonstrated and evaluated by using a computer simulation programs that we have developed on Matlab software.

In the initial simulations, we consider an eight-element uniform linear array. Three far-field signals ($M=3$) impinge initially on the antenna array from directions of -5° , 10° , and 25° , respectively. Assume that the ratio between the wavelength λ and the inter-element spacing d is 2. The noise is characterized as additive, white and Gaussian distributed with zero mean and variance $\sigma^2 = 0.1$. The number of snapshot is $K = 200$.

It should be to note also that some previous declared parameters will be changed in order to study other simulation scenarios, but the following assumptions are usually made:

- The antennas noise is uncorrelated with the signals.
- The number of sources is known and is less than the number of antennas.
- There is no correlation between sources.

The graphical result of Fig.1 represents the normalized spectrum versus the full angular range, from -90° to $+90^\circ$, of the Capon approach that contains main lobes identifying the angles of arrival. The positions of the lobes determine the estimated values of the arrival angles of Capon which is based, for its part, on the spectral analysis. However, the root-Capon approach does not provide graphs but it gives results as numerical values. The roots of this approach are marked on Fig.1 by black stars and its estimated angles of arrival are -5.1141° , 9.9798° , and 25.0254° .

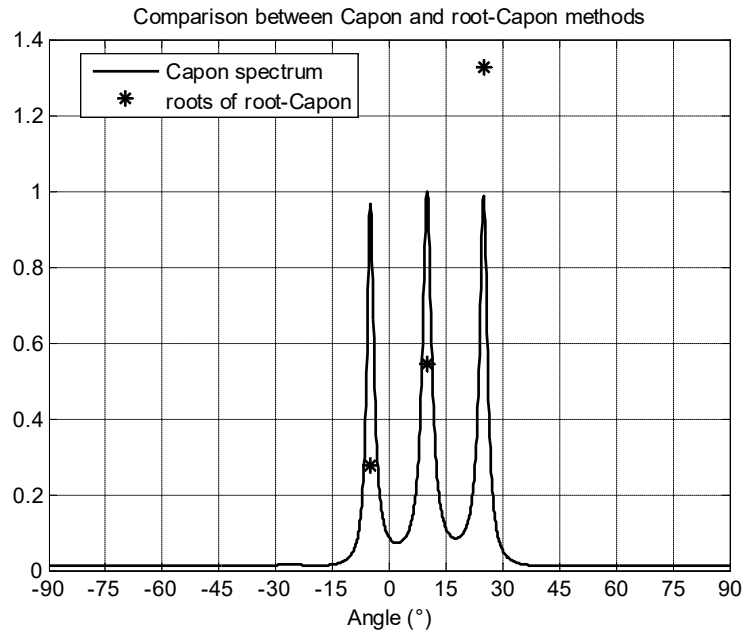


Fig. 1. Normalized Capon spectrum and roots found with root-Capon.

Additionally, the roots of the root-Capon polynomial are displayed in the complex plane, together with the unit circle, that allows us to visualize the effectiveness of this method in estimating the angles of arrival (Fig. 2). The angle of arrival of the sources is obtained from the phase of the roots whose magnitude is closed to 1. There are 3 roots having magnitude very close to 1 and corresponding to the 3 simulated sources.

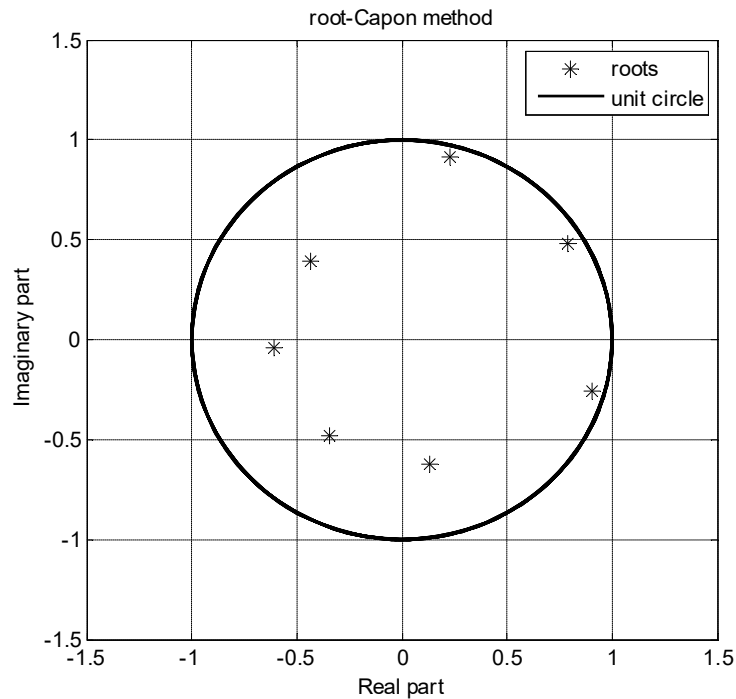


Fig. 2. Angle of arrival estimation with root-Capon method.

It is clearly shown from Fig. 3 that the two used spectral methods are able to find the directions of the three sources corresponding to -5° , 0° , and 25° , but with different estimation precisions that depend on the width of the peaks. The performances in term of angle of arrival estimation of the proposed modified-Capon algorithm are gradually improved with a multiplicative factor $MF=3$. This last realizes exactly an increase in the number of antennas. Thus, multiplying the number of antennas leads to widen the array, and this leads to improve the angle of arrival estimation performance because multiple antennas provide the diversity gain.

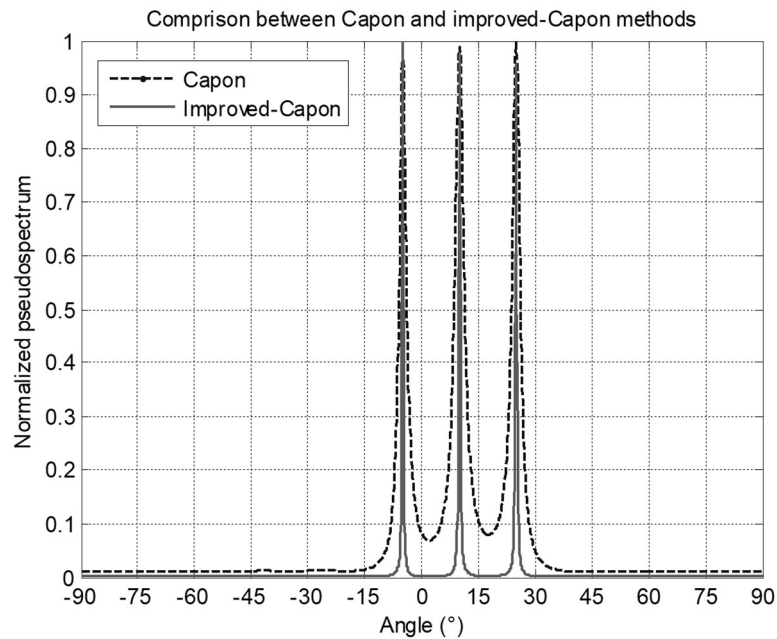


Fig. 3. Comparison between Capon and improved-Capon pseudo-spectrums.

Fig. 4 presents a comparison between the results obtained by the different studied methods of this paper. In this example, we have assumed three signals arriving from three sources, two of which are closely separated. It is clear that the Capon spectral-based method, as used here for this case, fail to locate the position of two closed sources. In fact, the graph of Capon illustrates only two lobes one of which is wide and covers two closely spaced sources which are located at -5° and 0° . But it is possible to improve the situation by using a correspondent modified version of Capon. The graph of this version indicates that all directions of sources are correctly estimated, and there are observed maxima on the curve at the positions of the three angles. In other words, the spectrum determines three peaks corresponding to the angles that we are looking for (-5° , 0° , and 25°). The peaks are very narrow and more selective, so a very precise localization that leads to notice the high resolution of the technique which can differentiate even very close sources.

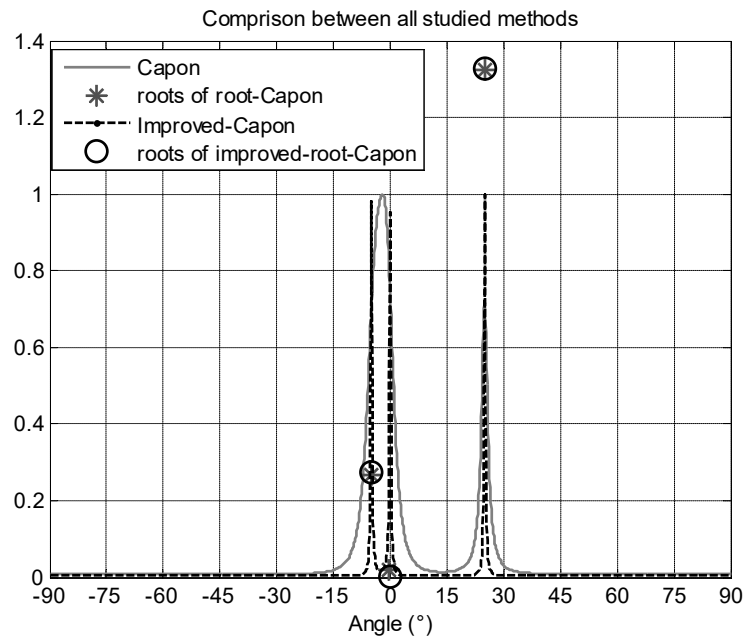


Fig. 4. Comparison between the studied algorithms when the DOAs are -5° , 0° , and 25° .

The polynomial versions offer good performances and smaller computational complexity than the pseudo-spectrum versions. The two polynomial algorithms are able to distinguish between two close signals while the classical Capon method failed to differentiate between them. In addition, the estimation error decreases if the separation between signals increases.

The complexity of the required operations in the scanning angle process stage to construct the spatial spectrum of the standard Capon method is $M(N^2 + N)$ for the number of multiplications and $M(N^2 + 1)$ for the number of additions, where M is the number of angles to scan and N is the number of antenna elements. The computational complexity of the improved-Capon slightly exceeds that of the standard Capon, and precisely in the multiplication operations which are $M(N^2 + N + 1)$. The two rooting polynomial methods, that are developed in our work, are proposed to reduce the computation complexity of their original algorithm versions. Then, the Improved-root-Capon method can be evaluated as efficient computationally.

In order to compare the effectiveness and estimation performance in terms of mean square error (MSE) of the proposed polynomial method “root-Capon” with its enhanced version “Improved-root-Capon”, the application of the two methods in different simulation scenarios allows us to present the results mentioned in tables (I, II and III).

In the simulation scenario of Table I, we assume the number of snapshots is fixed to 200 for three different signals arriving on ULA from different directions. This simulation compares the variations in performance of our two AOA rooting methods with a variation of signal-to-noise ratio (SNR). The MSE is calculated for each SNR and enrolled for our algorithms in Table I. The performance

estimation of each AOA method can be clearly seen in these results: as the value of SNR improves the MSE of the two presented methods decreases. However, the improved-root-Capon technique gives still the best estimation compared with the root-Capon even for low SNR regime.

TABLE I: PERFORMANCE OF THE ROOTING METHODS FOR VARYING ARRAY SIGNAL-TO-NOISE-RATIO.

SNR (dB)	Number of array elements=8, Number of snapshots=200, Sources are at (-5°,10°,25°)					
	Root-Capon			Improved-Root-Capon (MF=3)		
	-5°	10°	25°	-5°	10°	25°
0	0.6982°	0.3820°	0.2228°	0.0170°	0.0139°	0.0244°
5	0.2023°	0.1683°	0.1432°	0.0086°	0.0101°	0.0161°
10	0.1141°	0.0202°	0.0254°	0.0055°	0.0088°	0.0121°
20	0.0339°	0.0168°	0.0194°	0.0019°	0.0015°	0.0023°
30	0.0046°	0.0007°	0.0081°	0.0003°	0.0009°	0.0002°
50	0.0002°	0.0008°	0.0006°	0.0001°	0.0001°	0.0000°

We study the effect of changing the number of snapshots on the performance of the two rooting methods. In Table II, we simulate an ULA with 8 antenna elements and SNR = 10 dB for three signals arriving at (-5°, 10°, 25°). As expected, as the number of snapshots increases, the MSE of the both polynomial methods improves. Except the case of snapshots number less than the number of sources where the two methods fail to correctly estimate the directions of arrival. It also can be inferred that whatever the number of snapshots, the error of the improved-root-Capon is always poor as compared to that of root-Capon.

TABLE II. PERFORMANCE OF THE ROOTING METHODS FOR VARYING NUMBER OF SNAPSHOTS.

Snapshots	Number of array elements=8, SNR=10dB, Sources are at (-5°,10°,25°)					
	Root-Capon			Improved-Root-Capon (MF=3)		
	-5°	10°	25°	-5°	10°	25°
1	2.2179°	3.8377°	19.7489°	4.8044°	10.0810°	4.2898°
10	0.3125°	0.5474°	0.4183°	0.0677°	0.1662°	0.1349°
50	0.0561°	0.1633°	0.1463°	0.0596°	0.0931°	0.0227°
100	0.0610°	0.1360°	0.0434°	0.0297°	0.0100°	0.0108°
300	0.0327°	0.0395°	0.0385°	0.0148°	0.0052°	0.0011°
700	0.0095°	0.0102°	0.0159°	0.0010°	0.0002°	0.0007°

Table III summarizes the AOA estimation MSEs of our two rooting methods tested for different numbers of antenna elements. The SNR is assumed to be 10 dB and the snapshots number is 200. From this table, it can be observed that in all cases, the improved-root-Capon approach consistently outperforms the other proposed approach. The performance improvement of the both tested rooting methods is achieved at large number of antenna elements. In addition, the fact that improved-root-Capon outperforms the root-Capon can be explained by the true antennas number of improved-root-Capon which is greater than that of root-Capon.

TABLE III. PERFORMANCE OF THE ROOTING METHODS FOR VARYING NUMBER OF ARRAY ELEMENTS.

Antenna elements number	SNR=10dB, Number of snapshots=200, Sources are at (-5°,10°,25°)					
	Root-Capon			Improved-Root-Capon (MF=3)		
	-5°	10°	25°	-5°	10°	25°
4	0.0598°	0.5233°	0.1076°	0.0311°	0.0256°	0.0988°
6	0.0491°	0.1824°	0.3767°	0.0192°	0.0123°	0.0255°
10	0.0391°	0.0720°	0.0465°	0.0074°	0.0030°	0.0162°
12	0.0131°	0.0156°	0.0088°	0.0047°	0.0025°	0.0060°
14	0.0083°	0.0094°	0.0081°	0.0051°	0.0016°	0.0042°
16	0.0017°	0.0014°	0.0038°	0.0009°	0.0008°	0.0019°

While the multiplicative factor is an important term in the modification related to the method proposed in this work, and because the improved-root-Capon algorithm is chosen to be considered the best among the techniques studied here, it was worth following the influence of the variation of estimation error versus multiplicative factor value in case of a single source. The simulation results are displayed in Fig. 5. It can say, in brief, that the increase of the multiplicative factor value decreases considerably the error between theoretical and estimated angle values.

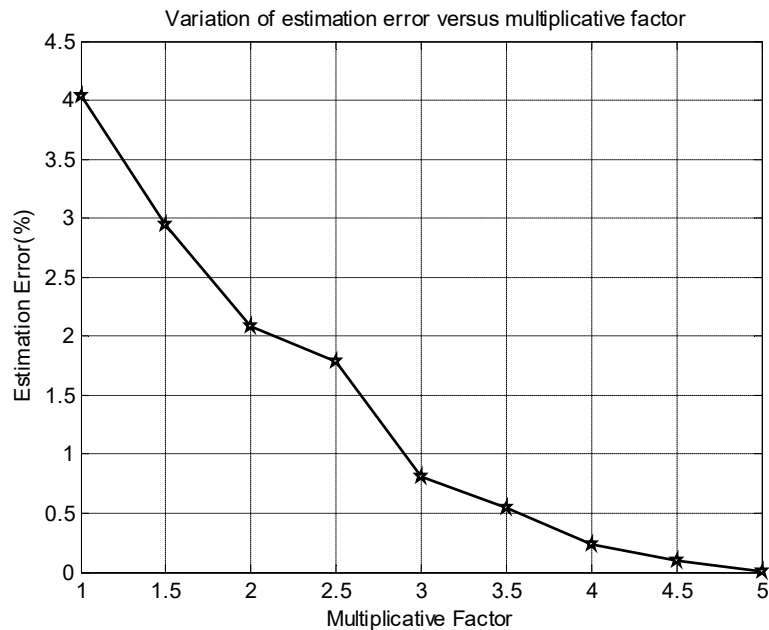


Fig. 5. Variation of estimation error versus multiplicative factor (MF) of the improved-root-Capon method.

By using a modified number of elements, the size of the antenna array remains the same, but the antenna processing stage, based on an arrival direction estimation algorithm, considers that the size of the antenna array is greater than the actual one. In other words, the advantage of the proposed algorithm is, on one hand, to increase virtually the number of antenna elements in order to achieve a significant improvement in terms of the algorithmic estimation resolution, and on another hand, to keep the same real number of the elements practically used in order to avoid other problems such as congestion or cost in the antenna system design that must meet compromises in terms of performance and implementation complexity.



V. CONCLUSION

The proposed variant of Capon can give more estimation accuracy than the conventional Capon method. This last which is a spectral-based method involves plotting the pseudo-spectrum against the angles and searching for the peaks, while root-Capon involves finding the roots of a polynomial which are numerical values.

The choice of the multiplicative factor value should be reasonable and suitable because, a high number of array elements can give, on one hand, true good resolution for Capon algorithm and reduced estimation error for root-Capon. However, on the other hand, this can lead to increase the processing time.

In concluding, we think it is safer say that our modified-algorithm, has clear performance improvements to offer as compared, in this work, to the other studied methods. In fact, we proposed the improved root-Capon as one of performing algorithms to reduce potentially the complexity of the standard Capon algorithm, increase its performance and its capability of resolution.

APPENDIX. PROOF OF IMPROVED CAPON AND IMPROVED ROOTING OF CAPON ALGORITHMS

For these two algorithms, we basically use the new array elements number N' which is derived from the modification mentioned in the subsection C of this paper.

The new $[N' \times 1]$ complex envelope representation of the received signals can be expressed as:

$$\mathbf{x}'(k) = \mathbf{A}'(\theta) \cdot \mathbf{s}(k) + \mathbf{v}'(k), \quad k = 1, \dots, K \quad (\text{A1})$$

Where $\mathbf{A}'(\theta)$ is the $[N' \times K]$ steering matrix, $\mathbf{v}'(k)$ is the $[N' \times 1]$ vector of the noise, and K denotes the number of snapshots.

The estimated covariance matrix $\widehat{\mathbf{R}}'_{xx}$ for the new vector $\mathbf{x}'(k)$ and the K snapshots number can be written as:

$$\widehat{\mathbf{R}}'_{xx} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}'(k) \mathbf{x}'^H(k) \quad (\text{A2})$$

Now, with the substitution of the new estimated covariance matrix $\widehat{\mathbf{R}}'_{xx}$ and the new steering matrix $\mathbf{A}'(\theta)$, we can easily obtain the following improved-Capon spectrum formula:

$$P_{im-capon}(\theta) = \frac{1}{\mathbf{a}'(\theta)^H \mathbf{R}'_{xx}^{-1} \mathbf{a}'(\theta)} \quad (\text{A3})$$

We can use the above expression to plot the improved-Capon pseudo-spectrum and then find the estimated AOA by searching the positions of the spectrum peaks.

For the improved polynomial rooting of Capon the calculation of roots also depends on the new values of $\hat{\mathbf{R}}'_{xx}$ and $\mathbf{A}'(\theta)$.

Quite simply, we firstly extract from equation (A3) the following polynomial rooting formula:

$$D(\theta) = \mathbf{a}'(\theta)^H \mathbf{R}'_{xx}^{-1} \mathbf{a}'(\theta) \quad (\text{A4})$$

Then, we follow exactly the same steps employed for the mathematical development of the root-Capon method as enounced through the subsection B of the present research, to exploit a new technique for AOA estimation.

Finally, the estimation of the AOAs can be deduced from an equation of this from:

$$\theta_i = -\sin^{-1} \left(\frac{\lambda}{2\pi d} \cdot \arg(\mathbf{z}_i) \right) \quad (\text{A5})$$

Where \mathbf{z}_i are the i closest roots to the unit circle.

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